

Logic Design HW1 Solution

1.

Decimal	Binary	Octal	Hexadecimal
316.1	~100111100.0001	~474.0631	~13C.1999
171.25	10101011.11	253.2	AB.4
23.671875	10111.101011	27.53	17.AC
60678	1110110100000110	166406	ED06

a.

整數:

$$(316)_{10} = 256 + 32 + 16 + 8 + 4 = 2^8 + 2^5 + 2^4 + 2^3 + 2^2$$

$$= (100111100)_2 = (474)_8 = (13C)_{16}$$

$$(0.1)_{10} \cong (0.0001)_2$$

小數:

例:在 Binary 只取小數第一位，將 Decimal(十進位)小數值*2 取整數位然後留下小數位再做下一位的相乘，即為 Binary 小數第一位數值。

$$(0.1)_{10} * 2 = 0.2 \text{ 則 } 0 \text{ 為 Binary 的小數第一位}$$

題目要求取小數四位

Sol:

Binary

$$(0.1)_{10} = (0.1)_{10} * 2 = (0.2)_{10} \dots\dots\dots \text{第一位取 } 0$$

$$(0.2)_{10} = (0.2)_{10} * 2 = (0.4)_{10} \dots\dots\dots \text{第二位取 } 0$$

$$(0.4)_{10} = (0.4)_{10} * 2 = (0.8)_{10} \dots\dots\dots \text{第三位取 } 0$$

$$(0.8)_{10} = (0.8)_{10} * 2 = (1.6)_{10} \dots\dots\dots \text{第四位取 } 1$$

$$(0.1)_{10} = (0.0001)_2$$

Octal:

$$(0.1)_{10} = (0.1)_{10} * 8 = (0.8)_{10} \dots\dots\dots \text{第一位取 } 0$$

$$(0.8)_{10} = (0.8)_{10} * 8 = (6.4)_{10} \dots\dots\dots \text{第二位取 } 6$$

$$(6.4)_{10} - (6)_{10} = (0.4)_{10} \dots\dots\dots \text{給下一位計算}$$

$$(0.4)_{10} = (0.4)_{10} * 8 = (3.2)_{10} \dots\dots\dots \text{第三位取 } 3$$

$$(3.2)_{10} - (3)_{10} = (0.2)_{10} \dots\dots\dots \text{給下一位計算}$$

$$(0.2)_{10} = (0.2)_{10} * 8 = (1.6)_{10} \dots\dots\dots \text{第四位取 } 1$$

$$(0.1)_{10} = (0.0631)_8$$

Hex:

$$(0.1)_{10} = (0.1)_{10} * 16 = (1.6)_{10} \dots\dots\dots \text{第一位取 } 1$$

$$(1.6)_{10} - (1)_{10} = (0.6)_{10} \dots\dots\dots \text{給下一位計算}$$

$$(0.6)_{10} = (0.6)_{10} * 16 = (9.6)_{10} \dots\dots\dots \text{第二位取 } 9$$

$$(9.6)_{10} - (9)_{10} = (0.6)_{10} \dots\dots\dots \text{給下一位計算}$$

$$(0.6)_{10} = (0.6)_{10} * 16 = (9.6)_{10} \dots\dots\dots \text{第三位取 } 9$$

$$(9.6)_{10} - (9)_{10} = (0.6)_{10} \dots\dots\dots \text{給下一位計算}$$

$$(0.6)_{10} = (0.6)_{10} * 16 = (9.6)_{10} \dots\dots\dots \text{第四位取 } 1$$

$$(0.1)_{10} = (0.1999)_{16}$$

b.

整數:

$$(10101011)_2 = 2^7 + 2^5 + 2^3 + 2^1 + 2^0 = (171)_{10}$$

$$(10_101_011)_2 = (253)_8 = (AB)_{16}$$

$$(0.11)_2 = (0.25)_{10} = (0.2)_8 = (0.4)_{16}$$

小數:

Sol:

Decimal

$$(0.01)_2 = (0 * \frac{1}{2} + 1 * \frac{1}{4})_{10} = (0.25)_{10}$$

Octal:

$$(0.25)_{10} = (0.25)_{10} * 8 = (2.0)_{10} \dots\dots\dots \text{第一位取 } 2$$

$$(0.01)_2 = (0.2)_8$$

Hex:

$$(0.25)_{10} = (0.25)_{10} * 16 = (4.0)_{10} \dots\dots\dots \text{第一位取 } 4$$

$$(0.01)_2 = (0.4)_{16}$$

c.

整數:

$$(27.53)_8 = (2 * 8^1 + 7 * 8^0 + 5 * 8^{-1} + 3 * 8^{-2})_{10} = (23.671875)_{10}$$

$$(27)_2 = (10111)_2 = (17)_{16}$$

$$(0.53)_8 = (0.101011)_2 = (0.AC)_{16}$$

小數:

Decimal:

$$(0.53)_8 = (5 * \frac{1}{8} + 3 * \frac{1}{64})_{10} = (0.671875)_{10}$$

Binary:

此部分可以將數值直接轉化為 2 進位

例: $(4)_8 = (100)_2$

在整數與小數上式轉換是可以互通的。

$(0.53)_8 = (0.101011)_2$

$$(5)_8 = (101)_2$$

$$(0.53)_8 = (0.101011)_2$$

$$(3)_8 = (011)_2$$

Hex:

則是將 Binary 的數值再轉換成 Hex 數值

取四個數值轉換為一個 Hex 數值

$$(0.10101100)_2 = (0.AC)_{16}$$

$$(1010)_2 = (A)_{16}$$

$$(0.10101100)_2 = (0.AC)_{16}$$

$$(1100)_2 = (C)_{16}$$

d.

$$(ED06)_{16} = (14 * 16^3 + 13 * 16^2 + 6 * 16^0)_{10} = (60678)_{10}$$

$$(ED06)_{16} = (1110_1101_0000_0110)_2 = (1_110_110_100_000_110)_2 \\ = (166406)_8$$

2.

$$9BF2 = (1001101111110010)_2 = (1_001_101_111_110_010)_2 = (115762)_8$$

3.

decode: T	encode: Atemp _{2:0}	encode: Btemp _{6:0}
68	000	0000000
70	001	0000001
72	010	0000011
74	011	0000111
76	100	0001111
78	101	0011111
80	110	0111111
82	111	1111111

Atemp_{2:0}:

$$(T-68)/2 = \sum_{i=0}^2 2^i TempA_i \quad \text{得出 Atemp}_{2:0} \text{ 即為答案。}$$

說明:

由於 T 是以 68 為基準且 2 個單位為一個 bit，故先將 $(T-68)/2$ ，可先得到一個十進位值，再將此十進位值轉成二進位值即為 Atemp_{2:0}。

Btemp_{6:0}:

$$(T-68)/2 = \sum_{i=0}^6 TempB_i \quad \text{得出 Btemp}_{6:0} \text{ 即為答案。}$$

說明:

T 值與 68 相差幾個 2，在 Btemp 即有幾個 bits 的 1。
例如 T=76， $(76-68)/2=4$ ，則會有 4 個 1，在前面補 0 使成為 0001111，即為答案。

4.

	ASCII	ASCII + Even parity
T	101_0100	1101_0100
s	111_0011	1111_0011
i	110_1001	0110_1001
n	110_1110	1110_1110
g	110_0111	1110_0111
-	010_1101	0010_1101
H	100_1000	0100_1000
u	111_0101	1111_0101
a	110_0001	1110_0001

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040	☞#32;	Space	64	40	100	☞#64;	␣	96	60	140	☞#96;	ˆ
1	1	001	SOH (start of heading)	33	21	041	☞#33;	!	65	41	101	☞#65;	A	97	61	141	☞#97;	a
2	2	002	STX (start of text)	34	22	042	☞#34;	"	66	42	102	☞#66;	B	98	62	142	☞#98;	b
3	3	003	ETX (end of text)	35	23	043	☞#35;	#	67	43	103	☞#67;	C	99	63	143	☞#99;	c
4	4	004	EOT (end of transmission)	36	24	044	☞#36;	\$	68	44	104	☞#68;	D	100	64	144	☞#100;	d
5	5	005	ENQ (enquiry)	37	25	045	☞#37;	%	69	45	105	☞#69;	E	101	65	145	☞#101;	e
6	6	006	ACK (acknowledge)	38	26	046	☞#38;	&	70	46	106	☞#70;	F	102	66	146	☞#102;	f
7	7	007	BEL (bell)	39	27	047	☞#39;	'	71	47	107	☞#71;	G	103	67	147	☞#103;	g
8	8	010	BS (backspace)	40	28	050	☞#40;	{	72	48	110	☞#72;	H	104	68	150	☞#104;	h
9	9	011	TAB (horizontal tab)	41	29	051	☞#41;	}	73	49	111	☞#73;	I	105	69	151	☞#105;	i
10	A	012	LF (NL line feed, new line)	42	2A	052	☞#42;	*	74	4A	112	☞#74;	J	106	6A	152	☞#106;	j
11	B	013	VT (vertical tab)	43	2B	053	☞#43;	+	75	4B	113	☞#75;	K	107	6B	153	☞#107;	k
12	C	014	FF (NP form feed, new page)	44	2C	054	☞#44;	,	76	4C	114	☞#76;	L	108	6C	154	☞#108;	l
13	D	015	CR (carriage return)	45	2D	055	☞#45;	-	77	4D	115	☞#77;	M	109	6D	155	☞#109;	m
14	E	016	SO (shift out)	46	2E	056	☞#46;	.	78	4E	116	☞#78;	N	110	6E	156	☞#110;	n
15	F	017	SI (shift in)	47	2F	057	☞#47;	/	79	4F	117	☞#79;	O	111	6F	157	☞#111;	o
16	10	020	DLE (data link escape)	48	30	060	☞#48;	0	80	50	120	☞#80;	P	112	70	160	☞#112;	p
17	11	021	DC1 (device control 1)	49	31	061	☞#49;	1	81	51	121	☞#81;	Q	113	71	161	☞#113;	q
18	12	022	DC2 (device control 2)	50	32	062	☞#50;	2	82	52	122	☞#82;	R	114	72	162	☞#114;	r
19	13	023	DC3 (device control 3)	51	33	063	☞#51;	3	83	53	123	☞#83;	S	115	73	163	☞#115;	s
20	14	024	DC4 (device control 4)	52	34	064	☞#52;	4	84	54	124	☞#84;	T	116	74	164	☞#116;	t
21	15	025	NAK (negative acknowledge)	53	35	065	☞#53;	5	85	55	125	☞#85;	U	117	75	165	☞#117;	u
22	16	026	SYN (synchronous idle)	54	36	066	☞#54;	6	86	56	126	☞#86;	V	118	76	166	☞#118;	v
23	17	027	ETB (end of trans. block)	55	37	067	☞#55;	7	87	57	127	☞#87;	W	119	77	167	☞#119;	w
24	18	030	CAN (cancel)	56	38	070	☞#56;	8	88	58	130	☞#88;	X	120	78	170	☞#120;	x
25	19	031	EM (end of medium)	57	39	071	☞#57;	9	89	59	131	☞#89;	Y	121	79	171	☞#121;	y
26	1A	032	SUB (substitute)	58	3A	072	☞#58;	:	90	5A	132	☞#90;	Z	122	7A	172	☞#122;	z
27	1B	033	ESC (escape)	59	3B	073	☞#59;	;	91	5B	133	☞#91;	[123	7B	173	☞#123;	{
28	1C	034	FS (file separator)	60	3C	074	☞#60;	<	92	5C	134	☞#92;	\	124	7C	174	☞#124;	
29	1D	035	GS (group separator)	61	3D	075	☞#61;	=	93	5D	135	☞#93;]	125	7D	175	☞#125;	}
30	1E	036	RS (record separator)	62	3E	076	☞#62;	>	94	5E	136	☞#94;	^	126	7E	176	☞#126;	~
31	1F	037	US (unit separator)	63	3F	077	☞#63;	?	95	5F	137	☞#95;	_	127	7F	177	☞#127;	DEL

Source: www.LookupTables.com

根據講義 01_Digital_Abstraction 第 34 頁的 ASCII code，能夠查詢 Chr 來獲得 16 進制，像是 T 的 ASCII code 16 進制為 54，轉換為二進制為 101_1000。因題目為 even parity 加上同位元後必須是偶數個 1，所以範例 101_1000 加上同位元就變成 1101_1000。此次題目為 Tsing-Hua 共 9 個字元，因此答案要有 9 組結果。

5.

Decimal	Binary	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	1110
6	0110	1010
7	0111	1011
8	1000	1001
9	1001	1000

a. Binary: $b_3b_2b_1b_0 \Rightarrow$ Gray code: $g_3g_2g_1g_0$

b. 0-4: $g_3=0$;

5-9: $g_3=1$

c. $g_i = b_{i+1} \oplus b_i$, \oplus : xor

Ex: $b_3b_2b_1b_0 = 0011$,

$$g_3 = 0, \quad g_2 = b_3 \oplus b_2 = 0, \quad g_1 = b_2 \oplus b_1 = 1, \quad g_0 = b_1 \oplus b_0 = 0$$

d. 上下鏡像貼上

Binary	Step1	Step2	Step3
0000	0	0000	0000
0001	0	0001	0001
0010	0	0011	0011
0011	0	0010	0010
0100	0	0110	0110
0101	1	1	1110
0110	1	1	1010
0111	1	1	1011
1000	1	1	1001
1001	1	1	1000