

1.

	1's complement	2's complement
10100111	01011000	01011001
10101011	01010100	01010101
01110011	10001100	10001101
11100101	00011010	00011011
01010101	10101010	10101011

2.

(a)

$$0101 + 1110 = 0011 \Rightarrow 3_{(10)}$$

-8 < 3 < 7, no overflow

(b)

$$0111110 + 1101011 = 0101001 \Rightarrow 41_{(10)}$$

-64 < 41 < 63, no overflow

3.

(a)

$$0101 - 0110 \Rightarrow 0101 + 1010 = 1111 \Rightarrow -1_{(10)}$$

(b)

$$10110 - 1100 = 10110 - 01100 \Rightarrow 22 + (-12), \text{轉 2's complement} \\ = 10110 + 10100 = 01010 \Rightarrow 10_{(10)}$$

(c)

$$1011110 - 1111110 \Rightarrow 94 + (-126), \text{轉 2's complement} \\ = 1011110 + 0000010 = 1100000 \Rightarrow -32_{(10)}$$

(d)

$$101010 - 101 = 101010 - 000101 \Rightarrow 42 + (-5), \text{轉 2's complement} \\ = 101010 + 111011 = 100101 \Rightarrow 37_{(10)}$$

4.

(a)

$$0101 - 0110 \Rightarrow 0101 + 1010 = 1111 \Rightarrow -1_{(10)},$$

no overflow(-8 < -1 < 7)

(b)

$$10110 - 1100 = 10110 - 11100 \Rightarrow -10 - (-4), \text{轉 2's complement} \\ \Rightarrow 10110 + 00100 = 11010 \Rightarrow -6_{(10)},$$

no overflow(-16 < -6 < 15)

(c)

$1011110 - 1111110 \Rightarrow -30 - (+2)$ , 轉 2's complement

$\Rightarrow 1011110 + 0000010 = 1100000 \Rightarrow -32_{(10)}$ ,

no overflow( $-64 < -32 < 63$ )

(d)

$101010 - 101 = 101010 - 111101 \Rightarrow -22 - (-3)$ , 轉 2's complement

$\Rightarrow 101010 + 000011 = 101101 \Rightarrow -19_{(10)}$ ,

no overflow( $-32 < -19 < 31$ )

5.

(a)

$0101 - 0110 \Rightarrow 0101 + 1010 = 1111 \Rightarrow -1_{(10)}$ ,

no overflow( $-8 < -1 < 7$ )

(b)

$10110 - 1100 \Rightarrow -6 - (-4)$ , 轉 2's complement

$\Rightarrow 1010 + 0100 = 1110 = -2_{(10)}$ ,

no overflow( $-8 < -2 < 7$ )

(c)

$1011110 - 1111110 \Rightarrow -30 - (-62)$ , 轉 2's complement

$\Rightarrow 1100010 + 0111110 = 0100000 = 32_{(10)}$ ,

no overflow( $-64 < 32 < 63$ )

(d)

$101010 - 101 \Rightarrow -10 - (-1)$ , 轉 2's complement

$\Rightarrow 10110 + 00001 = 10111 \Rightarrow -9_{(10)}$ ,

no overflow( $-16 < -9 < 15$ )

6.

$(+27) - (50) \Rightarrow 0011011 - 0110010 \Rightarrow 0011011 + 1001110 = 1101001 \Rightarrow -23_{(10)}$

$(-42) - (30) \Rightarrow 11010110 - 00011110 \Rightarrow 11010110 + 11100010 = 10111000 \Rightarrow -72_{(10)}$

7.

Sol 1

先畫出真值表

z <sub>3</sub>	z <sub>2</sub>	z <sub>1</sub>	z <sub>0</sub>	Z <sub>3</sub>	Z <sub>2</sub>	Z <sub>1</sub>	Z <sub>0</sub>
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	1
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	1
1	0	0	0	x	x	x	x
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

由真值表畫出 K-map，並求出 logic function。

Z<sub>3</sub> = 0

Z<sub>2</sub> = (z<sub>3</sub>'z<sub>2</sub> + z<sub>3</sub>z<sub>2</sub>' + z<sub>2</sub>z<sub>1</sub>'z<sub>0</sub>') or (z<sub>3</sub>'z<sub>2</sub> + z<sub>3</sub>z<sub>2</sub>' + z<sub>3</sub>z<sub>1</sub>'z<sub>0</sub>')

z <sub>1</sub> z <sub>0</sub> \z <sub>3</sub> z <sub>2</sub>	00	01	11	10
00		1	1	x
01		1		1
11		1		1
10		1		1

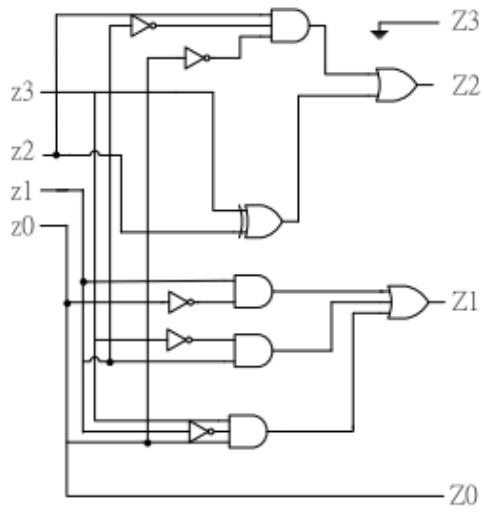
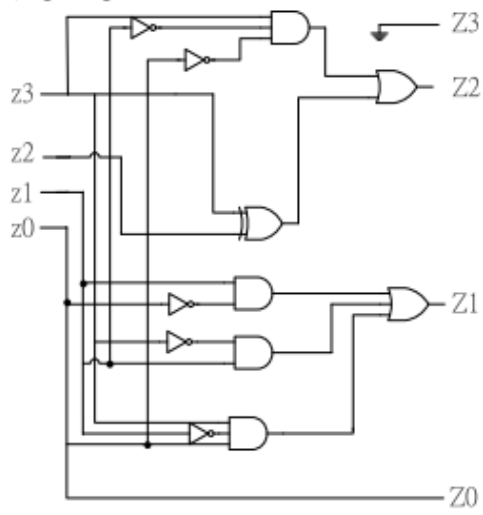
Z<sub>1</sub> = z<sub>1</sub>z<sub>0</sub>' + z<sub>3</sub>'z<sub>1</sub> + z<sub>3</sub>z<sub>1</sub>'z<sub>0</sub>

z <sub>1</sub> z <sub>0</sub> \z <sub>3</sub> z <sub>2</sub>	00	01	11	10
00				x
01			1	1
11	1	1		
10	1	1	1	1

Z<sub>0</sub> = z<sub>0</sub>

z <sub>1</sub> z <sub>0</sub> \z <sub>3</sub> z <sub>2</sub>	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

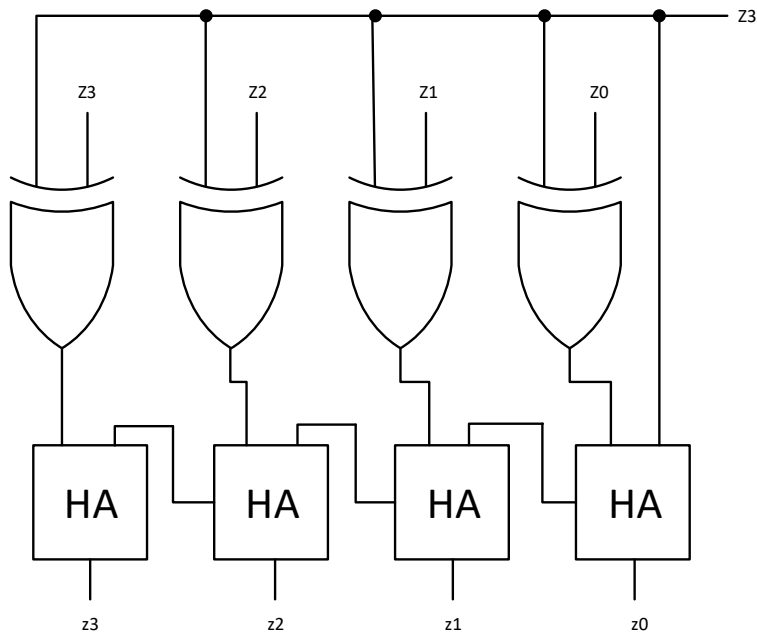
畫出 logic diagram



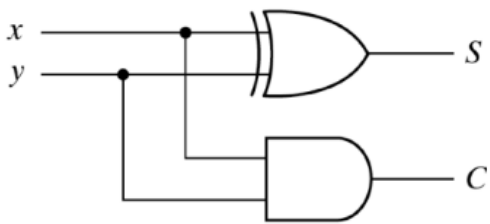
ps. 1000 為錯誤的輸入，若輸入 1000 結果會是 error

sol2.

$$Z = |z|$$



Half adder

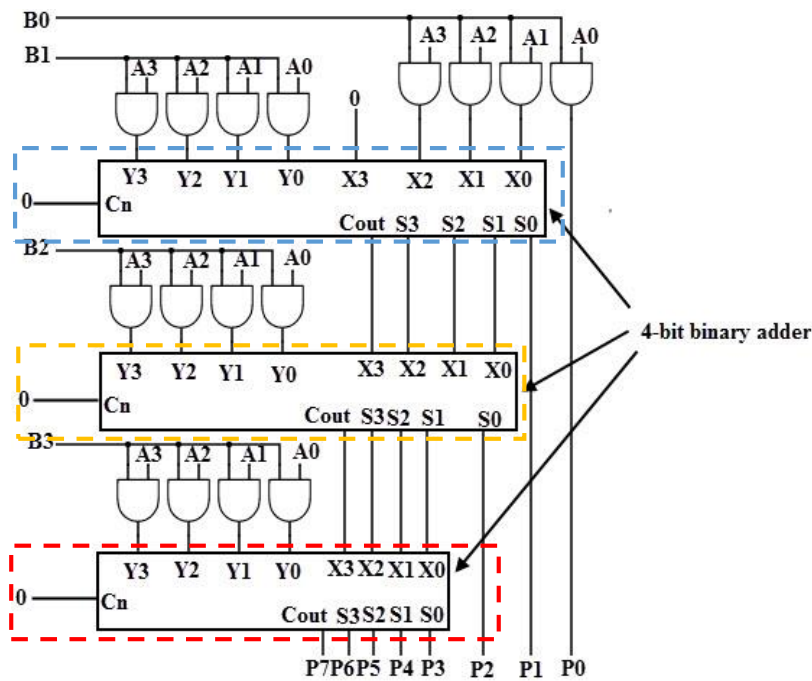


ps. 1000 為錯誤的輸入，若輸入 1000 結果會是 error

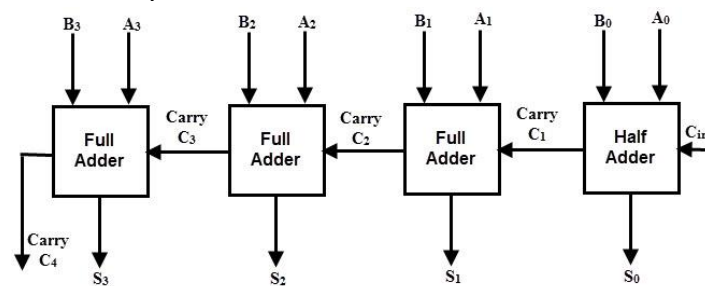
8.

4-bit x 4-bit multiplier (unsigned)

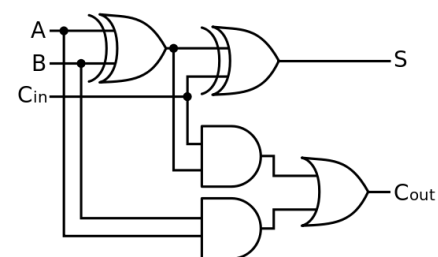
$$\begin{array}{r}
 \phantom{A3} A3 \phantom{A2} A1 \phantom{A0} \\
 \times \phantom{A3} B3 \phantom{B2} B1 \phantom{B0} \\
 \hline
 A3B0 \phantom{A2B0} \phantom{A1B0} A0B0 \\
 A3B1 \phantom{A2B1} \phantom{A1B1} A0B1 \\
 A3B2 \phantom{A2B2} \phantom{A1B2} A0B2 \\
 + A3B3 \phantom{A2B3} \phantom{A1B3} A0B3 \\
 \hline
 P7 \phantom{P6} \phantom{P5} \phantom{P4} \phantom{P3} \phantom{P2} \phantom{P1} P0
 \end{array}$$



4-bit binary adder

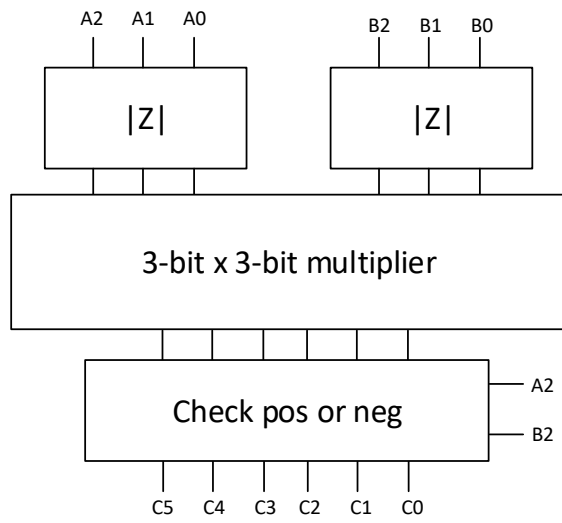


Full adder



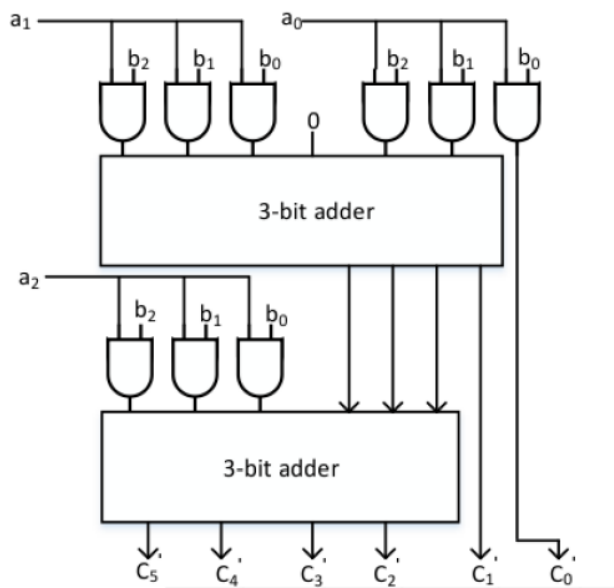
9.

Block diagram

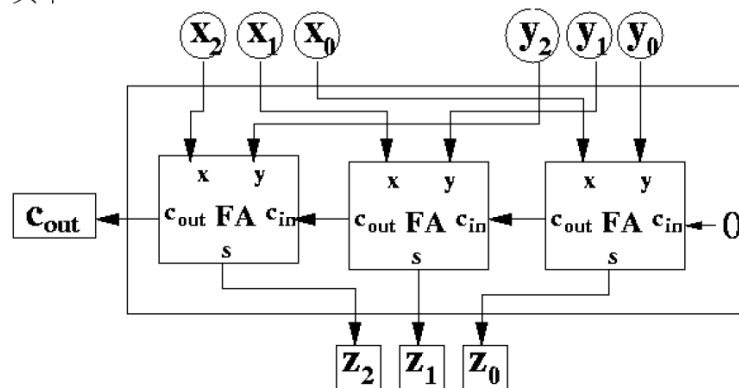


$|Z|$ (取絕對值)的 logic function 可參考第 7 題

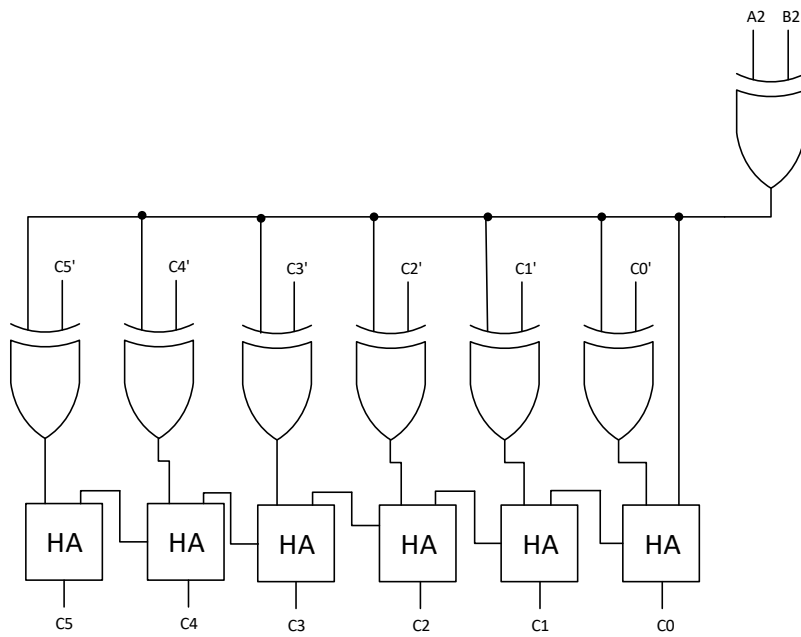
3-bit x 3bit multiplier



其中 3-bit adder



Check pos or neg



利用 A2 和 B2 判斷乘完的結果是否要變號



10.

### Design

```
module absolute_value_calculator(Z,z);
    input [3:0]Z;
    output [3:0]z;

    assign z = (Z[3])?(~Z+Z[3]):(Z);
endmodule
```

### Testbench

```
module test;
    reg [3:0]Z;
    wire [3:0]z;

    absolute_value_calculator avc(.Z(Z),.z(z));
    initial begin
        Z=4'b0000;
        #5 Z=4'b0001;
        #5 Z=4'b0010;
        #5 Z=4'b0011;
        #5 Z=4'b0100;
        #5 Z=4'b0101;
        #5 Z=4'b0110;
        #5 Z=4'b0111;
        #5 Z=4'b1001;
        #5 Z=4'b1010;
        #5 Z=4'b1011;
        #5 Z=4'b1100;
        #5 Z=4'b1101;
        #5 Z=4'b1110;
        #5 Z=4'b1111;
    end
endmodule
```

Z[3:0]	1111	0000	0001	0010	0011	0100	0101	0110	0111	1000	1010	1011	1100	1101	1110	1111
z[3:0]	0001	0000	0001	0010	0011	0100	0101	0110	0111	1100	1101	1110	1111	1111	1110	1111