2021 Midterm Reference Solution

1. (a) $x(t) \to \boxed{h(t)} \to y(t) \Rightarrow X(f) \to \boxed{H(f)} \to Y(f)$

$H(f) = e^{-j2\pi f\alpha}$ $\quad Y(f) = X(f)\cdot H(f) = X(f)e^{-j2\pi f\alpha}$

$y(t) = \mathcal{F}^{-1}\{Y(f)\} = \int_{-\infty}^{\infty} X(f)e^{-j2\pi f\alpha}\cdot e^{j2\pi ft}df = \int_{-\infty}^{\infty} X(f)\cdot e^{j2\pi f(t-\alpha)}df$

$= x(t-\alpha)$

$\therefore$ The signal is delayed by α

(b) (i) a real-valued baseband signal, transmittable, not a bandpass signal

(ii) a complex-valued baseband signal, Not transmittable

(iii) f_c is smaller than signal bandwidth, Not transmittable

(iv) The spectrum is not evenly symmetric with respect to the origin, it's not a valid bandpass signal

(V) The spectrum is evenly symmetric with respect to the origin, it's a transmittable bandpass signal.

$\therefore$ The answer is (V)

(c) By Carson's rule, $B_T = 2(\Delta f + W) = 2\Delta f(1+\frac{1}{\beta})$, Since there are still many frequency tones outside this spectral range, if we use this filter, we will lose information carried by these frequency tones. So the signal is distorted.

(d) For a Costas receiver, received signal is multiplied by a sinusoidal wave,

$\therefore s(t) \times \cos(2\pi f_c t) = A_c[1+k_a m(t)]\cos(2\pi f_c t)\cdot\cos(2\pi f_c t)$

$= A_c[1+k_a m(t)]\cdot\frac{1}{2}[1+\cos(4\pi f_c t)]$.

After LPF, we have $\frac{1}{2}A_c(1+k_a m(t))$, we can then use a DC block to recover the original signal as $\frac{1}{2}A_c k_a m(t)$. Thus it doesn't need the constraint of $|k_a m(t)| < 1$

2. (a) $y(t) = x(t)\otimes h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)\cdot e^{j2\pi f(t-\tau)}d\tau$

$= e^{j2\pi ft}\int_{-\infty}^{\infty} h(\tau)\cdot e^{-j2\pi f\tau}d\tau = e^{j2\pi ft}\cdot H(f)$.

$y(t) = H(f)\cdot e^{j2\pi ft} = H(f)\cdot x(t)$

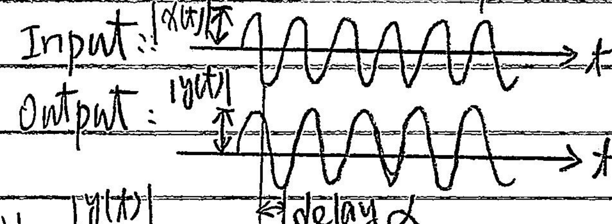
(b) From (a), $H(f) = \frac{y(t)}{e^{j2\pi f t}} = |H(f)| e^{j\beta(f)}$

$\Rightarrow |H(f)| = \frac{|y(t)|}{|e^{j2\pi f t}|} = |y(t)|$

Since the input is a sinusoidal wave, the output is also a sinusoidal wave, but with some delay α. $\therefore$ The output $y(t)$ can be expressed as $y(t) = |y(t)| \cdot e^{j2\pi f(t-\alpha)} = |y(t)| e^{-j2\pi f\alpha} \cdot e^{j2\pi f t} = |H(f)| e^{j\beta(f)} \cdot x(t)$

Then $\beta(f) = -2\pi f \alpha$.

$\therefore$ To measure $H(f)$, we can continuously tune the frequency of the input sinusoidal wave and then use an oscilloscope to monitor the output sinusoidal wave. Compare it with the input sinusoidal wave:



Then we have: $|H(f)| = \frac{|y(t)|}{|x(t)|}$

$\beta(f) = -2\pi f \alpha$

3, (a) $s(t) = A_c(a_0 + m(t))\cos(2\pi f_c t)$, $m(t) = A_m \cos(2\pi f_m t)$

Since there is no overmodulation: $\max\{|m(t)|\} = A_m \le a_0$

Let $A_m = a_0$, Then $s(t) = A_c a_0 \cos(2\pi f_c t) + A_c A_m \cos(2\pi f_m t) \cdot \cos(2\pi f_c t)$

$\therefore s(t) = A_c a_0 \cos(2\pi f_c t) + \frac{1}{2} A_c A_m \cos(2\pi(f_c - f_m)t) + \frac{1}{2} A_c A_m \cos(2\pi(f_c + f_m)t)$.

$\therefore P_s = \frac{1}{2}(A_c a_0)^2 + \frac{1}{2}(\frac{1}{2} A_c A_m)^2 + \frac{1}{2}(\frac{1}{2} A_c A_m)^2$ $(\because A_m \le a_0)$

$\le \frac{3}{4} A_c^2 a_0^2$

(b) $A_c = 2$ Volt. $P_m = 2W = \frac{1}{2} A_m^2 \Rightarrow A_m = 2$ Volt

$P_s = 22W = \frac{1}{2}(A_c a_0)^2 + (\frac{1}{2} A_c A_m)^2 = (\frac{1}{2}(2 \times a_0)^2 + 4)W$

$\therefore 4a_0^2 = 36 \Rightarrow a_0 = 3$ Volt.

(c) $a_0 = 3$ volt $> A_m = 2$ volt $\therefore$ It's not an over-modulated signal.

(d) $s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$

$S(f) = \mathcal{F}\{s(t)\} = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot [\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)]$

(e) $P_s = (\frac{A_c}{2})^2 \times 2 \times \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{1}{2} A_c^2$

↑ This number represents $\delta(f - f_c - n f_m)$ and $\delta(f + f_c + n f_m)$

If "2" is not shown in the answer or properly addressed, only partial credit can be given.

(f) Carrier has no power means $J_0(\beta)=0$

From the figure, $J_0(\beta)=0$ occurs at $\beta \cong 2.4\ (2.4048)$

$\therefore \Delta f \cong 2.4 f_m$

(g) at $\beta = 2.4$, $|J_n(2.4)| > 1$ occurs when $n \le 5$. ($J_5(2.4) = 0.016$)

$\therefore$ The bandwidth $B_T \simeq 2 \times 5 \times f_m = 100$ KHz ($f_m = 10$ KHz)

It's hard to see whether $J_5(2.4)$ is larger than 0.01 or not.

If students' answer says $n=4$, $B_T = 8 f_m = 80$ KHz can also get full credit.

4.(a) The impulse response is obtained by sending $\delta(t)$ into the system.

So we can get $h(t) = \delta(t) - \delta(t-\tau)$

(b) The corresponding $H(f) = 1 - e^{-j2\pi f\tau}$

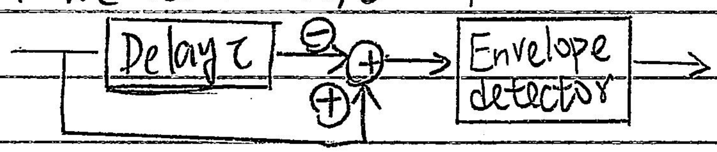
(c) If $2\pi f\tau$ is much less than 1, by Taylor's series, $e^{-j2\pi f\tau} \simeq 1 - j2\pi f\tau$

$\therefore H(f) \cong 1 - (1 - j2\pi f\tau) = j2\pi f\tau$

Thus output $Y(f) \cong j2\pi f\tau \cdot X(f) \Rightarrow y(t) \simeq \tau \cdot \frac{dx(t)}{dt}$

$\therefore$ we need $2\pi f\tau \ll 1$

(d)



5.(a) $x(t) = m(t) + A_c \cos(2\pi f_c t)$

(b) $y(t) = a_1 x(t) + a_2 x^2(t) = a_1 m(t) + a_1 A_c \cos(2\pi f_c t) + a_2 m^2(t)$

$+ 2a_2 m(t) \cdot A_c \cos(2\pi f_c t) + a_2 \cdot A_c^2 \cos^2(2\pi f_c t)$

$= a_1 m(t) + a_2 m^2(t) + \frac{1}{2} A_c^2 a_2 \quad \Leftarrow$ at baseband

$+ [a_1 A_c + 2a_2 A_c m(t)] \cos(2\pi f_c t) \quad \Leftarrow$ at f_c

$+ \frac{A_c^2}{2} a_2 \cos(2\pi 2 f_c t) \quad \Leftarrow$ at $2f_c$

(c) We wish to select the desired AM signal at f_c, thus we need to apply a BPF at f_c with at least $2W$ bandwidth to select the AM signal, so that $s(t) = [a_1 A_c + 2a_2 A_c m(t)] \cos(2\pi f_c t)$

(d) $s(t) = [a_1 A_c + 2a_2 A_c m(t)] \cos(2\pi f_c t) = a_1 A_c [1 + \frac{2a_2}{a_1} m(t)] \cos(2\pi f_c t)$

$= a_1 A_c [1 + k_a m(t)] \cos(2\pi f_c t)$

$\therefore$ The amplitude sensitivity is $\frac{2a_2}{a_1}$