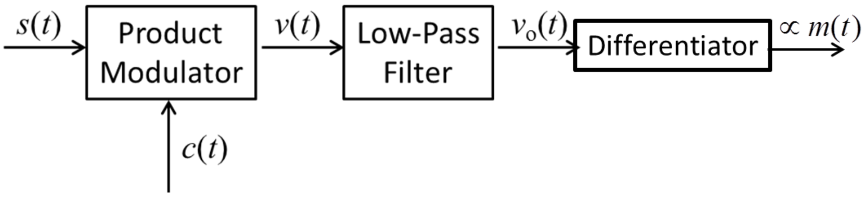
Homework #4

Due: 11/25

1. (40 %) A message $m(t)$ is employed to modulate a RF carrier at f_c in FM format.
   (a) (3 %) Please write down the expression of this passband signal, $s(t)$.
   (b) (4 %) This signal can be expressed in its phasor representation as: $s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}$.
   Please find the slow varying complex envelope $\tilde{s}(t)$.
   (c) (5 %) Assume we employ weak modulation scheme, which means the maximum phase deviation β is much less than unit. Then by applying Taylor series to the first order, we may obtain an approximation of $e^x = 1 + x$. Please find the expression of this passband signal $s(t)$.
   (d) (8 %) To detect this signal, we may use a product modulator associated with a low-pass filter to demodulate the signal. The configuration of this receiver can be illustrated as the following diagram. Please specify what kind of $c(t)$ you need to demodulate the signal and explain how it works.



   (e) (5 %) Now if we have the message $m(t)$ as $m(t) = \dfrac{\sin(100\pi t)}{\pi t}$ What is the bandwidth of message $m(t)$? Note you may need the following Fourier Transform pair: $sinc(t) \Leftrightarrow rect(f)$.
   (f) (5 %) If we define the total energy of $m(t)$ as $E = \int_{-\infty}^{\infty} |m(t)|^2 \, dt$, please find the message's total energy.
   (g) (5 %) Then we modulate it to obtain the transmitted FM signal $s(t)$ as
   $$s(t) = 3\cos(2\pi 10^6 t + 100\pi \int_0^t m(\tau)\,\mathrm{d}\tau).$$
   Is it still a weak FM signal? Why?
   (h) (5 %) Based on Carson's rule, please estimate the transmission bandwidth B_T of this signal.

2. (15 %) Consider a wide-band PM signal produced by a sinusoidal modulating wave, $A_m\cos(2\pi f_m t)$, using a modulator with a phase sensitivity k_p rad/volt.
   (a) (10 %) Show that if the maximum phase deviation of the PM signal is large compared with 1 radian, the bandwidth of the PM signal is linearly proportional to the modulation frequency f_m.
   (b) (5 %) Compare this characteristic of a wideband PM signal with the bandwidth of a wideband FM signal defined by Carson's rule.

3. (25 %) This problem illustrates design choices and limitations for certain FM detector designs. Consider an FM system where the modulated signal is

$$s(t) = 10\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right),$$

where the carrier frequency is f_c = 100 MHz. The modulating signal is $m(t) = 10\cos(2\pi f_m t)$, where f_m = 3 KHz.

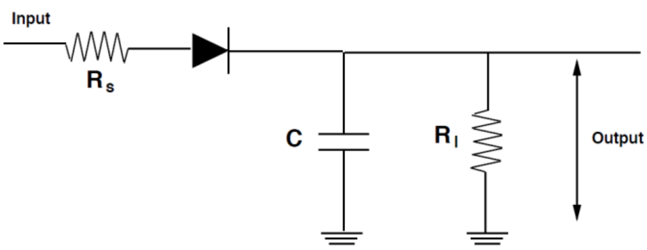
(a) (5 %) What is the maximum value of k_f such that $s(t)$ can be demodulated using an ideal differentiator followed by an envelope detector?

<u>For the remainder of this problem assume that $k_f = 10$.</u>

(b) (3 %) What is the approximate bandwidth of $s(t)$? Is this NBFM or WBFM?.

(c) (5 %) Find the instantaneous frequency $f_i(t)$ of $s(t)$. What are the maximum and minimum values of $f_i(t)$?

(d) (7 %) Suppose that you demodulate $s(t)$ using an ideal differentiator followed by an envelope detector. Assume a standard envelope detector as shown below, where the capacitor has capacitance $C = 10^{-9}$ F. Propose values for the source resistance R_s and load resistance R_l such that the output of the envelope detector is approximately equal to $c_1 + c_2 m(t)$ for some constants c_1 and c_2. Is it possible to use this detection method if $f_c \approx f_m$? Why or why not?



(e) (5 %) Suppose that you use a zero-crossing detector for $s(t)$. Find an expression for the minimum interval T for a zero-crossing detector such that there are at least four zero crossings in every interval T. Evaluate this expression.

4. (20 %) A composite angle modulated signal with carrier frequency at 10^6 Hz is expressed as:

$$s(t) = 10\cos[2\pi f_c t + 10\sin(2\pi \cdot 1000t) + 20\sin(2\pi \cdot 2000t)]$$

(a) (4 %) If this is an FM signal, what is the bandwidth of the message?

(b) (4 %) Find the average power of this angle modulated signal. Assume the load resistance is 1 Ω.

(c) (4 %) What is the instantaneous frequency?

(d) (4 %) At what time will we have the maximum frequency deviation Δf, and how much is it?

(e) (4 %) Find the transmitted bandwidth of this signal by Carson's rule.

Please note: Homework must be turned in by the beginning of class.
No late homework submission is allowable!