EE 3640 Communication Systems I — Chi-chao Chao
Spring 2023

Homework Assignment No. 3
Due 1:20pm, April 17, 2023

Reading: Haykin & Moher, Chapter 4.

Problems for Solution:

1. (a) Consider the PM signal

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

where $m(t)$ is the modulating signal. Determine the in-phase and quadrature components as well as the envelope and the phase of the PM signal.

(b) Repeat (a) for the FM signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right].$$

2. The sinusoidal modulating signal

$$m(t) = A_m \cos(2\pi f_m t)$$

is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier has frequency f_c and amplitude A_c.

(a) Determine the spectrum (Fourier transform) of the resulting phase-modulated signal, assuming that the maximum phase deviation $\beta_p = k_p A_m$ is small (and does not exceed 0.3 radians).

(b) Construct a phasor diagram for this modulated signal, and compare it with that of the corresponding narrow-band FM signal.

3. An FM signal with carrier amplitude $A_c = 1$ and modulation index $\beta = 1$ is transmitted through an ideal band-pass filter with mid-band frequency f_c and bandwidth $5f_m$, where f_c is the carrier frequency and f_m is the frequency of the sinusoidal modulating signal. Determine the amplitude spectrum of the filter output.

4. A carrier wave of frequency 99.7 MHz is frequency-modulated by a sinusoidal wave of amplitude 10 volts and frequency 20 kHz. The frequency sensitivity of the modulator is 10 kHz per volt.

(a) Determine the approximate transmission bandwidth of the FM signal by using Carson's rule.

(b) Determine the transmission bandwidth by considering only those side frequencies whose amplitudes exceed 1 percent of the unmodulated carrier amplitude. Use Table 4.1 in Haykin & Moher for this calculation.

(c) Repeat your calculations, assuming that the amplitude of the modulating signal is doubled.

5. An FM signal with a frequency deviation of 15 kHz at a modulation frequency of 10 kHz is applied to two frequency multipliers connected in cascade. The first multiplier triples the frequency and the second multiplier quadruples the frequency. Determine the frequency deviation and the modulation index of the FM signal obtained at the second multiplier output. What is the frequency separation of the adjacent side frequencies of this FM signal?

6. Consider the frequency demodulation scheme shown below in which the incoming FM signal $s(t)$ is passed through a delay line that produces a phase-shift of $\pi/2$ radians at the carrier frequency f_c. The delay-line output is subtracted from the incoming FM signal, and the resulting composite signal is then envelope-detected. This demodulator finds application in demodulating microwave FM signals. Assuming that

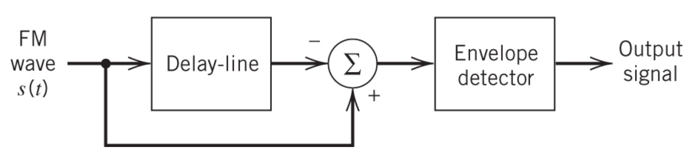
$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

analyze the operation of this demodulator when the modulation index β is less than unity and the delay T produced by the delay line is sufficiently small to justify making the approximations

$$\cos(2\pi f_m T) \approx 1$$

and

$$\sin(2\pi f_m T) \approx 2\pi f_m T.$$



7. Consider an FM signal $s(t)$ of carrier frequency f_c and amplitude A_c, which is produced by a modulating signal $m(t) = A_m \cos(2\pi f_m t)$, where $f_m \ll f_c$. Find an expression for the Hilbert transform $\hat{s}(t)$.

8. The *single-sideband version of angle modulation* is given by

$$s(t) = \exp[-\hat{\phi}(t)] \cos[2\pi f_c t + \phi(t)]$$

where $\hat{\phi}(t)$ is the Hilbert transform of the phase function $\phi(t)$ and f_c is the carrier frequency. Given the phase function

$$\phi(t) = \beta \sin(2\pi f_m t)$$

where β is the modulation index and f_m is the modulation frequency, derive the corresponding expression for the modulated wave $s(t)$. Explain that $s(t)$ contains no frequency components in the interval $-f_c < f < f_c$. (*Hint:* First find the complex envelope $\tilde{s}(t)$ and then expand $\tilde{s}(t)$ into a power series by using the expansion $\exp(x) = \sum_{n=0}^{\infty} x^n/(n!)$.)

Homework Collaboration Policy: I allow and encourage discussion or collaboration on the homework. However, you are expected to write up your own solution and understand what you turn in. Late homework is subject to a penalty of 5% to 40% of your total points.