Problem set #1 (Modern Physics) Solutions

03/29/2018 Provided by *Masahito Oh-e*

Solve the problems below. Describe the ways of thinking in English: only final solutions are not accepted. Make clear how you reach each solution.

Problem 1.

A spaceship is measured to be 100 m long while it is at rest with respect to an observer. If this spaceship now flies by the observer with a speed of 0.99c, what length will the observer find for the spaceship?

The proper length L_p of the ship is 100 m. The length L measured as the spaceship flies by at the speed of v = 0.99 c is

$$L = L_p \sqrt{1 - \frac{v^2}{c^2}} = (100\ \mathrm{m}) \sqrt{1 - \frac{(0.99\ c)^2}{c^2}} = 14\ \mathrm{m}$$

Set::write : $\sqrt{1 - \frac{v^2}{c^2}}$ L_pのタグTimesはProtectedです． ≫

14.1067

Problem 2.

Two trains are oppositely approching at the speed of 0.6c, respectively. Suppose an observer is in one of the train, what is the relative speed of the other train with respect to the observer?

Velocity addition :

Suppose two objects are moving at the speed of u and v with respect to each frame of reference, the velocity addition is expressed by

$$V = \frac{u + v}{1 + \frac{uv}{c^2}} = \frac{0.6\ c + 0.6\ c}{1 + 0.6^2} = \frac{15}{17}\ c$$

Set::write : $\frac{u + v}{1 + \frac{uv}{c^2}}$のタグTimesはProtectedです． ≫

0.882353 c

Problem 3.

An electron, which has a mass of 9.11 X 10^{-31}kg, moves with a speed of 0.750c. Find its relativistic momentum and compare this with the momentum calculated from the classical expression.

The relativistic momentum p of the mass of m and the speed v is expressed by

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{(9.11 \times 10^{-31}\ \text{kg})\ (0.75 \times 3.00 \times 10^8\ \text{m / s})}{\sqrt{1 - \frac{(0.75\ c)^2}{c^2}}} = 3.10 \times 10^{-22}\ \text{kg m / s}$$

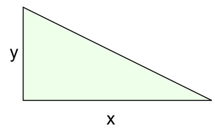
The incorrect classical expression would give

$mv = (9.11 \times 10^{-31}\ \text{kg}) \times (0.75 \times 3.00 \times 10^8\ \text{m / s}) = 2.05 \times 10^{-22}\ \text{kg m / s}$

Hence, for this case the correct relativistic result is 50% greater than the classical result.

Problem 4.

A spaceship in the form of a triangle flies by an observer at 0.950c. When the ship is measured by an observer at rest with respect to the ship, the distances x and y are found to be 50.0 m and 25.0 m, respectively. What is the shape of the ship as seen by an observer who sees the ship in motion along the direction?



The observer sees the horizontal length of the ship to be contracted to a length of

$$L = L_p \sqrt{1 - \frac{v^2}{c^2}} = (50\ \text{m}) \sqrt{1 - \frac{(0.95\ c)^2}{c^2}} = 15.6\ \text{m}$$

Set::write : $\sqrt{1 - \frac{v^2}{c^2}}$ 14.1067 $\frac{3.09893\times10^{-22}\ \text{kg m}}{\text{s}}$ のタグTimesはProtectedです・ »

15.6125 m

The 25-m vertical height is unchanged because it is perpendicular to the direction of relative motion between the observer and the spaceship.

Problem 5.

Muons are unstable elementary particles that have a charge equal to that of an electron and a mass 207 times that of the electron. Muons are naturally produced by the collision of cosmic radiation with atoms at a height of several thousand meters above the surface ofthe Earth. Muons have a lifetime of only 2.2 μs when measured in a reference frame at rest with respect to them. If we take 2.2μs (proper time) as the average lifetime of a muon and assume that its speed is close to the speed of light, we would find that these particles could travel a distance of about 650 m before they decayed. Hence, they could not reach the Earth from the upper atmosphere where they are produced. But we do observe some of the muons reach the earth. Explain why muons can reach the earth.

The phenomenon of time dilation explains this effect. Relative to an observer on Earth, the muons have a lifetime equal to $\gamma\tau$, where τ=2.2 μs is the lifetime in a frame of reference traveling with the muons. For example, for v= 0.99c, $\gamma \approx 7.1$ and $\gamma\tau \approx$ 16 μs. Hence, the average distance traveled as measured by an observer on Earth is $\gamma v \tau \approx$ 4700 m. Because of time dilation, the muons' lifetime is longer as measured by the Earth observer. 4700 m still looks a short distance, but it is way longer than 650 m. We can find out that slight difference of the speed of muons that is close to the speed of light makes great differences of distance.

$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.99\,c)^2}{c^2}}}$$

7.08881

Problem 6.

There is a twin, A and B. Suppose when they are 20 years old, A makes a round trip to the Centaurus α star that is 4.4 light-years far away from the earth by spacecraft at the speed of v=0.99c. When A returns the earth and see B, how old is each of them?

Time for the round trip with respect to the Earth is 4.4×2/0.99=8.9 (years). ∴ B: 28.9 years old.

From B, time of A goes by $\sqrt{1 - 0.99^2}$ =0.141 times as fast as that of B.Therefore, A is in the spaceship for 8.9×0.141=1.25 (years). ∴ A: 21.25 years old