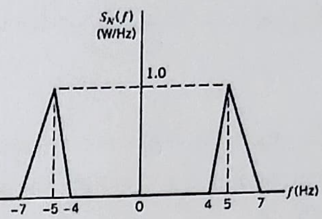
EE364000 Communication System I, Final Exam

Jan. 10, 2022

Note: You have to give your solution as clear and detailed as possible. If there's anything not clear in your solution, you'll only receive partial credits!!

1. (20 %) Quiz:
   (a) (5 %) For a fair coin, if we throw it 10 times and get the result as {H, H, T, H, T, H, H, T, T, H}. Is this sample set an ergodic process? Why?
   (b) (5 %) A narrowband noise is shown as the left figure. Please sketch the cross-density ($S_{N_I N_Q}(f)/j$) of the in-phase and quadrature-phase components.



   (c) (5 %) Is $R(\tau)=\begin{cases}1, & -1\le\tau\le1\\ 0, & \text{elsewhere}\end{cases}$ a valid autocorrelation function for a random communication signal?
   (d) (5 %) In FM demodulation, we enjoy f^2 noise reduction effect in baseband. If we transmit PM signal, can we have this f^2 noise reduction advantage? Why?

2. (25 %) For a coherent receiving system shown in Fig. P2, the input lower sideband signal of a SSB AM signal is expressed as $s(t)=A_c m(t)\cos(2\pi f_c t)+A_c\hat{m}(t)\sin(2\pi f_c t)$, where $\hat{m}(t)$ is the Hilbert transform of the message $m(t)$ with $\hat{M}(f)=-j\,\text{sgn}(f)M(f)$. Assume the noise is additive white Gaussian noise (AWGN), detection is in coherent receiving, the message power is P with bandwidth W and the BPF has the spectral range of the corresponding SSB AM signal, please find
   (a) (12 %) $x(t)$, $v(t)$ and $y(t)$
   (b) (4 %) Output SNR.
   (c) (3 %) Input SNR
   (d) (3 %) Channel SNR
   (e) (3 %) FOM

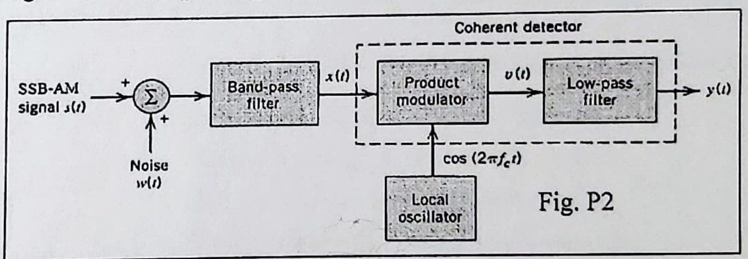


Fig. P2

3. (15 %) A random process is given as $Z(t)=X(t)\cos(2\pi f_c t+\Theta)$, where $X(t)$ is a wide-sense stationary random process with $\mu_X=0$ and $\mathbf{E}\left[X^2(t)\right]=\sigma_X^2$.
   (a) (8 %) If Θ is not random, but a deterministic number, say $\Theta=\theta_1$. Is this process wide-sense stationary?
   (b) (7 %) If Θ is a random variable uniformly distributed in $[-\pi,\pi]$, Is this process wide-sense stationary?

4. (20 %) For an FM demodulator, assume the message is a sinusoidal wave: $m(t)=A_m\cos(2\pi f_m t)$. Let the noise be an AWGN with zero mean and a power spectral density of $N_0/2$. After FM demodulation,

$(SNR)_O=\dfrac{3A_c^2k_f^2P}{2N_0W^3}$, and $(FOM)_{FM}=\dfrac{3k_f^2P}{W^2}$, where A_c is carrier amplitude, k_f is frequency sensitivity, P is message power and W is message bandwidth.

(a) (6 %) If ρ is the carrier to noise ratio at transmission band and assume transmission bandwidth B_T is much larger than message bandwidth W, please express $(SNR)_O$ as a function of ρ, B_T and W.

(b) (8 %) Following (a), please illustrate two methods to enhance $(SNR)_O$ of an FM signal. Which approach is more efficient and why?

(c) (6 %) If we wish an FM signal outperforms a conventional AM signal in terms of FOM, how much transmission bandwidth to message bandwidth ratio of the FM should be?

5. (30 %) A baseband filtered noise system is shown below

$$w(t) \longrightarrow \boxed{\text{LPF}, h(t)} \longrightarrow n(t)$$

in which, the impulse response of the LPF is expressed as: $h(t) = 20e^{-5t}u(t)$ and $u(t)$ is a unit step function, $w(t)$ is a white Gaussian noise with zero mean and single-sided power spectral density 2 x 10^{-2} W/Hz.

(a) (4 %) Find the frequency response of the LPF.

(b) (5 %) Find the power spectral density of $n(t)$.

(c) (5 %) Find the autocorrelation function of $n(t)$.

(d) (3 %) Find the mean of $n(t)$.

(e) (4 %) Find the variance of $n(t)$.

(f) (3 %) Is $n(t)$ still a white noise? Why?

(g) (3 %) Is $n(t)$ still a Gaussian noise? Why?

(h) (3 %) Determine the probability function of $n(t)$.

Note: In this exam, you may need the following formulas:

I.
$$\begin{cases} \sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)] \\ \cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)] \\ \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)] \\ \sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \end{cases}$$

II. Fourier Transform pair: $\exp(-a|t|) \Leftrightarrow \dfrac{2a}{a^2+(2\pi f)^2}$