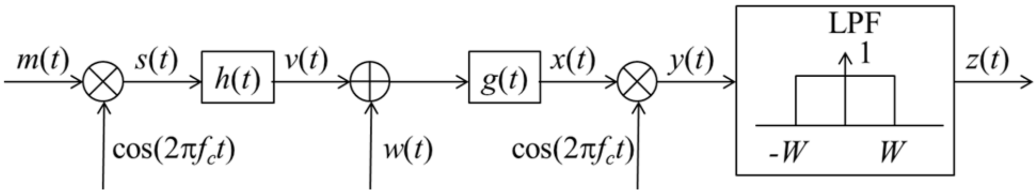
Homework #5

Due: 12/30

1. (30 %) Consider the following system:



Assume the noise $w(t)$ is white noise with power spectral density (PSD) $S_W(f)$ = $N_0/2$ = 0.2 mW/Hz and the PSD, $S_M(f)$ of the message $m(t)$ is defined as:

$$S_M(f) = \begin{cases} 12 \text{ mW/Hz} & |f| \le 0.3W \\ 4 \text{ mW/Hz} & 0.3 \le |f| \le W \\ 0 & |f| \ge W \end{cases}$$

The frequency response of the channel $h(t)$ is defined as:

$$H(f) = \begin{cases} 0.2 & f_c - 0.3W \le |f| \le f_c + 0.3W \\ 1 & \text{elsewhere} \end{cases}$$

(a) (4 %) Please find the power of $m(t)$.
(b) (4 %) Please find the power of $s(t)$.
(c) (6 %) Please plot the PSD of $v(t)$.
(d) (6 %) Assume the system is noise-free, i.e., $w(t)$ = 0, and $G(f) = 1$, please find the PSD of $z(t)$.
(e) (5 %) If the output of $z(t) = m(t)$ in the absence of noise, i.e., $w(t)$ = 0, please find $G(f)$.
(f) (5 %) Assume $w(t) \neq 0$ and $G(f)$ is what you found in (e), please find the PSD of $z(t)$ in the absence of the signal $s(t)$.

2. (35 %) Assume a noise $n_1(t)$ is stationary with a power spectral density shown in Fig. 1, that is $S_{n1}(f) = \delta(f) + \text{rect}(f/2f_0)$. Another noise process $n_2(t)$ is related with $n_1(t)$ by:

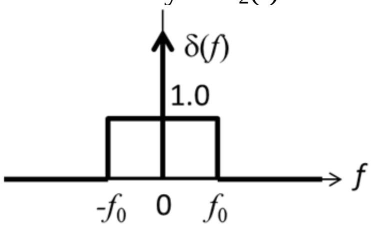
$$n_2(t) = n_1(t)\cos(2\pi f_c t + \theta) - n_1(t)\sin(2\pi f_c t + \theta),$$

where f_c is carrier frequency and θ is the value of a random variable Θ uniformly distributed within $(0, 2\pi)$.

(a) (10 %) Please find and sketch the autocorrelation function of $n_1(t)$.
(b) (5 %) If we claim that the signals are uncorrelated when the two sampling points are approaching to $\pm\infty$. Then from (a), you can find at some specific sampling rates, the sampled signals are also uncorrelated. What sampling rates will give uncorrelated samples of $n_1(t)$? Why? You have to give an explanation to get the full credit.
(c) (5 %) Show that the cross correlation of $n_1(t)$ and $n_2(t)$ is 0. Assume that random variables N_1 and Θ are statistically independent.
(d) (5 %) Demonstrate that $n_2(t) = \sqrt{2} n_1(t)\cos(2\pi f_c t + \pi/4 + \theta)$.
Note: $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$.
(e) (5 %) Using (d) to find the autocorrelation function of $n_2(t)$ in terms of the

autocorrelation of $n_1(t)$: $R_{N_1}(\tau)$, where τ is the time difference.

(f) (5 %) Plot power spectral density of $n_2(t)$.



3. (25 %) A random process, defined by $X(t) = A(t)\cos(2\pi f_c t + \theta)$, is applied to an integrator which produces an output of $Y(t)$ as $Y(t) = \int_{t-T}^{t} X(\tau)d\tau$.

(a) (5 %) Please find the corresponding impulse response of the employed integrator. **Note:** You'll get 0 credit if you only give the answer without any justification.

(b) (8 %) Suppose that the carrier frequency f_c is a constant, $A(t)$ is a WSS random process independent of θ, and θ is a random variable uniformly distributed in $[0.\ 2\pi]$. We denote the power spectral density of $A(t)$ by $S_A(f)$. Show that the power spectral density $S_Y(f)$ is given by

$$S_Y(f) = \frac{1}{4}\left[S_A(f - f_c) + S_A(f + f_c)\right]T^2 \mathrm{sinc}^2(Tf).$$

(c) (6 %) Suppose that the carrier frequency f_c is a constant, $A(t) = A$, where A is a Gaussian distributed random variable with zero mean and variance of σ_A^2, and $\theta = 0$. Find the mean and variance of the output $Y(t)$ at a particular time t_k.

(d) (6 %) Based on the assumption in (c), is $Y(t)$ stationary? Why?

4. (10 %) Let $X(t)$ be a zero-mean stationary Gaussian random process with the power spectral density $S_X(f) = 5\mathrm{rect}\left(\frac{f}{1000}\right)$. Determine the probability density function of the random variable $X(t = 3)$.

Please note: Homework must be turned in by the beginning of class.
No late homework submission is allowable!