EE 3640 Communication Systems I — Chi-chao Chao
Spring 2023

Homework Assignment No. 5
Due 1:20pm, May 29, 2023

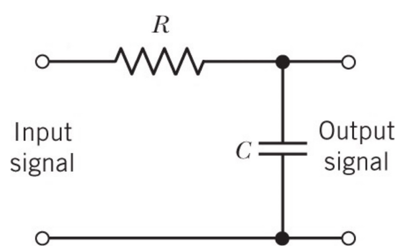
Reading: Haykin & Moher, Chapter 6.

Problems for Solution:

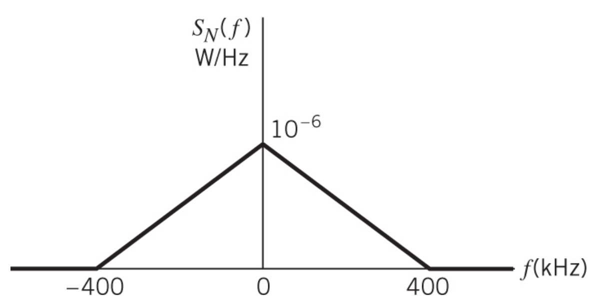
1. The sample function

$$x(t) = A_c \cos(2\pi f_c t) + w(t)$$

is applied to the low-pass RC filter shown below. The amplitude A_c and frequency f_c of the sinusoidal component are constants, and $w(t)$ is a white Gaussian noise of zero mean and power spectral density $N_0/2$. Find an expression for the output signal-to noise ratio with the sinusoidal component of $x(t)$ regarded as the signal of interest.



2. A DSB-SC modulated signal is transmitted over a noisy channel, with the power spectral density of the noise being as shown below. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assuming that the average power of the modulated wave is 10 watts, determine the output signal-to-noise ratio of the receiver.



3. The power spectral density of the white noise measured at the front end of an AM receiver is 10^{-3} watt per Hertz. The modulating wave is sinusoidal, with a carrier power of 80 kilowatts, and a sideband power of 10 kilowatts per sideband. The message bandwidth is 4 kHz.

   (a) Assuming the use of an envelope detector in the receiver, determine the output signal-to-noise ratio of the system when the carrier-to-noise ratio is high.

(b) By how many decibels is this system inferior to a DSB-SC modulation system?

4. An unmodulated carrier of amplitude A_c and frequency f_c and band-limited white noise are summed and then passed through an ideal envelop detector. Assume the noise power spectral density to be of height $N_0/2$ and bandwidth $2W$, centered about the carrier frequency f_c. Determine the output signal-to-noise ratio for the case when the carrier-to-noise ratio is high. (*Hint:* The unmodulated carrier is regarded as the signal of interest.)

5. Consider a phase modulation (PM) system, with the modulated wave defined by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

where k_p is the phase sentivity and $m(t)$ is the message signal with bandwidth W and average power P. Consider the receiver model discussed in class, where the phase demodulator consists of a phase detector followed by a baseband low-pass filter. The additive noise $w(t)$ is white Gaussian of zero mean and power spectral density $N_0/2$. The phase detector is assumed ideal, i.e., $v(t) = \theta(t)$ if its input $x(t)$ is $R(t)\cos[2\pi f_c t + \theta(t)]$. Also assume that the carrier-to-noise ratio at the detector input is high.

   (a) Determine the output signal-to-noise ratio.

   (b) Determine the figure of merit of the system.

6. Suppose that the transfer functions of the pre-emphasis and de-emphasis filters of an FM system are given as follows:

$$H_{\mathrm{pe}}(f) = k\left(1 + \frac{jf}{f_0}\right)$$

$$H_{\mathrm{de}}(f) = \frac{1}{k}\left(\frac{1}{1 + jf/f_0}\right).$$

The scaling factor k is to be chosen so that the average power of the emphasized message signal is the same as that of the original message signal $m(t)$.

   (a) Find the value of k that satisfies the above requirement for the case when the power spectral density of the message signal $m(t)$ is

$$S_M(f) = \begin{cases} S_0/[1 + (f/f_0)^2], & -W \le f \le W \\ 0, & \text{elsewhere.} \end{cases}$$

   (*Hint:* Note the indefinite integral $\int 1/(a^2 + b^2x^2)\,dx = (1/(ab))\tan^{-1}(bx/a)$.)

   (b) What is the corresponding value of the improvement factor I produced by using this pair of pre-emphasis and de-emphasis filters? (*Hint:* Note the indefinite integral $\int x^2/(a^2 + b^2x^2)\,dx = (x/b^2) - (a/b^3)\tan^{-1}(bx/a)$.)

Homework Collaboration Policy: I allow and encourage discussion or collaboration on the homework. However, you are expected to write up your own solution and understand what you turn in. Late homework is subject to a penalty of 5% to 40% of your total points.