1. (16%) Write the truth table of the following Boolean functions and express each function in sum-of-minterms and product-of-maxterms.

   (a) (x + y'z')(w + xy')

   (b) w'x'y + wyz + wx'z' + x'yz

(a)

w	x	y	z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

SOM:

$F = \Sigma(4, 5, 8, 12, 13, 14, 15)$

$F = w'xy'z' + w'xy'z + wx'y'z' + wxy'z' + wxy'z + wxyz' + wxyz$ #

POM:

$F = \Pi(0, 1, 2, 3, 6, 7, 9, 10, 11)$

$F = (w+x+y+z)(w+x+y+z')(w+x+y'+z)(w+x+y'+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x+y+z')(w'+x+y'+z)(w'+x+y'+z')$ #

(b)

w	x	y	z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

SOM:

$F = \Sigma(2, 3, 8, 10, 11, 15)$

$= w'x'yz' + w'x'yz + wx'y'z' + wx'yz' + wx'yz + wxyz$ #

POM:

$F = \Pi(0, 1, 4, 5, 6, 7, 9, 12, 13, 14)$

$= (w+x+y+z)(w+x+y+z')(w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x+y+z')(w'+x'+y+z)(w'+x'+y+z')(w'+x'+y'+z)$ #

2. (12%) What are the literal cost and gate input cost of the following Boolean function?

   (a) (x + y'z')(w + xy')

   (b) w'x'y + (z' + x'y')

Ans:

(a) literal cost = 6

gate input cost = 10

L (Literal Cost) = 6

G (Gate Input Cost) = 6 + 4 = 10

GN (Gate Input Cost with NOTs) = 10 + 2 = 12

(b) literal cost = 6

gate input cost = 9

L (Literal Cost) = 6

G (Gate Input Cost) = 6 + 3 = 9

GN (Gate Input Cost with NOTs) = 9 + 4 = 13

3. Consider the following truth table.

x	y	z	F(x,y,z)	G(x,y,z)
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

(a) (14%) Write their corresponding Boolean expression F and G in sum-of-minterms and product-of-maxterms.

(b) (10%) Draw the logic diagram using only NAND and NOT gates.

(a) SOM:

$$F = \Sigma_m(0,2,3,4,7)$$
$$= \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + xyz$$

$$G = \Sigma_m(1,3,4,6)$$
$$= \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + xy\bar{z}$$

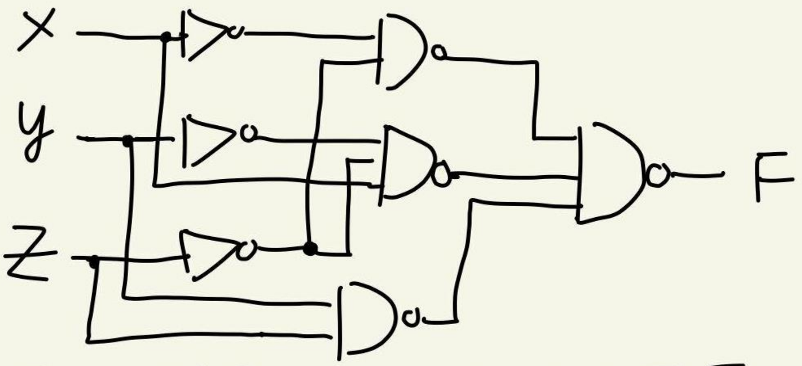
POM:

$$F = \Pi M(1,5,6)$$
$$= (x+y+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)$$

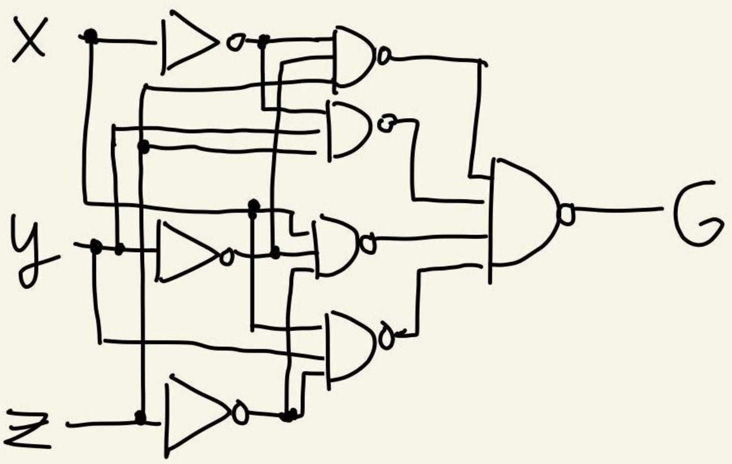
$$G = \Pi M(0,2,5,7)$$
$$= (x+y+z)(x+\bar{y}+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})$$

(b)

$$F = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + xyz$$
$$= \bar{x}\bar{z} + yz + x\bar{y}\bar{z}$$
$$= ((x'z')'(yz)'(xy'z')')'$$



$$G = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + xy\bar{z}$$
$$= ((x'y'z)'(x'yz)'(xy'z')'(xyz')')'$$



4. (12%) F = (x + y + z')(x + z)(y' + z). Find the complement of the function F, and find FF' and F + F'.

①

$$F' = ((x+y+z')(x+z)(y'+z))'$$
$$= (x+y+z')' + (x+z)' + (y'+z)'$$
$$= x'y'z + x'z' + yz'$$

#

② $FF' = 0$

③ $F + F' = 1$

5. (12%) Use DeMorgan's theorem to remove the complement outside the braces.

(a) ((x + w')y' + w'z + (yz)'(x+y+z))'

(b) (x'y + y(x+z))'

(a) ((x + w')y' + w'z + (yz)'(x+y+z))'

=((x+w')y')' (w'z)' ((yz)'(x+y+z))' = ((x+w')'+y) (w+z') (yz+(x+y+z)')

= (x'w+y)(w+z')(yz+x'y'z')

(b) (x'y + y(x+z))'

=(x'y)' (y(x+z))' = (x+y')(y'+(x+z)') = (x+y')(y'+x'z')

6. (12%) Convert F to the other normal form and standard forms of sum-of-products and product-of-sums. $F(x, y, z) = \Sigma(2, 3, 5, 7)$

$$F(x,y,z) = \Sigma(2,3,5,7)$$
$$= x'yz' + x'yz + xy'z + xyz$$
$$= x'y(z'+z) + xz(y'+y)$$
$$= x'y + xz \text{ ※ (SOP)}$$
$$F(x,y,z) = \Sigma(2,3,5,7) = \Pi(0,1,4,6)$$
$$= (x+y+z)(x+y+z')(x'+y+z)(x'+y'+z)$$
$$= (x+y)(x'+z) \text{ ※ (POS)}$$

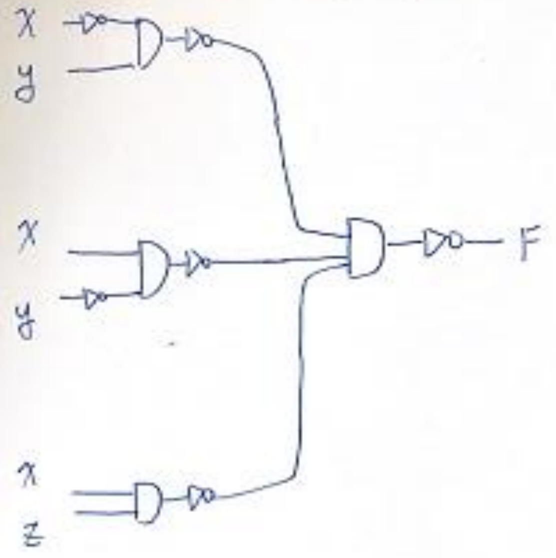
7. Implement the function F. F(x, y, z) = x'y + xy' + xz.

(a) (6%) Use AND and NOT gates only.

(b) (6%) Use NOR and NOT gates only.

(a) AND NOT

$$F = \overline{\overline{\bar{x}y} \cdot \overline{x\bar{y}} \cdot \overline{xz}}$$



(b) NOR NOT

$$F = \overline{(\overline{x+y}) + (\overline{x+y+z})}$$