2021 Homework 3 Reference Solution

1.

(a) $s(t) = a A_m A_c \cos(2\pi(f_c+f_m)t) + A_m A_c(1-a)\cos(2\pi(f_c-f_m)t)$.

Since there is no component at carrier frequency f_c, i.e., no $\cos(2\pi f_c t)$ term, so it's a carrier-suppressed signal. The answer is Yes!

(b) The corresponding spectrum:

$$S(f) = \frac{1}{2} a A_m A_c [\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))] + \frac{1}{2} A_m A_c (1-a)[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))]$$

The amplitude @ f_c+f_m is $\frac{1}{2} a A_m A_c$ and at $f_c - f_m$ is $\frac{1}{2} A_m A_c (1-a)$. If $a \neq 1/2$, $\frac{1}{2} a A_m A_c \neq \frac{1}{2} A_m A_c (1-a)$

The spectrum is not symmetric with respect to f_c.

$\therefore$ The slow varying envelope of this signal is not real.

(c) For VSB, if $|f_1 - f_c| = |f_2 - f_c|$ and $|S(f_1)| + |S(f_2)| = $ constant, it's a VSB signal. From $s(t)$, we can see that if let $f_1 = f_c + f_m$, $f_2 = f_c - f_m$, we have $|f_1 - f_c| = |f_2 - f_c| = f_m$. and $|S(f_1)| + |S(f_2)| = \frac{1}{2} a A_m A_c + \frac{1}{2}(1-a) A_m A_c = A_m A_c$ is a constant, So it's a VSB signal.

(d)

$$\begin{aligned}
s(t) &= a A_m A_c \cos(2\pi(f_c+f_m)t) + A_m A_c (1-a)\cos(2\pi(f_c-f_m)t) \\
&= Re\{ a A_m A_c e^{j2\pi(f_c+f_m)t} + A_m A_c (1-a) e^{j2\pi(f_c-f_m)t} \} \\
&= Re\{ (a A_m A_c e^{j2\pi f_m t} + A_m A_c (1-a) e^{-j2\pi f_m t}) e^{j2\pi f_c t} \} \\
&= Re\{ A_m A_c [a e^{j2\pi f_m t} + (1-a) e^{-j2\pi f_m t}] e^{j2\pi f_c t} \} \\
&= Re\{ \tilde{s}(t) e^{j2\pi f_c t} \}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \tilde{s}(t) &= A_m A_c (a e^{j2\pi f_m t} + (1-a) e^{-j2\pi f_m t}) \\
&= A_m A_c [a \cos(2\pi f_m t) + j a \sin(2\pi f_m t) + (1-a)\cos(2\pi f_m t) - j(1-a)\sin(2\pi f_m t)] \\
&= A_m A_c [\cos(2\pi f_m t) + j(2a-1)\sin(2\pi f_m t)] \\
&= m_I(t) + j m_Q(t)
\end{aligned}$$

$\Rightarrow$ In-Phase: $m_I(t) = A_m A_c \cdot \cos(2\pi f_m t)$

Quadrature-phase: $m_Q(t) = (2a-1) A_m A_c \sin(2\pi f_m t)$

(e) If it's a DSB-SC signal, upper side band = lower side band and $s(t) = m(t)\cos(2\pi f_c t)$, then we should let $a = 1/2$, so that $m_Q(t) = 0$ and $s(t) = A_m A_c \cos(2\pi f_m t)\cos(2\pi f_c t)$

(f) For USB signal, amplitude at $f_c - f_m$ should be 0.

$\therefore a = 1$

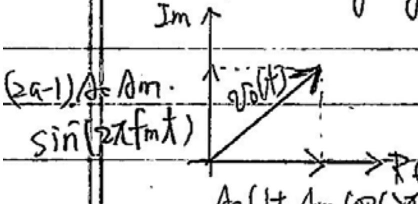
(g) For LSB signal, amplitude @ $f_c + f_m$ should be 0,

$\therefore a = 0$

(h) Let $v(t) = A_c \cos(2\pi f_c t) + s(t)$

$$= \mathrm{Re}\left\{ [A_c + A_m A_c \cos(2\pi f_m t) + j(2a-1) A_m A_c \sin(2\pi f_m t)] e^{j2\pi f_c t} \right\}$$

So for the received $v(t)$, the real part of the slow varying envelope is $A_c(1 + A_m \cos(2\pi f_m t))$ and the imaginary part of the slow varying envelope is $(2a-1) A_m A_c \sin(2\pi f_m t)$



$\therefore$ The envelope is

$$v_0(t) = \sqrt{[A_c(1 + A_m \cos(2\pi f_m t))]^2 + (A_c A_m (2a-1) \sin(2\pi f_m t))^2}$$

$$= A_c(1 + A_m \cos(2\pi f_m t)) \sqrt{1 + \left(\frac{A_m (2a-1) \sin(2\pi f_m t)}{1 + A_m \cos(2\pi f_m t)} \right)^2}$$

(i) The targeting output of the envelope detector is $A_c(1 + A_m \cos(2\pi f_m t))$ $= A_c(1 + m(t))$

The distortion factor is $\sqrt{1 + \left(\frac{A_m (2a-1) \sin(2\pi f_m t)}{1 + A_m \cos(2\pi f_m t)} \right)^2}$.

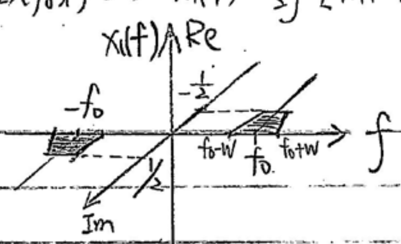
(j) If the output is distortionless, then the distortion factor should be 1, which means

$$\sqrt{1 + \left(\frac{A_m (2a-1) \sin(2\pi f_m t)}{1 + A_m \cos(2\pi f_m t)} \right)^2} = 1 \Rightarrow A_m (2a-1) \sin(2\pi f_m t) = 0$$

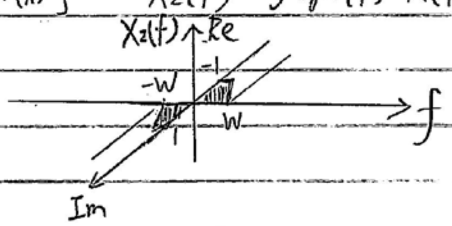
$\therefore a = 1/2$

2.

(a) $x_1(t) = x(t) \sin(2\pi f_0 t) \Rightarrow X_1(f) = \frac{1}{2j}[X(f-f_0) - X(f+f_0)]$

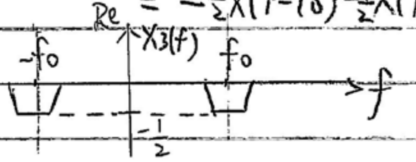


(b) $x_2(t) = \mathcal{H}\{x(t)\}$ $\quad X_2(f) = -j\,\mathrm{sgn}(f) \cdot X(f)$



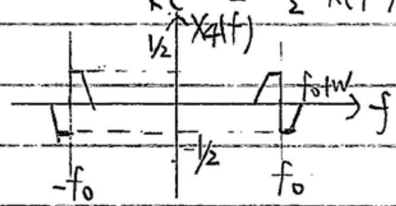
(c) $x_3(t) = \mathcal{H}\{x_1(t)\}$ $\quad X_3(f) = -j\,\mathrm{sgn}(f) \cdot \frac{1}{2j}[X(f-f_0) - X(f+f_0)]$

$$= -\tfrac{1}{2}X(f-f_0) - \tfrac{1}{2}X(f+f_0)$$



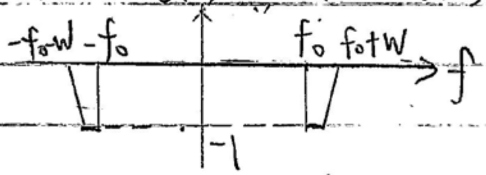
(d) $x_4(t) = x_2(t)\cdot\sin(2\pi f_0 t) \Rightarrow X_4(f) = \frac{1}{2j}[X_2(f-f_0) - X_2(f+f_0)]$

$\text{Re} \quad = -\frac{1}{2}X(f-f_0)\,\text{sgn}(f-f_0) + \frac{1}{2}X(f+f_0)\,\text{sgn}(f+f_0)$



(e) $x_5(t) = x_4(t) + x_3(t) \Rightarrow X_5(f) = X_4(f) + X_3(f) = -\frac{1}{2}X(f-f_0)[1+\text{sgn}(f-f_0)]$

$-\frac{1}{2}X(f+f_0)[1-\text{sgn}(f+f_0)]$

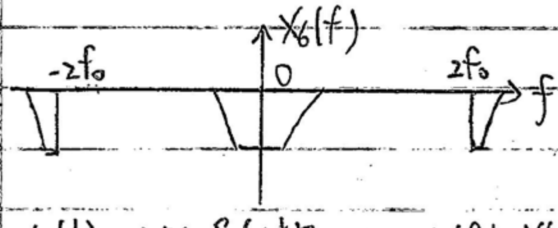


(f) $x_6(t) = x_5\cdot 2\cos(2\pi f_0 t) \Rightarrow X_6(f) = -\frac{1}{2}X(f)[1+\text{sgn}(f)] - \frac{1}{2}X(f-2f_0)[1+\text{sgn}(f-2f_0)]$

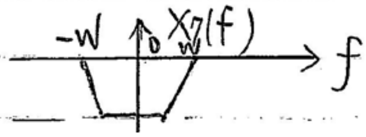
$-\frac{1}{2}X(f)[1-\text{sgn}(f)] - \frac{1}{2}X(f+2f_0)[1-\text{sgn}(f+2f_0)]$

$= -X(f) - \frac{1}{2}X(f-2f_0)[1+\text{sgn}(f-2f_0)]$

$-\frac{1}{2}X(f+2f_0)[1-\text{sgn}(f-2f_0)]$



(g) $x_7(t) = \text{LPF}\{x_6(t)\} \Rightarrow X_7(f) = -X(f)$



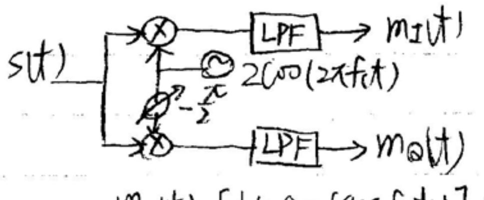
3.

(a) The upper sideband and lower sideband of this signal is not complex conjugated to each other, thus the corresponding slow varying baseband signal's spectrum is not symmetrically distributed with respect to $f=0$.
$\therefore$ It's not a real-valued baseband signal.

(b) NO. The VSB signal requires $S(f-f_c) + S(f+f_c) = 1$, thus $S(f_c) = \frac{1}{2}$.
However, in this spectrum, $S(f_c) = 1 \Rightarrow$ It's not a VSB signal.

(c) $s(t) = \text{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\} = \text{Re}\{[m_I(t) - j\,m_Q(t)][\cos(2\pi f_c t) + j\sin(2\pi f_c t)]\}$

$= m_I(t)\cos(2\pi f_c t) + m_Q(t)\sin(2\pi f_c t)$.



$m_I(t)$ can be obtained by

$s(t)\cdot 2\cos(2\pi f_c t) = m_I(t)\cdot 2\cos^2(2\pi f_c t)$

$+ m_Q(t)\,2\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)$.

$= m_I(t)\,[1+\cos(4\pi f_c t)] + m_Q(t)\cdot\sin(4\pi f_c t)$

After LPF, we obtain $m_I(t)$.

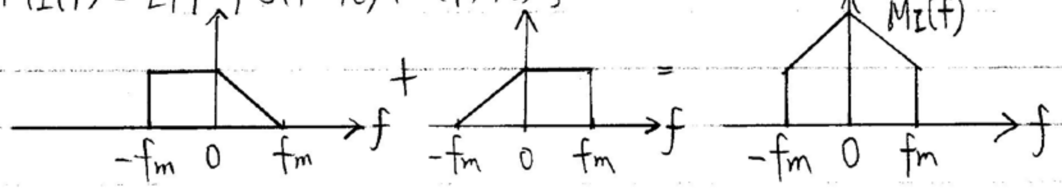
$m_Q(t)$ can be obtained by $s(t)\cdot 2\sin(2\pi f_c t) = m_I(t)\cdot 2\cos(2\pi f_c t)\sin(2\pi f_c t)$

$+ m_Q(t)\cdot\sin(2\pi f_c t)\cdot 2\sin(2\pi f_c t) = m_I(t)\sin(4\pi f_c t) + m_Q(t) - m_Q(t)\cos(4\pi f_c t)$

After LPF, we obtain $m_Q(t)$

(d) To obtain $m_I(t)$, we multiply $s(t)$ by $2\cos(2\pi f_c t)$ and use a LPF to filter it, then we may write

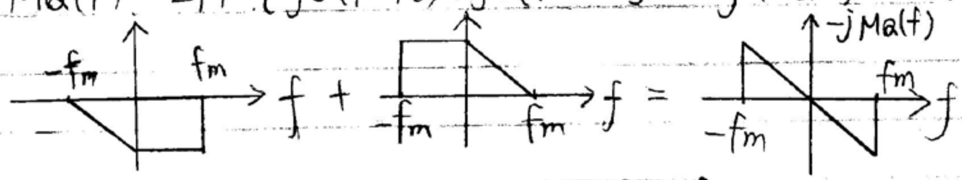
$M_I(f) = LPF\{S(f-f_c)+S(f+f_c)\}$



∴ The resulting $M_I(f) = \begin{cases} 0 & f<-f_m \\ 1+\frac{1}{f_m}(f+f_m) & -f_m\le f\le 0 \\ 1-\frac{1}{f_m}(f-f_m) & 0\le f\le f_m \\ 0 & f>f_m\end{cases} = rect\left(\frac{f}{2f_m}\right)+tri\left(\frac{f}{f_m}\right)$

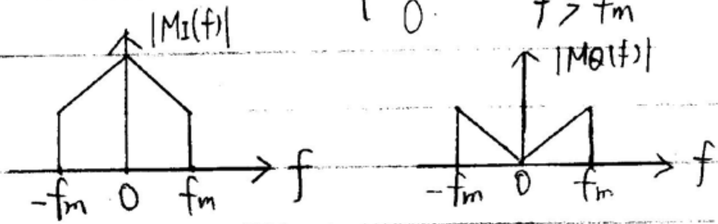
To obtain $m_Q(t)$, we multiply $s(t)$ by $2\sin(2\pi f_c t)$ and use a LPF to obtain $m_Q(t)$ at baseband. Note: $2\sin(2\pi f_c t) \Leftrightarrow -j[\delta(f-f_c)-\delta(f+f_c)]$

∴ $M_Q(f) = LPF\{-jS(f-f_c)+jS(f+f_c)] \Rightarrow \frac{1}{j}M_Q(f) = -S(f-f_c)+S(f+f_c)$



∴ The resulting $M_Q(f) = \begin{cases} 0 & f<-f_m \\ -j\frac{f}{f_m} & -f_m\le f\le f_m \\ 0 & f>f_m\end{cases} = -j\frac{f}{f_m}rect\left(\frac{f}{2f_m}\right)$

∴



(e) $m_I(t) = \mathcal{F}^{-1}\{M_I(f)\} = 2f_m\, sinc(2f_m t) + f_m\, sinc^2(f_m t)$

$m_Q(t) = \mathcal{F}^{-1}\{M_Q(f)\} = \mathcal{F}^{-1}\{-j\frac{f}{f_m}rect\left(\frac{f}{2f_m}\right)\}$

$= -\frac{1}{2\pi f_m}\mathcal{F}^{-1}\{j2\pi f\cdot rect\left(\frac{f}{2f_m}\right)\} = -\frac{1}{2\pi f_m}\frac{d}{dt}\left(2f_m\, sinc(2f_m t)\right)$

$= -\frac{1}{\pi}\frac{d}{dt}\left[\frac{\sin(2\pi f_m t)}{2\pi f_m t}\right] = -\frac{1}{\pi}\left[\frac{\cos(2\pi f_m t)}{t} - \frac{1}{2\pi f_m}\frac{\sin(2\pi f_m t)}{t^2}\right]$