Problem set #2 (Modern Physics)

04/16/2018 Provided by *Masahito Oh-e*

Solve the problems below. Describe the ways of thinking in English: only final solutions are not accepted. Make clear how you reach each solution.

Problem 1.
Calculate photon energies of the edges of the visible range spectrum at λ=380 nm, and λ=770 nm, respectively, with the units of J and eV.

Photon energy ϵ is expressed as $\epsilon = h\nu = \frac{hc}{\lambda}$, where h is the Planck's constant, c the speed of light, ν the frequency of light, and λ the wavelength of light. Therefore, for each wavelength λ=380 nm and λ=770 nm, the energies are calculated as

$$\epsilon 380nm = \frac{(6.626\times10^{-34})\times(3.0\times10^{8})}{380\times10^{-9}}$$

5.23105×10^{-19}

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5.231052631578949`*^-19/(1.6*(10^-19))
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$5.23105\times10^{-19}/(1.6/10\text{^}19)$

3.26941

$$\epsilon 380nm = \frac{(6.626\times10^{-34})\times(3.0\times10^{8})}{770\times10^{-9}}$$

2.58156×10^{-19}

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2.58156×10^-19/(1.6*(10^-19))
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$2.58156\times10^{-19}/(1.6/10\text{^}19)$

1.61347

ϵ(380 nm) = 5.2×10^{-19} J =3.3 eV
ϵ(780 nm) = 2.6×10^{-19} J =1.6 eV

Problem 2.
Show that the Planck spectral distribution formula leads to the experimentally observed Stefan law for the total radiation emitted by a blackbody at all wavelengths. Stefan law: $E_{total} = aT^4$ $W\cdot m^{-2}\cdot K^{-4}$, where T is the temperature. Use the relation $\int_0^\infty \frac{x^3}{(e^x-1)} dx = \frac{\pi^4}{15}$.

According to Planck's law, the total energy density is expressed as $u(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(h\nu/kT)-1}$, where h is the Planck's constant, c the speed of light, ν the frequency of light, T the temperature, and k the Boltzmann constant. Since Stefan's law is an expression for the total power per unit area radiated at

all frequencies, we must integrate the expression for u(v) over all frequencies. If we make the change of variables x=hv/kT, we find
dx=h/kT(dv), and

$$u=\int_0^\infty u(v)\,dv=\frac{8\pi}{c^3}\frac{k^3T^3}{h^2}\int_0^\infty\frac{(hv/kT)^3}{\exp(hv/kT)-1}\,dv=\frac{8\pi}{c^3}\frac{k^3T^3}{h^2}\frac{kT}{h}\int_0^\infty\frac{x^3}{\exp(x)-1}\,dx=\frac{8\pi k^4T^4}{c^3h^3}\int_0^\infty\frac{x^3}{\exp(x)-1}\,dx=\frac{8\pi^4k^4}{15c^3h^3}T^4$$

Problem 3.
Suppose that light of total intensity 1.0 μW/cm^2 falls on a clean iron sample 1.0 /cm^2 in area. Assume that the iron sample reflects 96% of the light and that only 3.0% of the absorbed energy lies in the violet region of the spectrum above the threshold frequency.
(a) What intensity is actually available for the photoelectric effect?
(b) Assuming that all the photons in the violet region have an effective wavelength of 250 nm, how many electrons will be emitted per second?
(c) Calculate the current in the photo-tube in amperes.
(d) If the cutoff frequency is f_0 = 1.1 ×10^{15} Hz, find the work function, ϕ, for iron.
(e) Find the stopping voltage for iron if photo-electrons are produced by light with λ=250 nm.

(a) Because only 4.0% of the incident energy is absorbed, and only 3.0% of this energy is able to produce photo-electrons, the intensity available is

$$I = (0.030)\times(0.040)I_0=(0.030)\times(0.040)\times(1.0\ \mu W/cm^2)=1.2\ nW/cm^2.$$

(b) For an efficiency of 100%, one photon of energy, hv, will produce one electron, so

# of electrons /s =

$$\frac{1.2\times10^{-9}\,W}{hv}=\frac{\lambda\times1.2\times10^{-9}}{hc}=\frac{250\times10^{-9}\times1.2\times10^{-9}}{6.626\times10^{-34}\times3.0\times10^{8}}$$

1.50921×10^{9}

# of electrons / s = 1.5 × 10^9

(c) The current i can be calculated by i=e×(# of electrons/s)

$$i=(1.602\times10^{-19}\ C)\times(1.5\times10^{9}\ electrons/s)=2.4\times10^{-10}\ A$$

(d) Work function ϕ can be expressed as hf_0. Therefore,

$$\phi=hf_0=(4.14\text{x}10^{15}\ eV\cdot s)(1.1\times10^{15}\ s^{-1})=4.5\ eV$$

(e) From the photoelectric equation, the energy of photo-electrons can be calculated by

$$e\,V_s=hf-\phi=\frac{hc}{\lambda}-\phi=\frac{(4.14\times10^{15}\ eV\cdot s)\times(3.0\times10^{8}\ m/s)}{250\times10^{-9}\ m}-4.5\ eV=0.46\ eV$$

Problem 4.
X-rays of wavelength $\boldsymbol{\lambda}$=0.200 nm are aimed at a block of carbon. The scattered x-rays are observed at an angle of 45.0˚ to the incident beam. Calculate the increased wavelength of the scattered x-rays at this angle.

The change in wavelength expected by a photon that is scattered through the angle θ by a particle of rest mass m is expressed by

$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$. Tanking θ=45°, we find

$\Delta\lambda = \frac{6.63\times10^{-34}\,Js}{(9.11\times10^{31}\,\text{kg})(3.00\times10^{8}\,m/s)}(1 - \cos 45.0°) = 7.11\times10^{-13}$ m = 0.00071 nm

Hence, the wavelength of the scattered x-ray at this angle is

$\lambda = \Delta\lambda + \lambda_0$= 0.200711 nm

Problem 5.

An electron of charge q and mass m is accelerated from rest through a small potential difference V=50 V. Calculate de Broglie wavelength $\boldsymbol{\lambda}$, assuming that the particle is nonrelativistic.

When a charge is accelerated from rest through a potential difference V, its gain in kinetic energy, $\frac{1}{2}mv^2$, must equal the loss in potential energy qV. That is, $\frac{1}{2}mv^2 = qV$. Because p = mv, we can express this in the form

$\frac{p^2}{2m} = qV$ or $p = \sqrt{2\,mqV}$

Substituting this expression for p into the de Broglie relation λ= h/p gives

$\lambda = (h/p) = \frac{h}{\sqrt{2\,mqV}}$. Therefore, the de Broglie wavelength of an electron accelerated through 50 V is

$\lambda = h/\text{Sqrt}[2\,mqV] = \frac{6.63\times10^{-34}\,J\cdot s}{\sqrt{2\,(9.11\times10^{-31}\,\text{kg})(1.6\times10^{-19}\,C)(50\,V)}} = 1.7\times10^{-10}$ m = 1.7 Å

Problem 6.

Consider a 2 μg mass traveling with a speed of 10 cm/s. If the particle's speed is uncertain by 1.5%, what is its uncertainty in position? Use the uncertainty relation: $\Delta x \Delta p_x \geq h$.

With m=2 × 10^{-6} g = 2 × 10^{-9} kg and v = 10 cm/s = 0.1 m/s, the uncertainty in speed is

Δv= (0.015)(0.1 m/s) =1.5 × 10^{-3} m/s.

Thus, the uncertainty in position is given by

$\Delta x = \frac{h}{\Delta p} = \frac{h}{m\Delta v} = \frac{6.63\times10^{-34}\,J\cdot s}{(2\times10^{-9}\,\text{kg})(1.5\times10^{-3}\,m/s)} = 2.21\times10^{-22}\,m$.