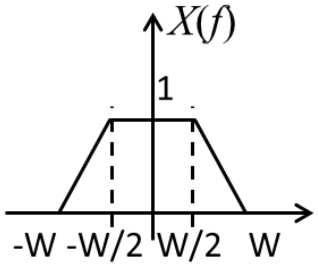
Homework #3

Due: Nov. 15

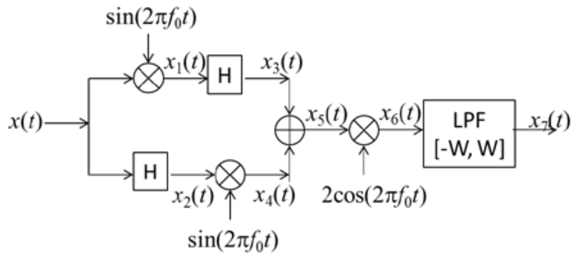
1. (40 %) For a sinusoidal message $m(t) = A_m \cos(2\pi f_m t)$, if we modulate it on an RF carrier with a carrier frequency f_c and get the transmitted signal $s(t)$ expressed as: $s(t) = aA_mA_c \cos[2\pi(f_c + f_m)t] + A_mA_c(1 - a) \cos[2\pi(f_c - f_m)t]$, where $0 \le a \le 1$.
   (a) (3 %) Is it a carrier-suppressed signal? Why?
   (b) (3 %) It is obvious that the amplitude of the transmitted signal is not a constant, so we can categorize this signal as an AM signal. Generally speaking, if $a \neq 0.5$, the slow varying envelope of this signal is not a real-valued signal. Why?
   (c) (5 %) In fact, this is a VSB signal! Please explain why this signal is VSB according to the characteristics of a VSB signal.
   (d) (7 %) Moreover, the transmitted signal $s(t)$ is neither a pure upper sideband (USB) nor a lower sideband (USB) signal. Please find its in-phase and quadrature-phase components of the corresponding complex envelope signal.

   However, by adjusting the value of a, such a transmitted signal can be in different modulation formats.
   (e) (4 %) Please find the value of a such that this signal is a DSB-SC signal.
   (f) (3 %) What is value of a to let the signal become an USB signal?
   (g) (3 %) How about LSB signal, what is value of a?
   (h) (6 %) To simplify the receiver structure, we add an unmodulated carrier, $A_c \cos(2\pi f_c t)$, to the signal. Thus the transmitted signal becomes $s(t) + A_c \cos(2\pi f_c t)$. At the receiver, we can simply use an envelope detector to detect this signal. What is the output of the envelope detector? Note, the envelope detector detect the slow varying amplitude of the composite signal: A_c + (slow varying envelope) in complex plane.
   (i) (3 %) From (e), please identify which one is the distortion factor.
   (j) (3 %) Following (f), under what condition can we demodulate the message without distortion?

2. (35 %) A low-pass (baseband) signal $x(t)$ has a Fourier transform as shown in the following Fig. (a). This signal is applied to the system shown in Fig. (b). The blocks marked H represent Hilbert transform. Assume $W << f_c$. Please find their corresponding amplitude spectra $X_1(f)$ to $X_7(f)$ of $x_1(t)$ to $x_7(t)$.

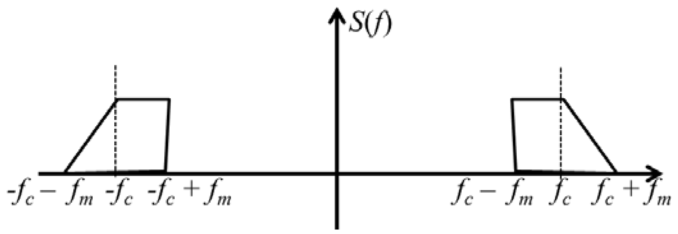


(a)



(b)

3. (35 %) The spectrum of an AM band-pass signal is shown as follows:

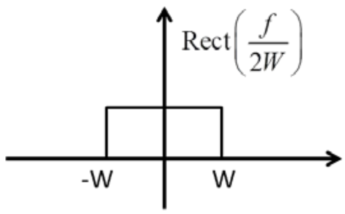


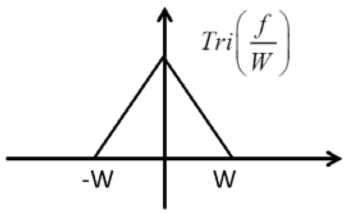
(a) (5 %) Apparently, the corresponding baseband message is not real-valued. Please explain why?

(b) (5 %) Is this signal a VSB signal at a carrier frequency of f_c? Why? You need to give some explanation to get the full credit.

(c) (5 %) Since the corresponding baseband signal is not a real-valued signal, we may assume it's complex envelope signal is expressed as $\tilde{s}(t) = m_I(t) - jm_Q(t)$. Please design a receiver that can be employed to recover the corresponding $m_I(t)$ and $m_Q(t)$.

(d) (12 %) Please calculate and plot the spectra of the baseband $|M_I(f)|$ and $|M_Q(f)|$ in frequency domain. You need to write down $M_I(f)$ and $M_Q(f)$ in terms of $S(f)$ to get the full credit. You may need to use the following function definitions:





(e) (8 %) Please calculate the real-valued $m_I(t)$ and $m_Q(t)$ of this signal in time domain. You may need this Fourier transform property: $\frac{d}{dt}g(t) \Leftrightarrow j2\pi f G(f)$ and $\text{sinc}(2Wt) \Leftrightarrow \frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$

Please note: Homework must be turned in by the beginning of class.
No late homework submission is allowable!