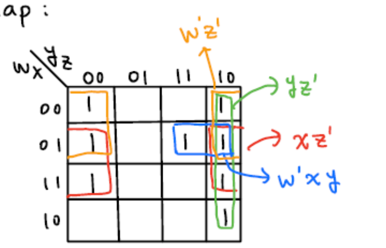
1. (20%) Simplify the following Boolean functions or expression using map method:
   (a) $F(w,x,y,z) = \Pi(1,3,5,8,9,11,13,15)$
   (b) G(a,b,c) = ac′+a′b′c+bc

(林彥岑)

1. (a) $F(w,x,y,z) = \Pi(1,3,5,8,9,11,13,15)$

$= \Sigma(0,2,4,6,7,10,12,14)$

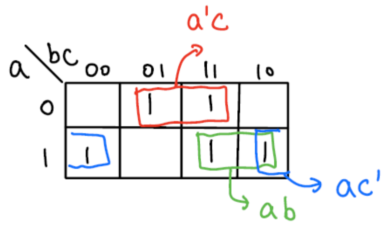
k-map:



$\therefore F = w'z' + yz' + xz' + w'xy$ #

(b)

$G = ac' + a'b'c + bc$

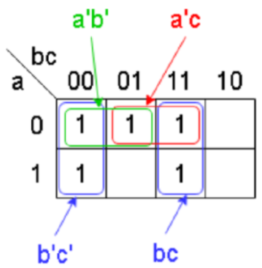


Simplified $G = a'c + ac' + ab$ #

2. (20%) Simplify the following Boolean functions by first finding the essential prime implicants (Please indicate the essential prime implicants and prime implicants):
   (a) $F(a,b,c) = \sum(0,1,3,4,7)$
   (b) F(w,x,y,z) = (x+y′z′)(w+xy′),

(楊博舜)

(a) $F(a,b,c) = \sum(0,1,3,4,7)$



EPI:
- For $ab'c'$, the only PI is $b'c'$, so $b'c'$ is an EPI.
- For abc, the only PI is bc, so bc is an EPI.

PI:
- $b'c'$
- bc
- $a'c$
- $a'b'$

$\Rightarrow F(a,b,c) = b'c' + bc + a'c$

(b) $F(w,x,y,z) = (x + y'z')(w + xy')$

$= wx + xy' + wy'z' + xy'z'$

(呂易縉)

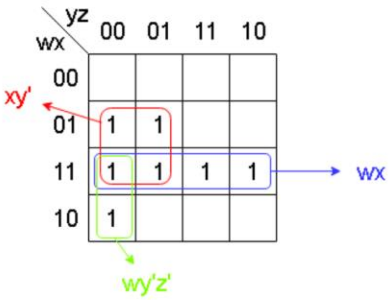
$wx \rightarrow 1100, 1101, 1110, 1111$

$xy' \rightarrow 0100, 0101, 1100, 1101$

$wy'z' \rightarrow 1000, 1100$

$xy'z' \rightarrow 0100, 1100$

Fill the K-map:



EPI:
- For $wxyz$, the only PI is wx, so wx is an EPI.
- For $w'xy'z$, the only PI is xy', so xy' is an EPI.
- For $wx'y'z'$, the only PI is $wy'z'$, so $wy'z'$ is an EPI.

PI:
- wx
- xy'
- $wy'z'$

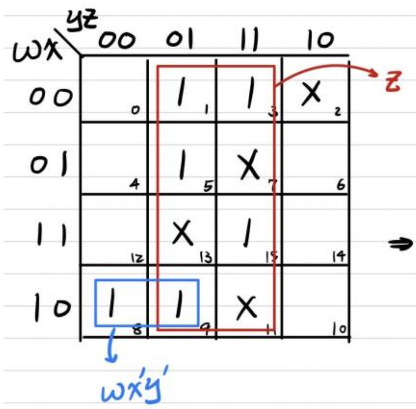
$\Rightarrow F(w,x,y,z) = wx + xy' + wy'z'$

3. (20%) Simplify the following Boolean function F, together with the don't-care conditions d, and then express the simplified function in sum of products:

$$F(w,x,y,z) = \Pi(0,2,4,6,7,10,11,12.13,14), \mathrm{d} = \sum(2,7,11,13)$$

$F(w,x,y,z) = \Pi(0,2,4,6,7,10,11,12,13,14)$

$\Rightarrow \Sigma(1,3,5,8,9,15)$



$\Rightarrow F(w,x,y,z) = wx'y' + z.$

4. (10%) Simplify the following expression, and implement it with two-level NAND gates:

$$F(A,B,C) = \Pi(2,5,6)$$

(林致佑)

4.

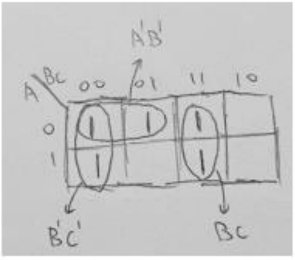
$F(A,B,C) = \prod(2,5,6)$

$= \sum(0,1,3,4,7)$

$= A'B' + B'C' + BC$

$= ((A'B' + B'C' + BC)')'$

$= ((A'B')'(B'C')'(BC)')'$

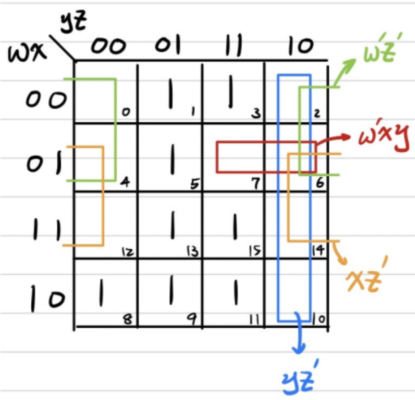


A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

5. (10%) Simplify the following expression, and implement it with two-level NOR gates:

$$F(w,x,y,z) = \sum(1,3,5,8,9,11,13,15)$$

(徐浩庭)



$F' = w'xy + w'z' + xz' + yz'$

$F = (F')'$ OR-AND

$= (w'xy + w'z' + xz' + yz')'$

$= (w + x' + y')\cdot(w+z)\cdot(x'+z)\cdot(y'+z)$

$\therefore F = ((w+x'+y')')'\cdot((w+z)')'\cdot((x'+z)')'\cdot((y'+z)')'$

$= ((w+x'+y')' + (w+z)' + (x'+z)' + (y'+z)')'$

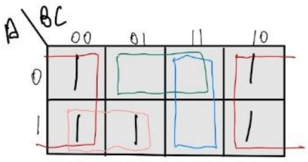
NOR - NOR

6. (20%) Simplify the following Boolean function F, using the two-level forms (a) AND-OR-Inverter, (b) OR-AND-Inverter logic diagrams

$$F(A,B,C) = \sum(0,2,4,5,6)$$

(賴聖耘)

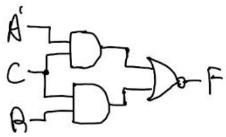
6. $F(A,B,C) = \sum(0,2,4,5,6)$



① AOI

$F' = A'C + BC$

$F = (F')' = (A'C + BC)'$



② OAI

$F' = (C' + AB')'$

$= (C')'(AB')'$

$= C(A'+B)$

$F = (F')' = (C(A'+B))'$