Problem set #3 (Modern Physics: 6 problems)

05/14/2018 Provided by *Masahito Oh-e*

Solve the problems below. Describe the ways of thinking in English: only final solutions are not accepted. Make clear how you reach each solution.
Due is on May 24 (Thu). Delayed submission will not be accepted without any reasonable reason.

Problem 1.

Time-dependent Schrödinger equation: $\hat{\mathcal{H}}\Psi = i\hbar \frac{\partial}{\partial t}\Psi$

If $\Psi_1 = \psi_1(x,y,z)\exp(-2\pi\nu_1 t)$, $\Psi_2 = \psi_2(x,y,z)\exp(-2\pi\nu_2 t)$, ⋯⋯ are solutions of the Schrödinger equation above, show that the linear combination of Ψ_1, Ψ_2 , ⋯⋯is also the solution of the Schrödinger equation.

Problem 2.

Show that

(a) $[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$

(b) $[x^2, p_x] = 2i\hbar x$

(Hints: The operator of the momentum $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$, $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$)

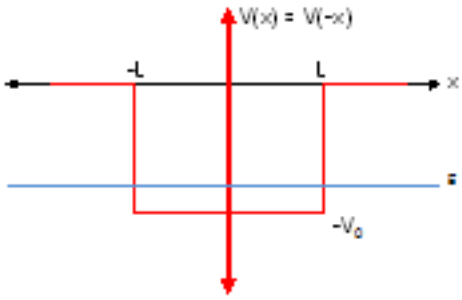
Problem 3.

(a) If a wave function is $\psi(x) = \frac{N}{x^2+a^2}$, calculate the normalization constant N.

(b) Given a wave function to be $\psi(x) = \left(\frac{\pi}{\alpha}\right)^{-\frac{1}{4}}\exp\left(-\frac{\alpha^2 x^2}{2}\right)$, calculate the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$.

(Hints: Use $\int_0^\infty \sqrt{x}\, e^{-x}\, dx = \frac{\sqrt{\pi}}{2}$, if it is needed.)

Problem 4.



(1) Write forms of $\Psi(x)$ in the three domains for odd $\Psi(x)$.

(2) Write a boundary condition for continuity of Ψ.

(3) Write a boundary condition for continuity of $\partial\Psi$.

(4) Show that you get $k = -l\cot(lL)$. $k = \frac{\sqrt{-2mE}}{\hbar}$, $l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$

Problem 5.

A particle is trapped in a one dimensional potential given by = $kx^2/2$. At a time t = 0 the state of the

particle is described by the wave function $\psi =$
$C_1 \psi_1 + C_2 \psi_2$, where ψ is the eigen function belonging to the eigen value E_n.
What is the expected value of the energy?

Problem 6.

Consider a particle described by a wavefunction: $\psi = e^{ix} + 2\,i e^{3\,ix}$ $(-\pi \leq x \leq \pi)$

(a) Normalize the wavefunction.

(b) If you precisely measure the momentum of the state expressed by the wavefunction ψ, what values can you obtain and what probability cay you get, respectively? (Hints: See the wavefunction ψ as the superposition of the two waves. Deduce the momentum of each wave. The operator of the momentum $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$)