

Homework #2

Due: Oct. 28

1. (20 %) For a signal expressed as: $f(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT_b}{T_p}\right)$, where $T_b = 5T_p$.
- (a) (5 %) Please plot this signal in time domain.
 - (b) (10 %) Please calculate the Fourier Transform of this signal.
 - (c) (5 %) Please sketch the spectrum of this signal.

2. (30 %) Consider a stable linear time invariant (LTI) and causal system that is characterized by the differential equation:

$$\frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

where $x(t)$ and $y(t)$ are input and output of this system, respectively.

- (a) (10 %) Calculate the transfer function.
- (b) (10 %) Laplace transform is defined as $L\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$. In this system, we may apply Laplace transform instead of Fourier transform with the replacement of $j2\pi f$ by s in the answer you've obtained in (a). Please explain why this operation is valid in this system.
- (c) (10 %) Apply Inverse Laplace transform to obtain the impulse response of this system.

3. (35 %) A bandpass signal is expressed as:

$$s(t) = 50 \cos(22000\pi t) + 100 \cos(20000\pi t) + 150 \cos(18000\pi t)$$

- (a) (5 %) Find and plot the spectrum of $s(t)$.
- (b) (5 %) What is the carrier frequency, f_c ?
- (c) (5 %) What is the transmission bandwidth of this signal?
- (d) (5 %) Let $\tilde{s}(t)$ be the slow varying complex envelope of this band-pass signal. Find and express $\tilde{s}(t)$ as $\tilde{s}(t) = m_I(t) + jm_Q(t)$.
- (e) (5 %) Find and plot the spectrum of $\tilde{s}(t)$.
- (f) (5 %) Find and plot the spectrum of $|M_I(f)|$.
- (g) (5 %) Find and plot the spectrum of $|M_Q(f)|$.

4. (15 %) For a linear time-invariant system with an impulse response of $h(t) = 2\text{sinc}^2(2t)$ and the input signal $x(t) = \text{sinc}(4t)$.

- (a) (5 %) Please plot the frequency response of the system.
- (b) (10 %) Please find the output signal $y(t)$.

Please note: Homework must be turned in by the beginning of class.
No late homework submission is allowable!

Homework Policy

You can discuss the homework problems with any number of students currently taking the course, the teaching assistants, and the instructor. However, solutions should not be exchanged. You should make sure that you understand what you turn in, and should of course write up every word of the solution by yourself. It is OK to compare your final answer with others currently enrolled in the course, but you should fix up any error by your own effort. If these sentences are still vague, just tell yourself "I shall not take unfair advantage of any other student" and this should answer other policy-related questions you have in your mind.