Introduction

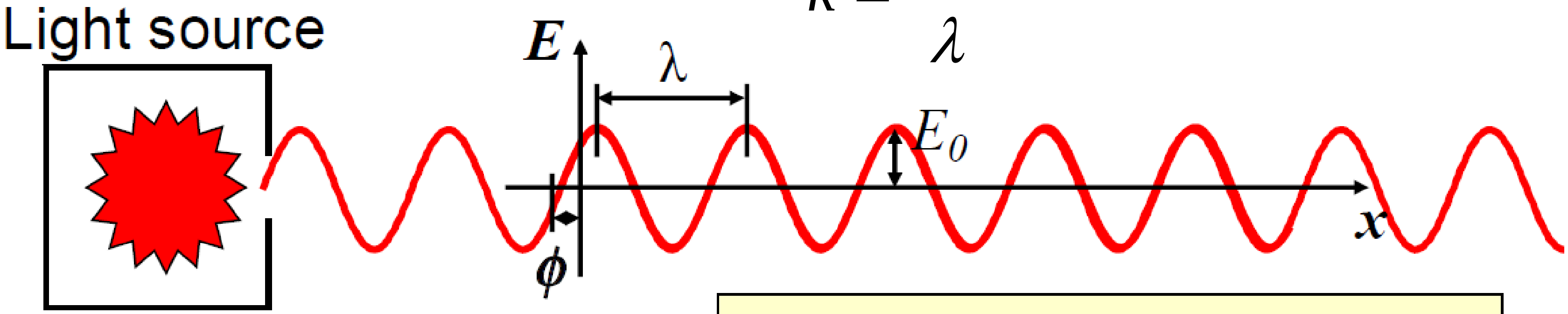
Particle-wave duality (of light and matter)!
It seem confusing when to use the wave or the particle representation of the very same physical entity. But in the end it simply depends on the experiment ('question' or 'measurement')! We will see several experiments that will help you understand this exciting concept.

Light as waves

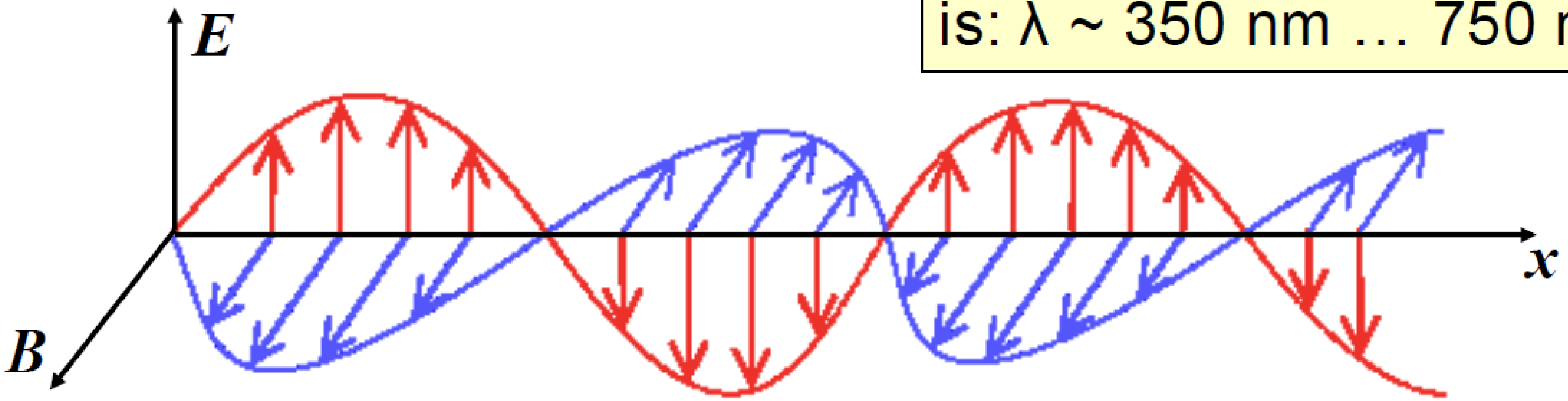
$$E(x,t) = E_0 \sin(kx - \omega t + \phi)$$

$$\lambda = \frac{2\pi c}{\omega}, \quad \omega = 2\pi f = \frac{2\pi}{T}$$

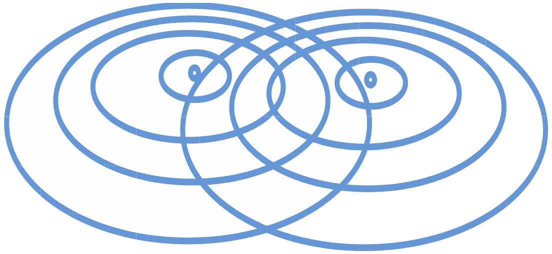
$$k = \frac{2\pi}{\lambda}$$

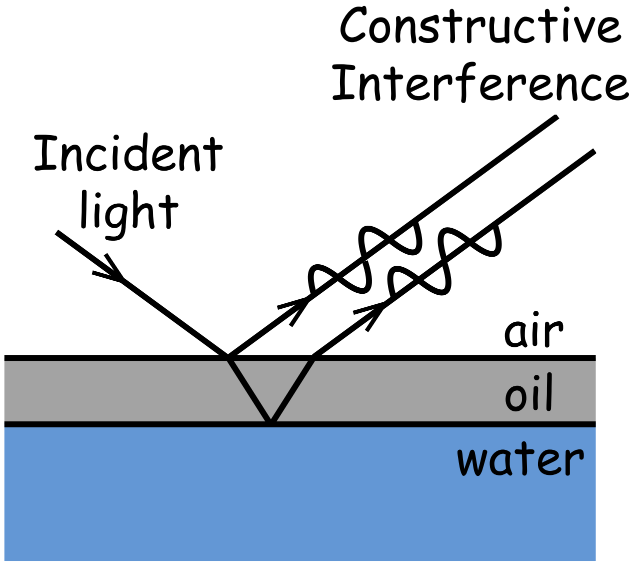


Wavelength λ of visible light is: λ ~ 350 nm … 750 nm.



What are wave-like properties?





Constructive Interference

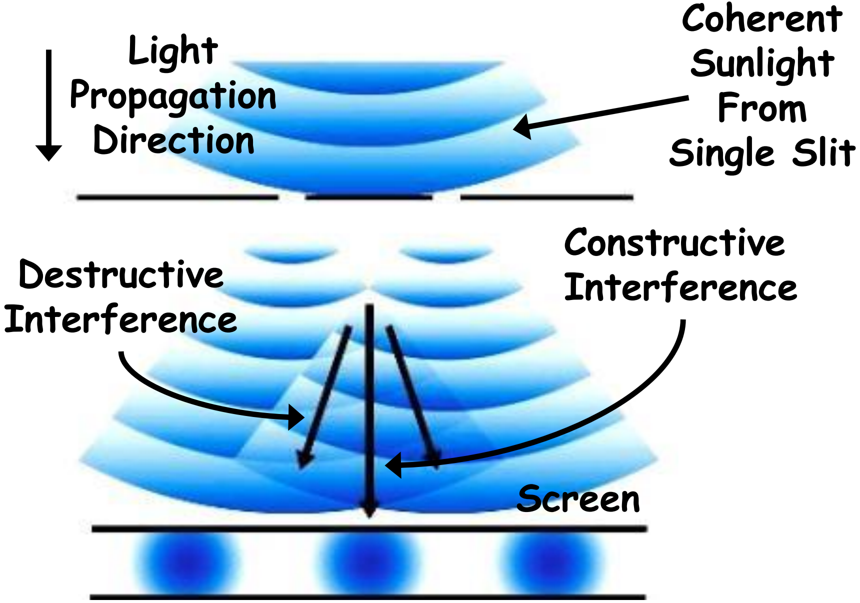
Incident light

air

oil

water

Young's Double Slit Experiment



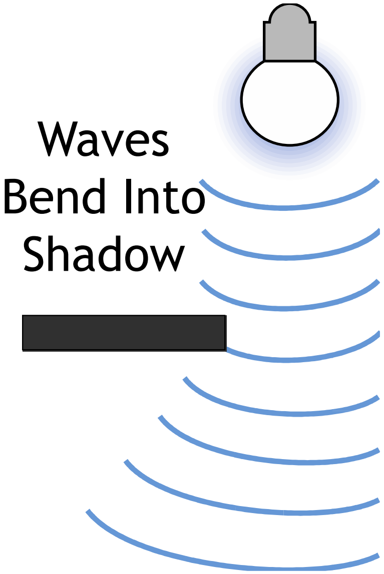
Light Propagation Direction

Coherent Sunlight From Single Slit

Destructive Interference

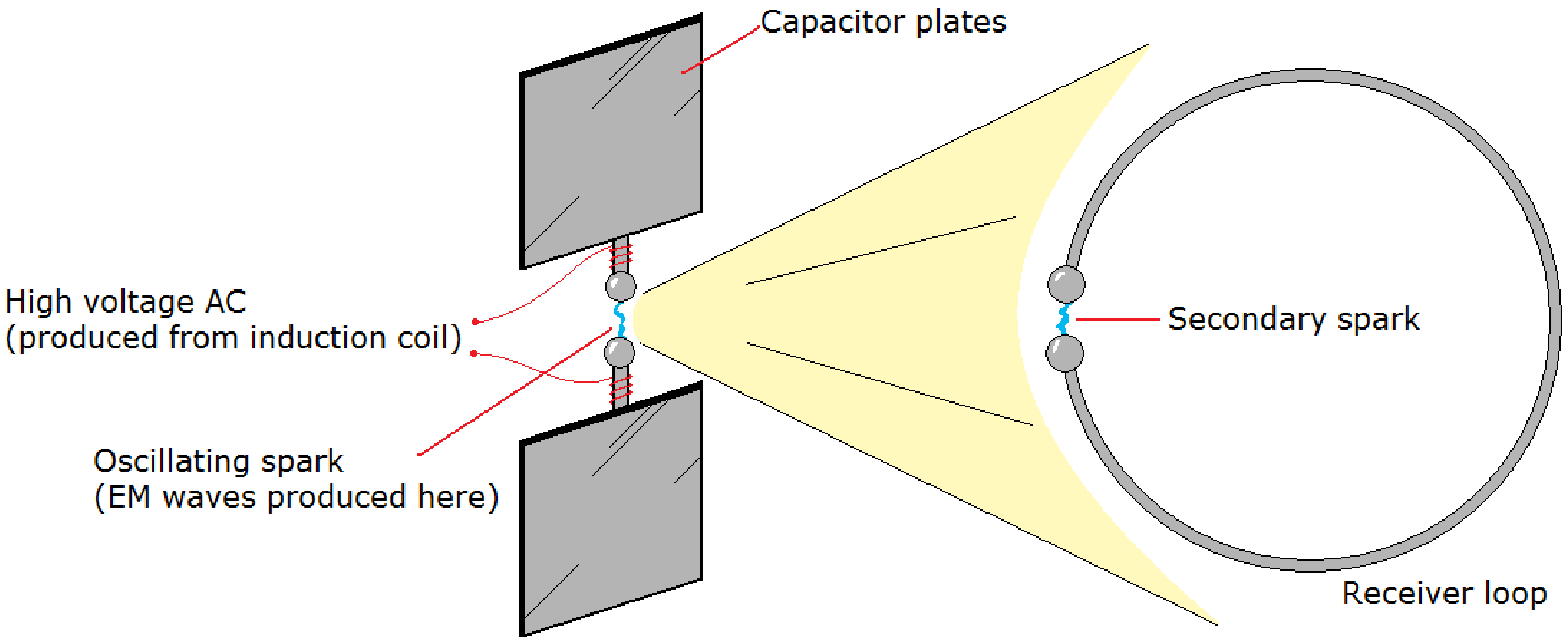
Constructive Interference

Screen

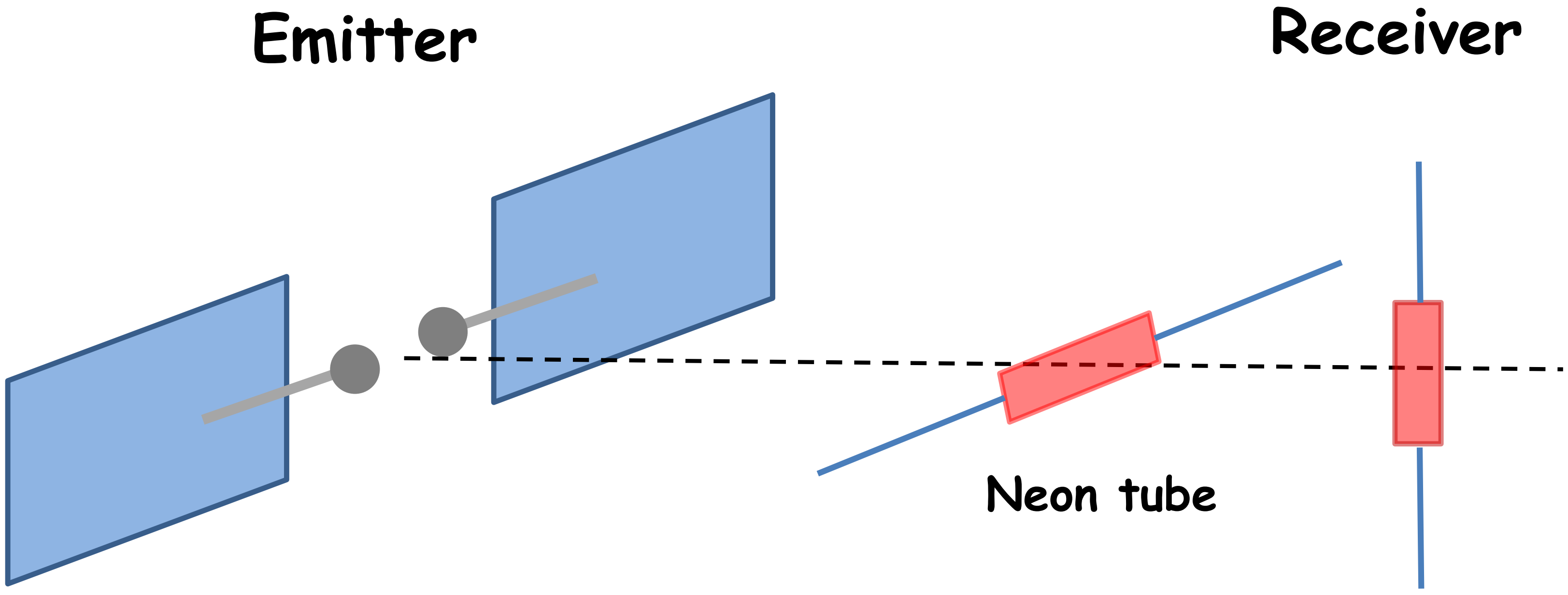


Waves Bend Into Shadow

Hertz experiments



Quiz 1

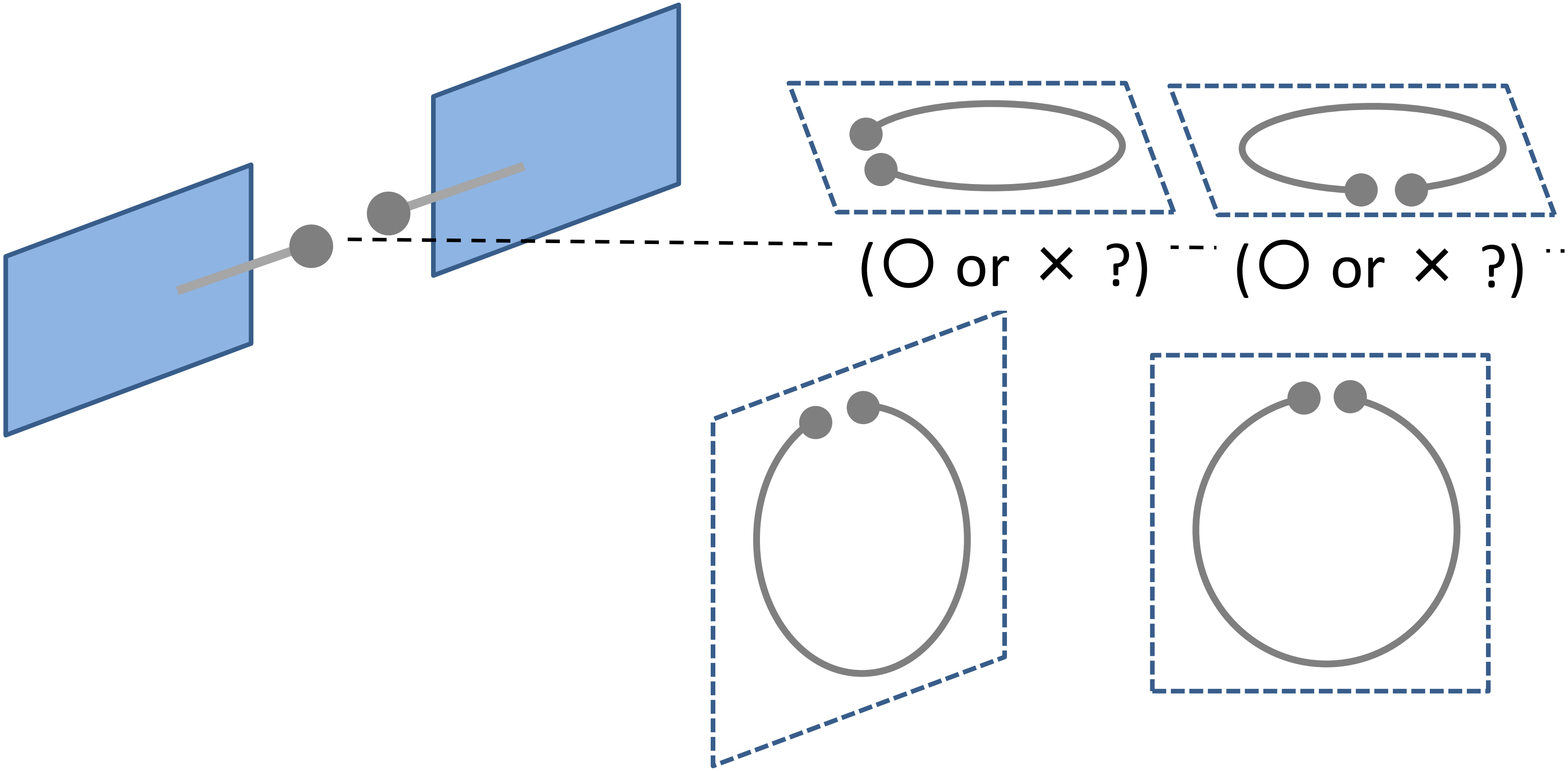


(○ or × ?) (○ or × ?)

Quiz 2

Emitter

Receiver



(○ or × ?)

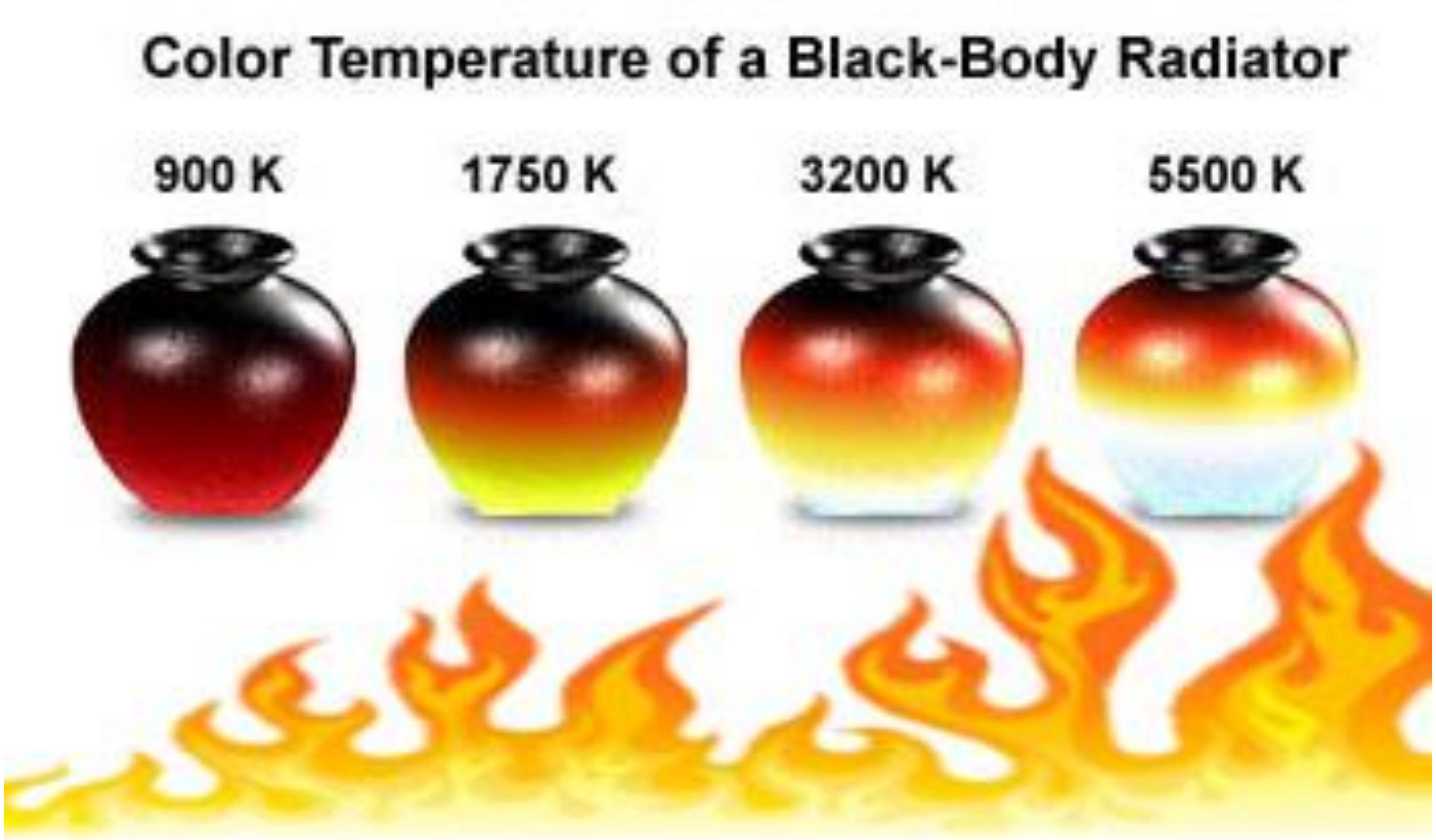
(○ or × ?)

(○ or × ?)

(○ or × ?)

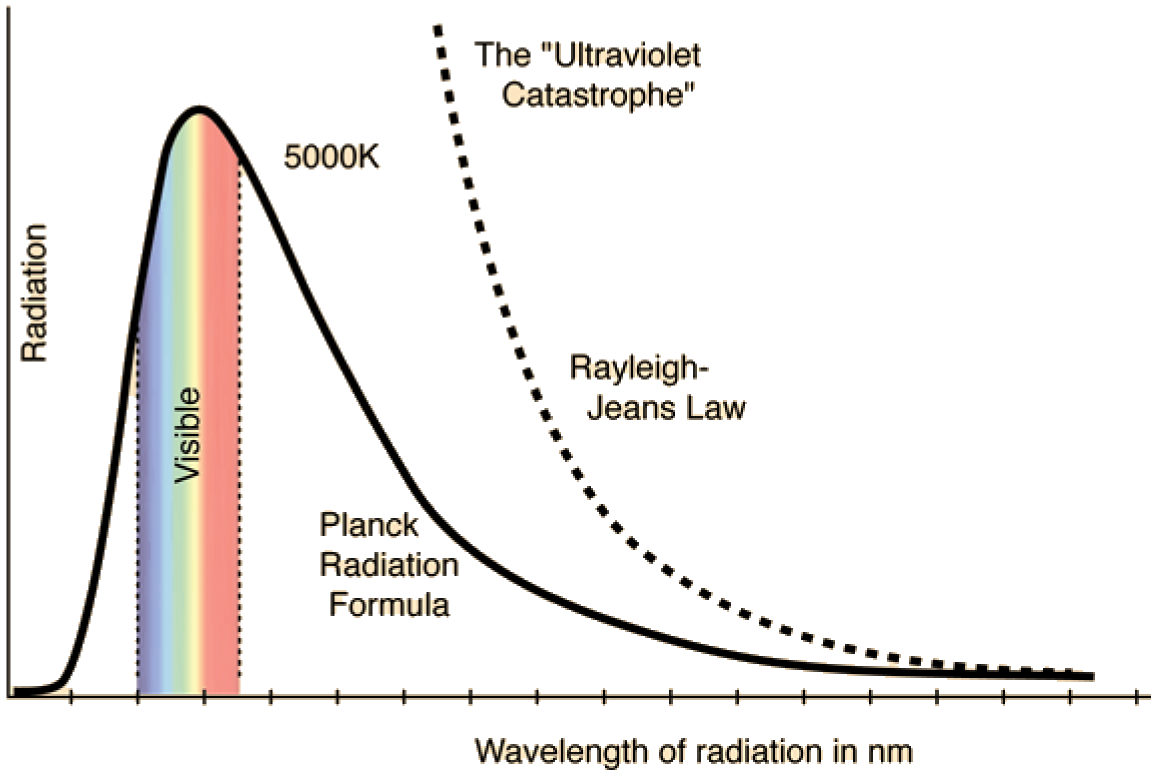
Blast furnace

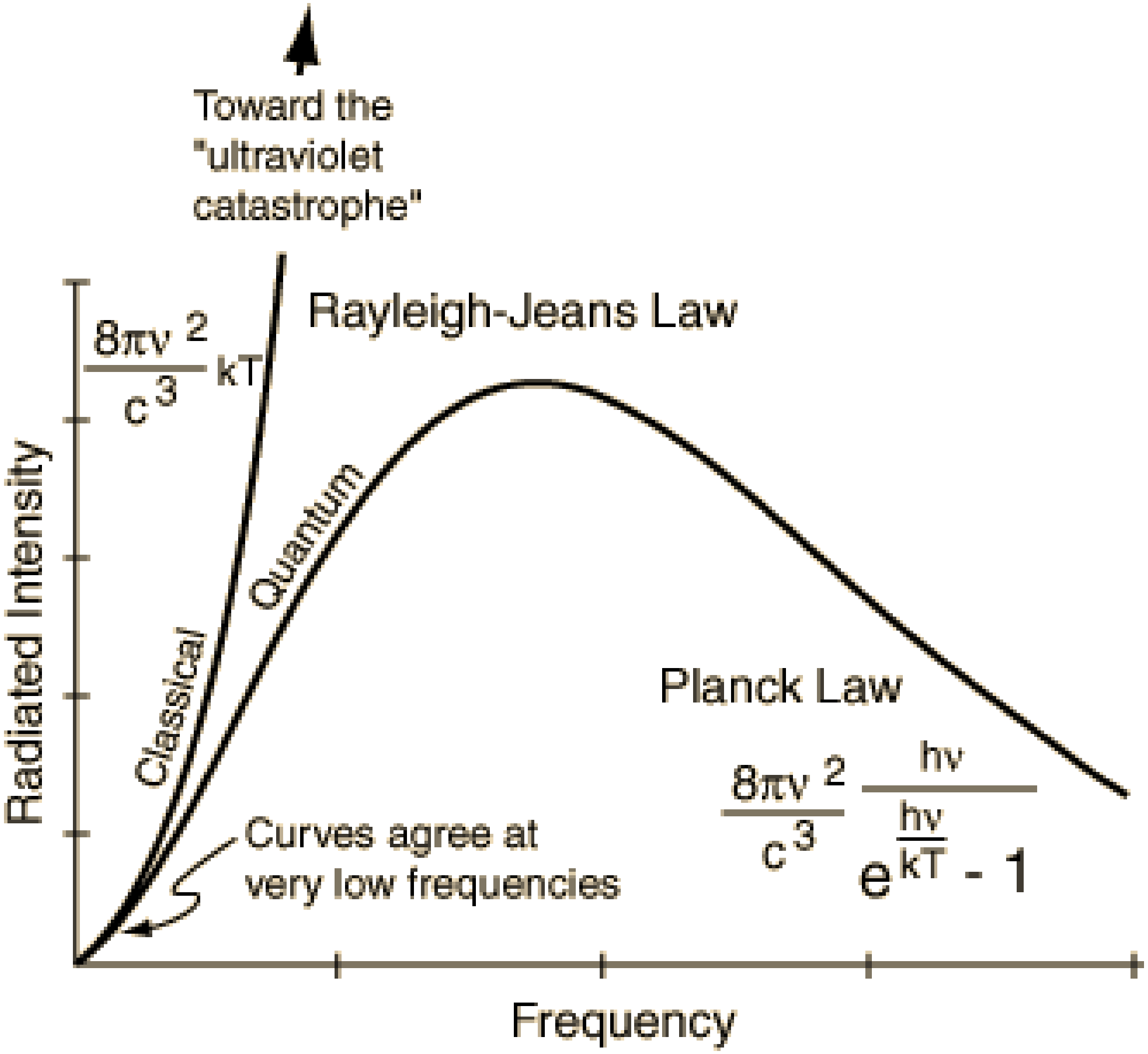
Color and temperature



Ultraviolet catastrophe

The **ultraviolet catastrophe**, also called the **Rayleigh–Jeans catastrophe**, was the prediction of late 19th century/early 20th century classical physics that an ideal blackbody at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases.

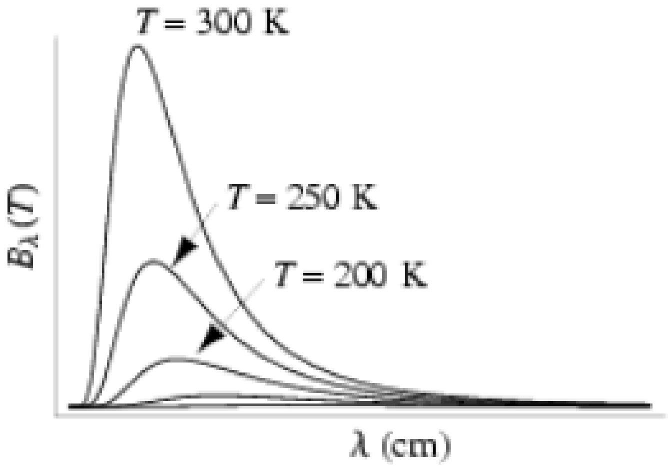



Toward the "ultraviolet catastrophe"
Rayleigh-Jeans Law
$\frac{8\pi\nu^2}{c^3}kT$
Radiated Intensity
Classical
Quantum
Planck Law
$\frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$
Curves agree at very low frequencies
Frequency

We have two forms.

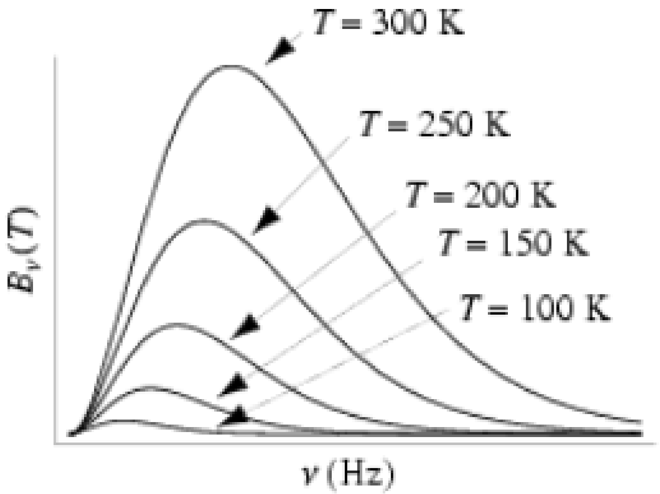
- As a function of wavelength.

$$\boxed{u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{hc}{\lambda kT}}-1}}$$



- And as a function of frequency

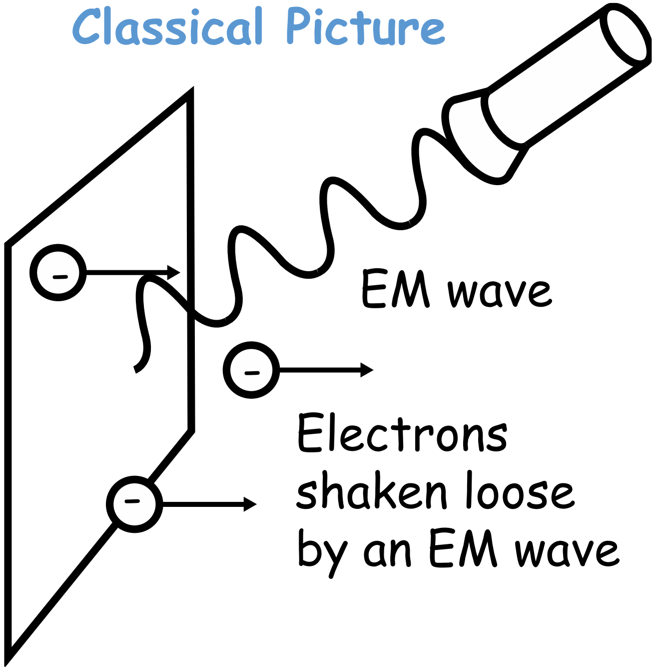
$$\boxed{u_\upsilon d\upsilon = \frac{8\pi h\upsilon^3}{c^3}\frac{d\upsilon}{e^{\frac{h\nu}{kT}}-1}}$$



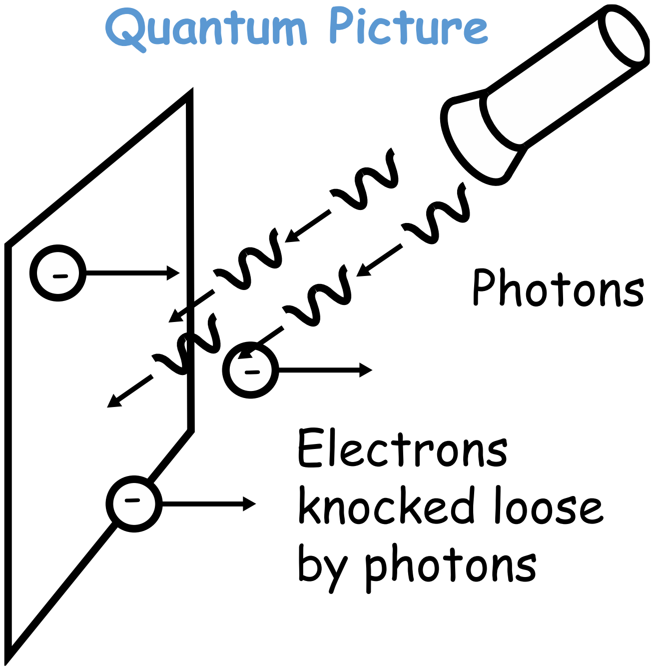
Photoelectric effect

- ✔ When light is incident on certain metallic surfaces, electrons are emitted from the surface.
  - This is called the *photoelectric effect*
  - The emitted electrons are called *photoelectrons*
- ✔ The effect was first discovered by Hertz
- ✔ The successful explanation of the effect was given by Einstein in 1905.
  - Received Nobel Prize in 1921 for paper on electromagnetic radiation, of which the photoelectric effect was a part

Classical Picture

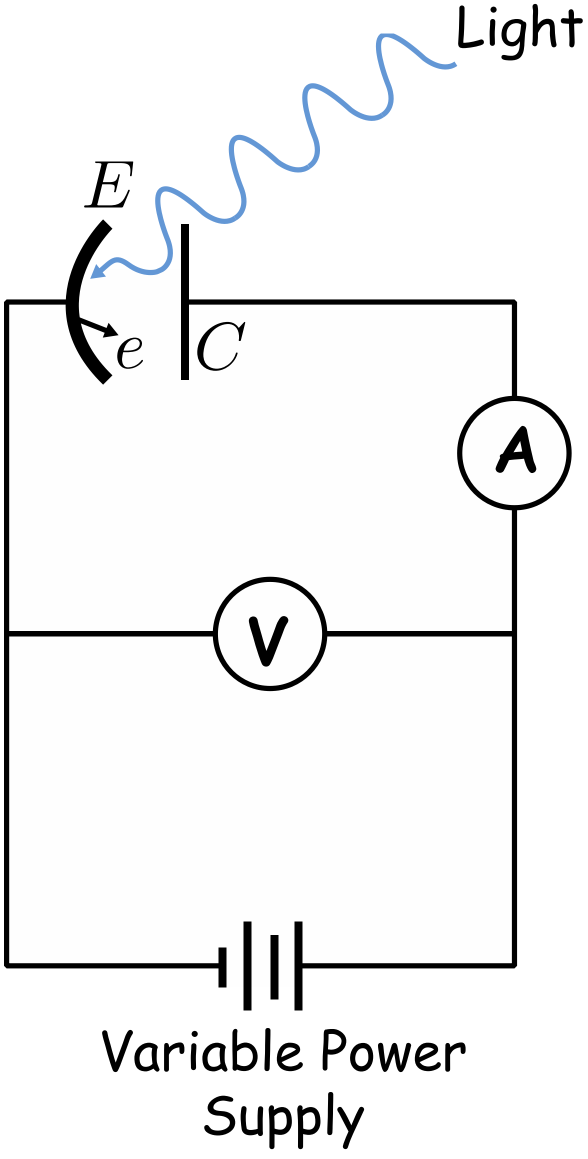


Quantum Picture



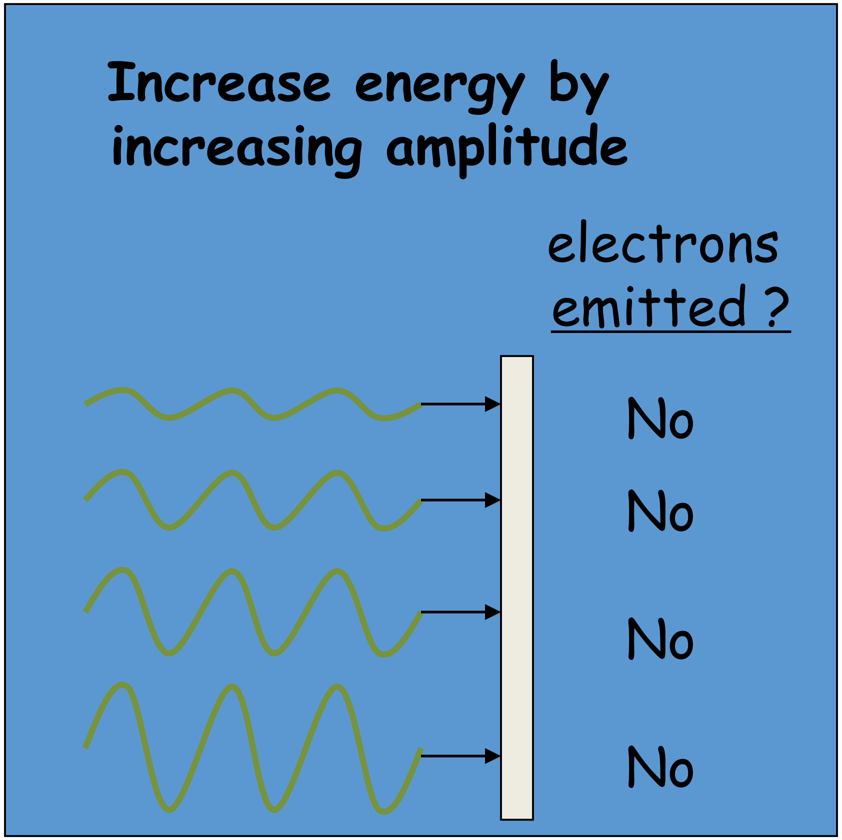
Photoelectric Effect Schematic

- ✔ When light strikes E, photoelectrons are emitted
- ✔ Electrons collected at C and passing through the ammeter are a current in the circuit
- ✔ C is maintained at a positive potential by the power supply



Observation of the Photoelectric Effect … a Quantum Phenomenon

"Classical" Method



Increase energy by increasing amplitude

electrons emitted ?

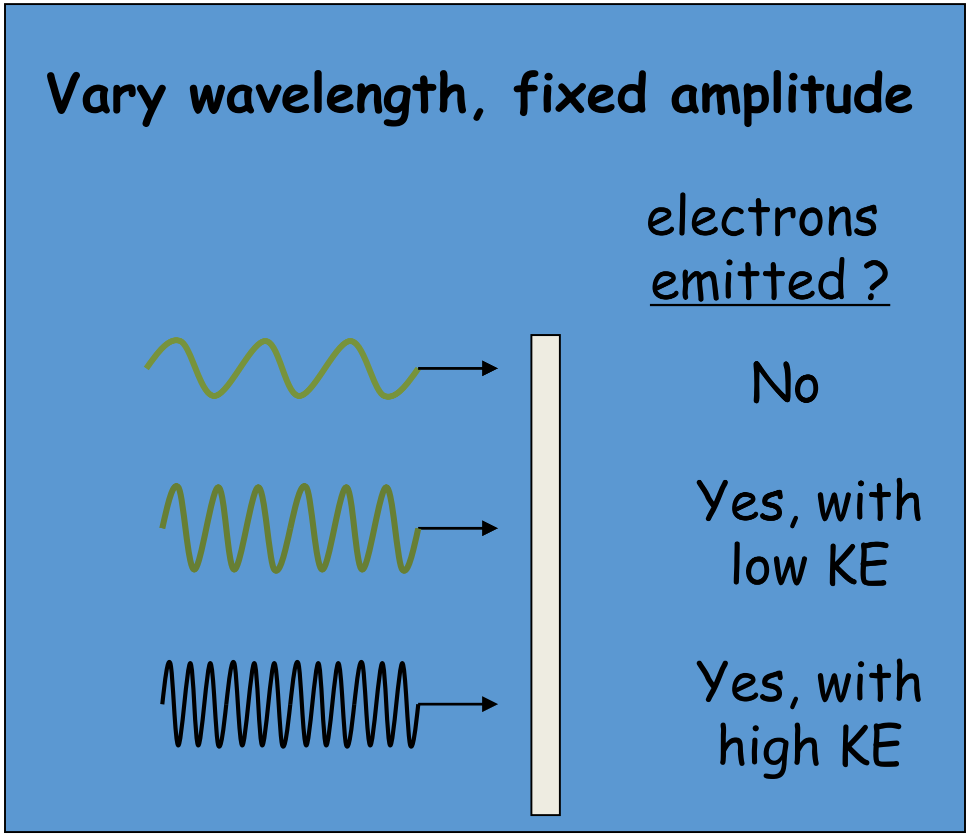
No

No

No

No

What if we try this ?



Vary wavelength, fixed amplitude

electrons emitted ?

No

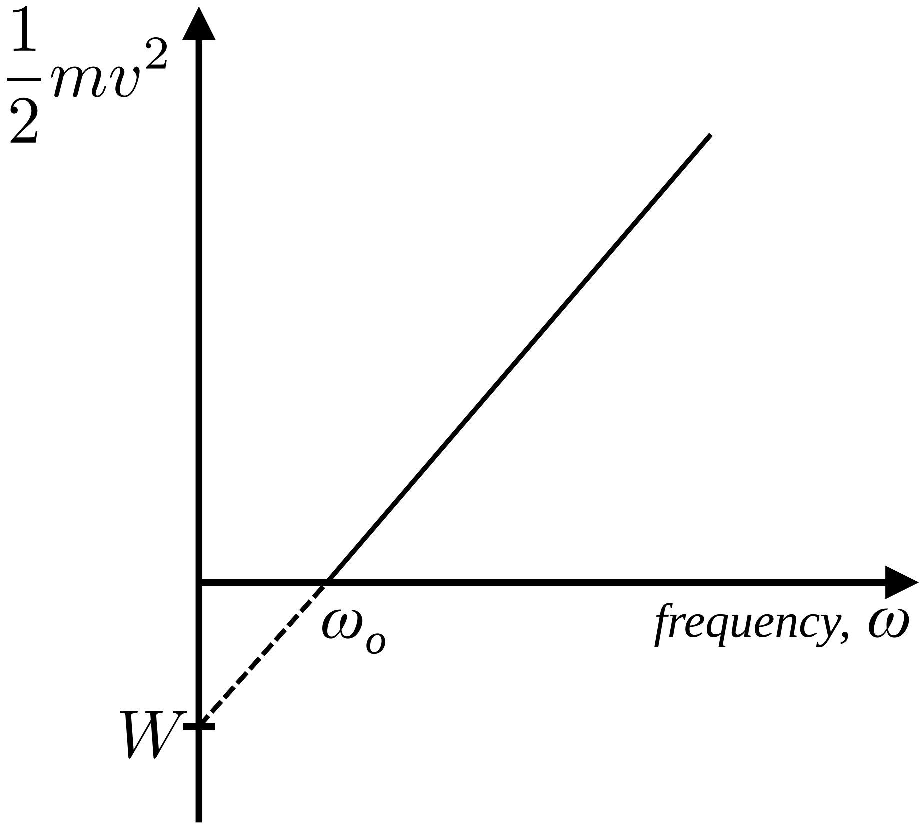
Yes, with low KE

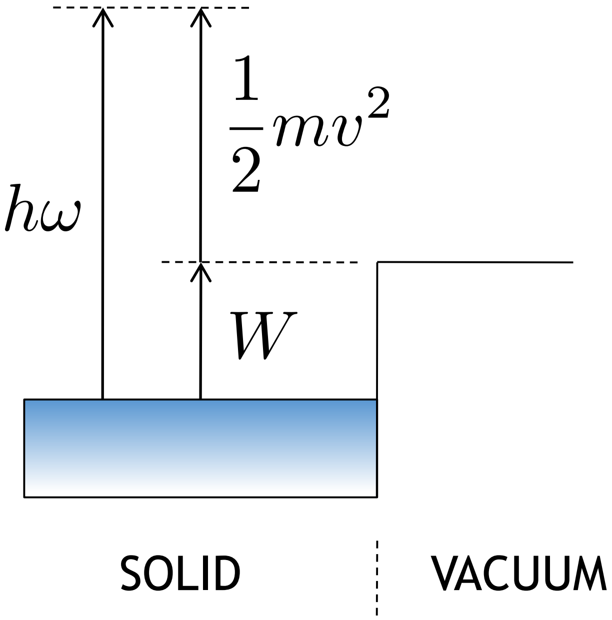
Yes, with high KE

No electrons were emitted until the frequency of the light exceeded a critical frequency, at which point electrons were emitted from the surface !

(Recall: small λ ➔ large ν)

Electron Energy as a Function of Frequency





$$\hbar\omega = W + \frac{1}{2}mv^2$$

PHOTON ENERGY

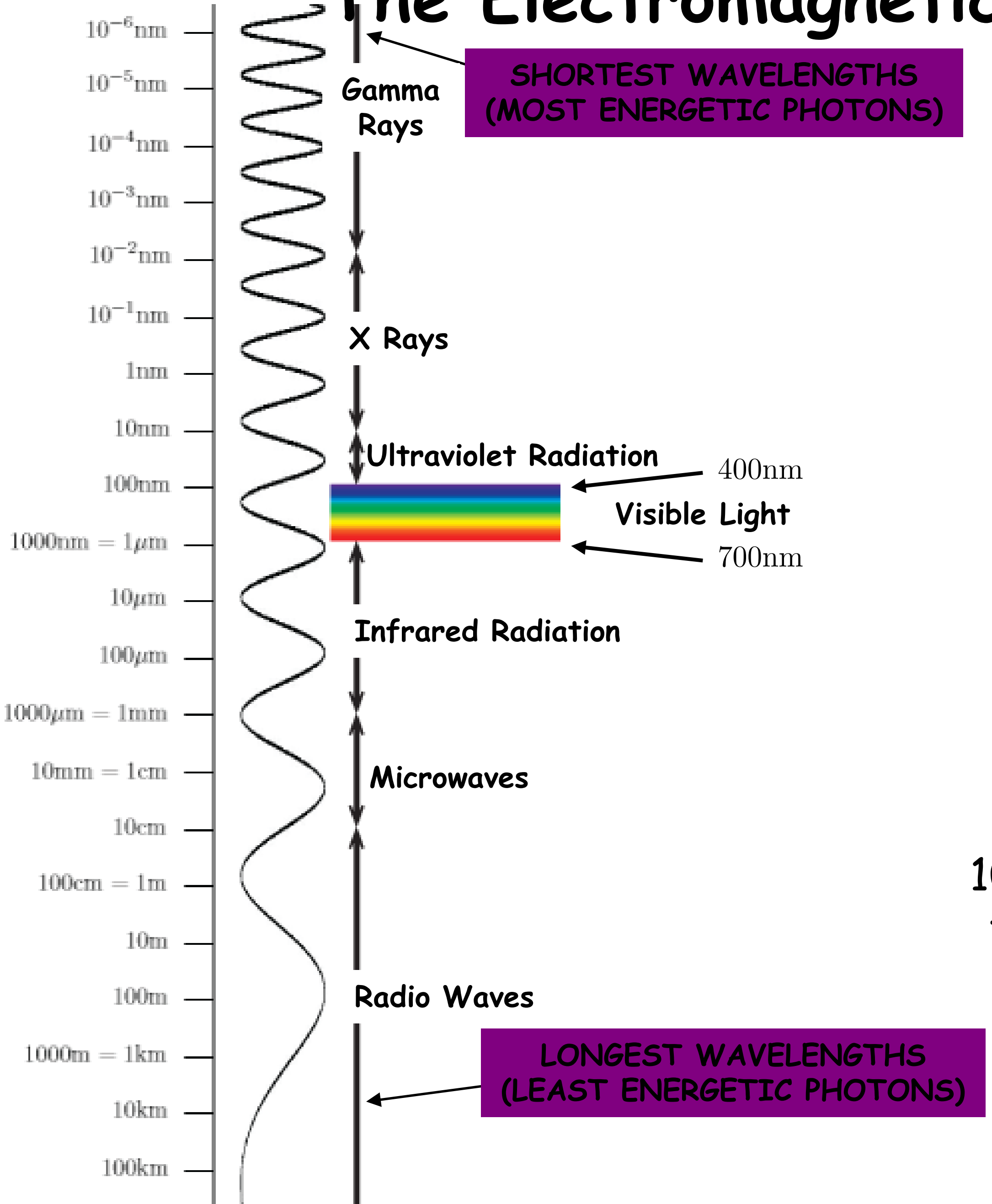
BINDING ENERGY OF ELECTRON

ELECTRON KINETIC ENERGY

$$h = 6.626 \times 10^{-34}\,[J \cdot s]$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\,[J \cdot s]$$

The Electromagnetic Spectrum



SHORTEST WAVELENGTHS
(MOST ENERGETIC PHOTONS)

LONGEST WAVELENGTHS
(LEAST ENERGETIC PHOTONS)

According to **quantum theory**, a **photon** has an **energy** given by

$$E = h\nu = \frac{hc}{\lambda} = \hbar\omega$$

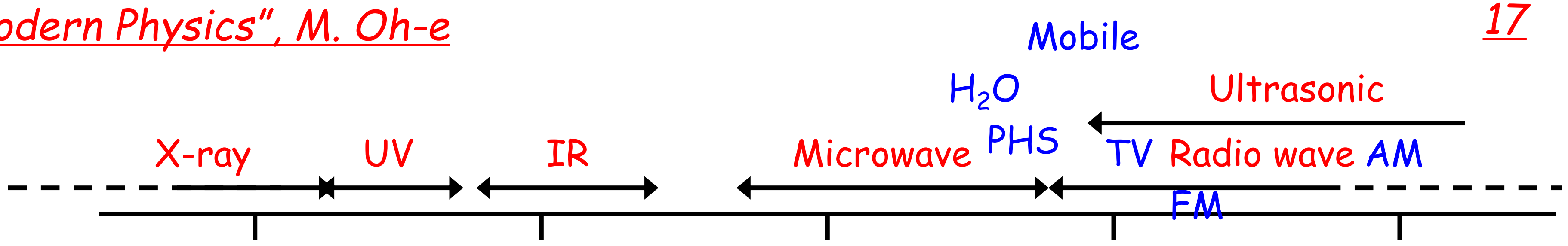
$$h = 6.6 \times 10^{-34}\,[\mathrm{J \cdot s}]$$

(Planck's constant)

$$\hbar = 1.05 \times 10^{-34}\,[\mathrm{J \cdot s}]$$

10 photons have an energy equal to ten times that of a single photon

$$E[eV] = \frac{1239.84}{\lambda[nm]}$$



Mobile

H_2O

PHS

Ultrasonic

X-ray

UV

IR

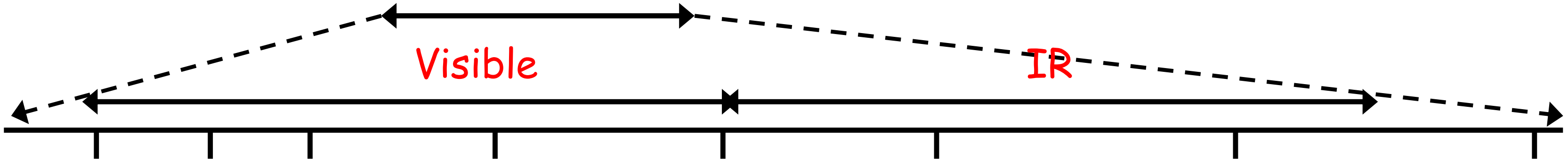
Microwave

TV Radio wave AM

FM

2.45GHz

λ	1nm	1um	1mm	1m	1km
ν	300PHz	300THz	300GHz	300MHz	300kHz
E	1.24keV	1.24eV	1.24meV	1.24ueV	1.24neV
K	10,000kcm^{-1}	10,000cm^{-1}	10 cm^{-1}	10mcm^{-1}	10ucm^{-1}



Visible

IR

λ	200	380	400	450	550	780	1um	3.3um	10um	100um
ν X10^{14}		7.8	7.5	6.6	5.5	3.8	300THz		30THz	3THz
E		3.3	3.1	2.8	2.3	1.6	1.24		0.124	0.0124
K	50000		25000				10,000	3000	1000	100

$$
\begin{aligned}
1\ \mathrm{eV} &= 1.6022\times10^{-19}\ \mathrm{J}\\
&= 8065.5\ \mathrm{cm}^{-1}\\
&= 2.148\times10^{14}\ \mathrm{Hz}\\
&= 1.24\ \mu\mathrm{m}\\
&= 11600\ \mathrm{K}
\end{aligned}
$$

$$
\begin{aligned}
1\ \mathrm{cm}^{-1} &= 1.9865\times10^{-23}\ \mathrm{J}\\
&= 0.124\ \mathrm{meV}\\
&= 30\ \mathrm{GHz}\\
&= 1.43822\ \mathrm{K}
\end{aligned}
$$

$$1\ \mathrm{meV} = 8\ \mathrm{cm}^{-1}$$

$$\omega = 2\pi\nu = \sqrt{\frac{k}{m}}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\lambda = 2\pi c/\omega$$

$$\nu\ [10^{15}\mathrm{Hz}] = 300/\lambda[\mathrm{nm}] \qquad E\ [\mathrm{keV}] = 1.24/\lambda[\mathrm{nm}]$$

Thermal energy at room temperature

$$
\begin{aligned}
k_B T &= 4\times10^{-21}\ \mathrm{J}\\
&= 25\ \mathrm{meV}\\
&= 200\ \mathrm{cm}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
k_B T &= 1.053\times10^{-21}\ \mathrm{J}\\
&= 6.625\ \mathrm{meV}\\
&= 53\ \mathrm{cm}^{-1} \quad @77\mathrm{K}
\end{aligned}
$$

Photon energy

$$
\begin{aligned}
\hbar\omega &= 1.05\times10^{-34}\ J\cdot s\times 3\times10^{15}\ s^{-1}\\
&\approx 3\times10^{-19}\ J\\
&\approx 3\times10^{-12}\ erg
\end{aligned}
$$

Intensity

Classical Intensity

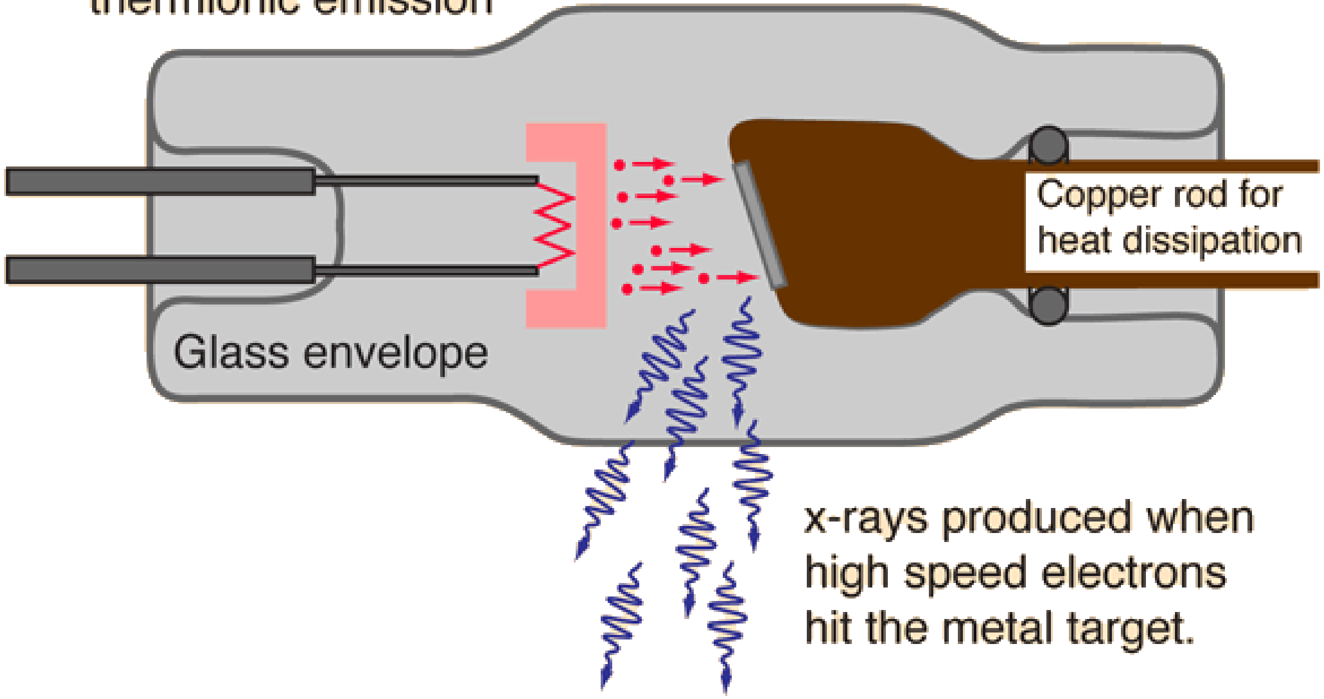
$$\vec{S} = \vec{E} \times \vec{H} \quad \Longrightarrow \quad \frac{\mathrm{Watts}}{\mathrm{cm}^2}$$

Intensity in terms of Photons

$$\left|\vec{S}\right| = \frac{n\hbar\omega}{\tau A} \quad \Longrightarrow \quad \frac{\mathrm{photons}}{\mathrm{sec\ cm}^2}\,\frac{\mathrm{J}}{\mathrm{photon}} = \frac{\mathrm{Watts}}{\mathrm{cm}^2}$$

Generation of X-ray

Inverse phenomena of the photoelectric effect

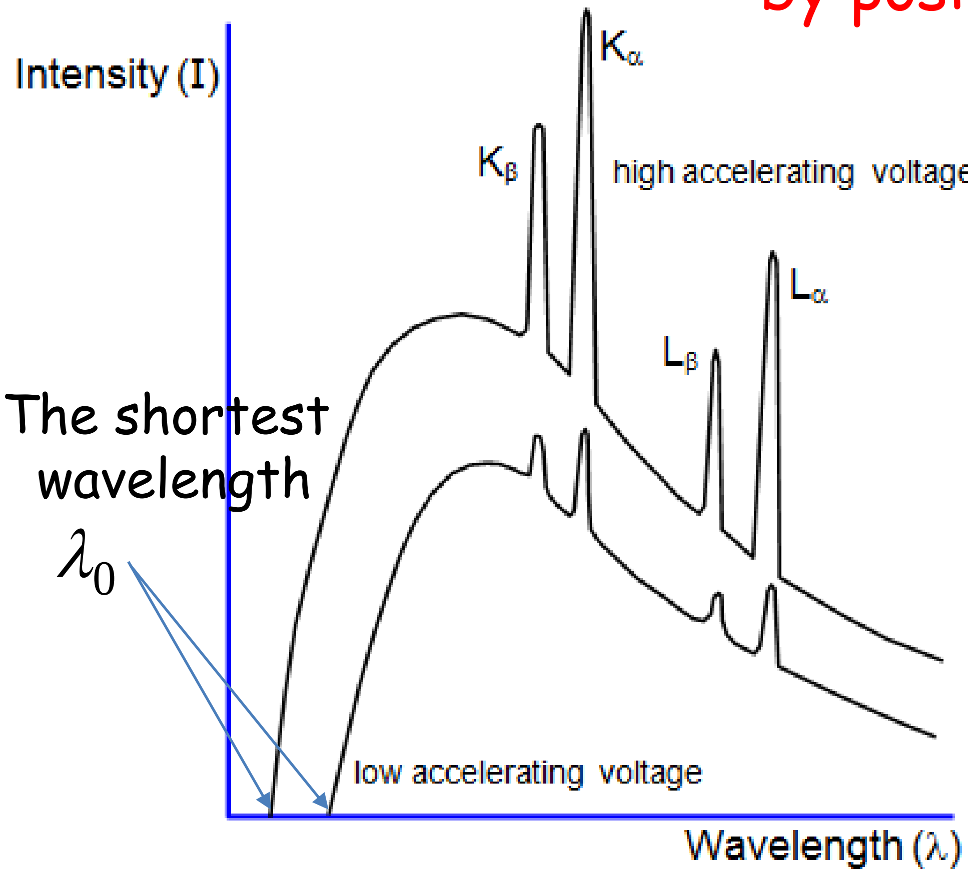


Heated filament emits electrons by thermionic emission

Electrons are accelerated by a high voltage.

x-rays produced when high speed electrons hit the metal target.

X-ray spectrum



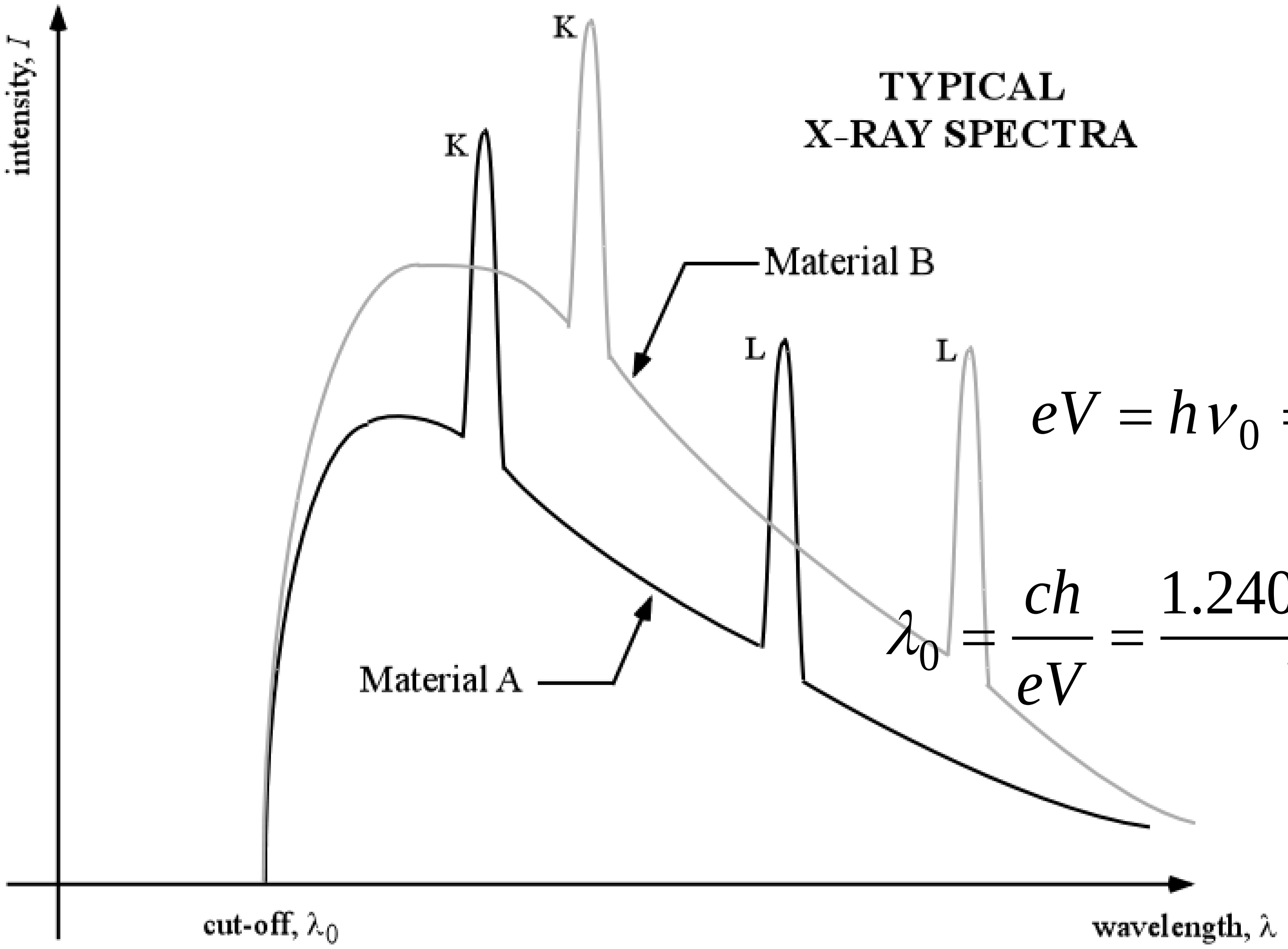
✔ Incident electrons are decelerated by positive nuclei in the anode.

✔ Some of the KE is converted into electromagnetic photons. This is known as the braking radiation.

✔ The photons are continuous range of energies.

$$eV = h\nu_0 = \frac{ch}{\lambda_0}$$

$$\lambda_0 = \frac{ch}{eV} = \frac{1.240\times10^{-6}}{V} \quad (m)$$

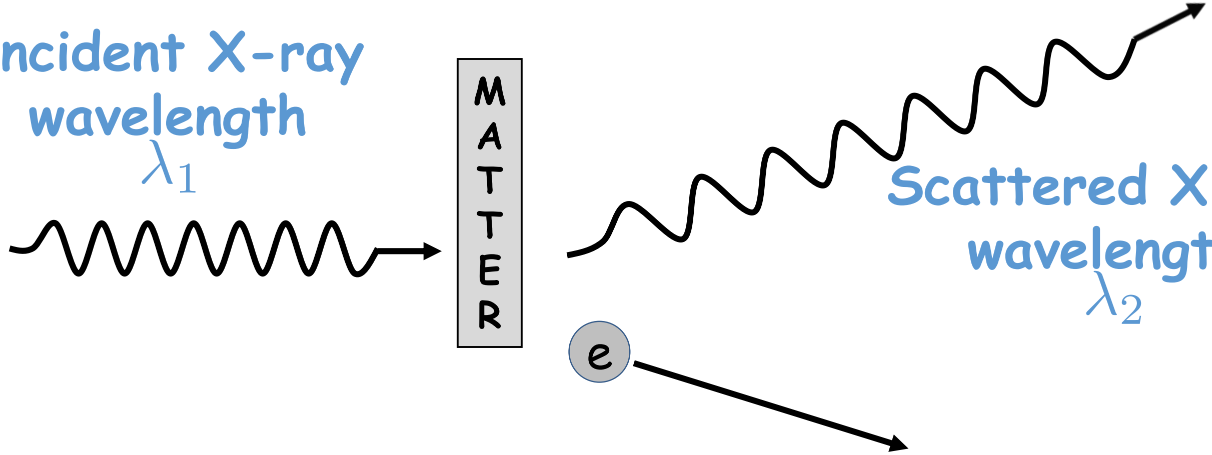


$$eV = h\nu_0 = \frac{ch}{\lambda_0}$$

$$\lambda_0 = \frac{ch}{eV} = \frac{1.240\times10^{-6}}{V} \quad (m)$$

The Compton Effect

In 1924, A. H. Compton performed an experiment where X-rays impinged on matter, and he measured the scattered radiation.



Incident X-ray wavelength λ_1

Scattered X-ray wavelength λ_2

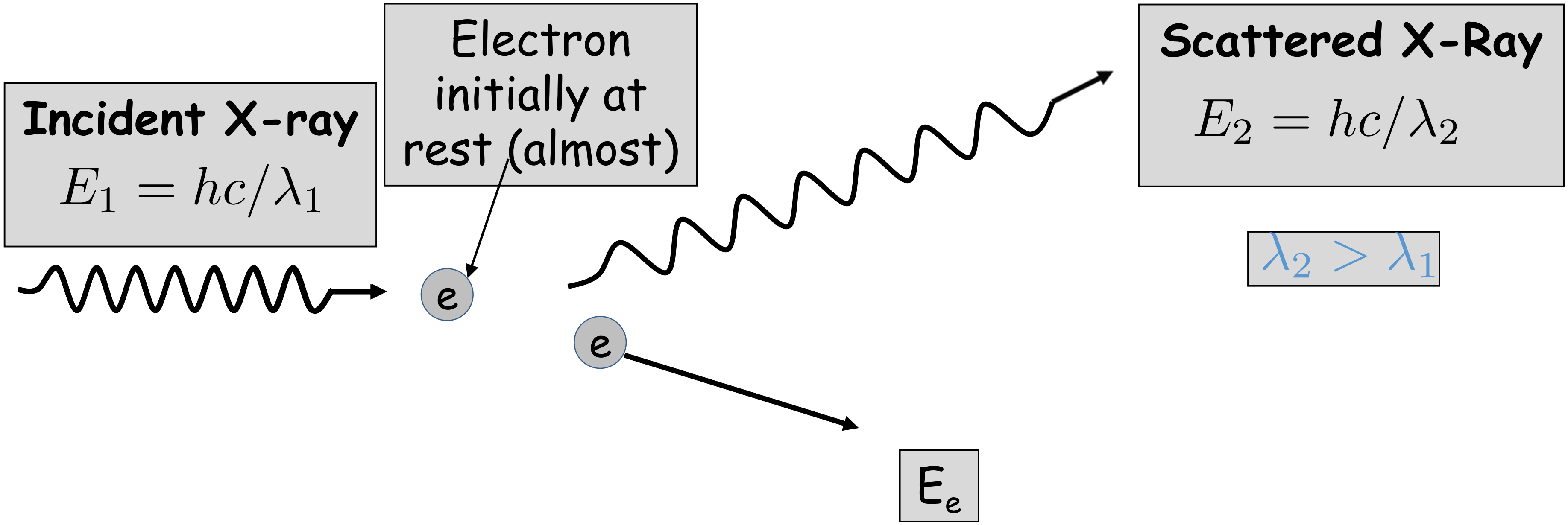
$\lambda_1 > \lambda_2$

Electron comes flying out

<u>Problem:</u> According to the wave picture of light, the incident X-ray should give up some of its energy to the electron, and emerge with a lower energy (*i.e.*, the amplitude is lower), but should have $\lambda_1 = \lambda_2$.

It was found that the scattered X-ray did not have the same wavelength !

Quantum Picture of light



Incident X-ray

$E_1 = hc/\lambda_1$

Electron initially at rest (almost)

Scattered X-Ray

$E_2 = hc/\lambda_2$

$\lambda_2 > \lambda_1$

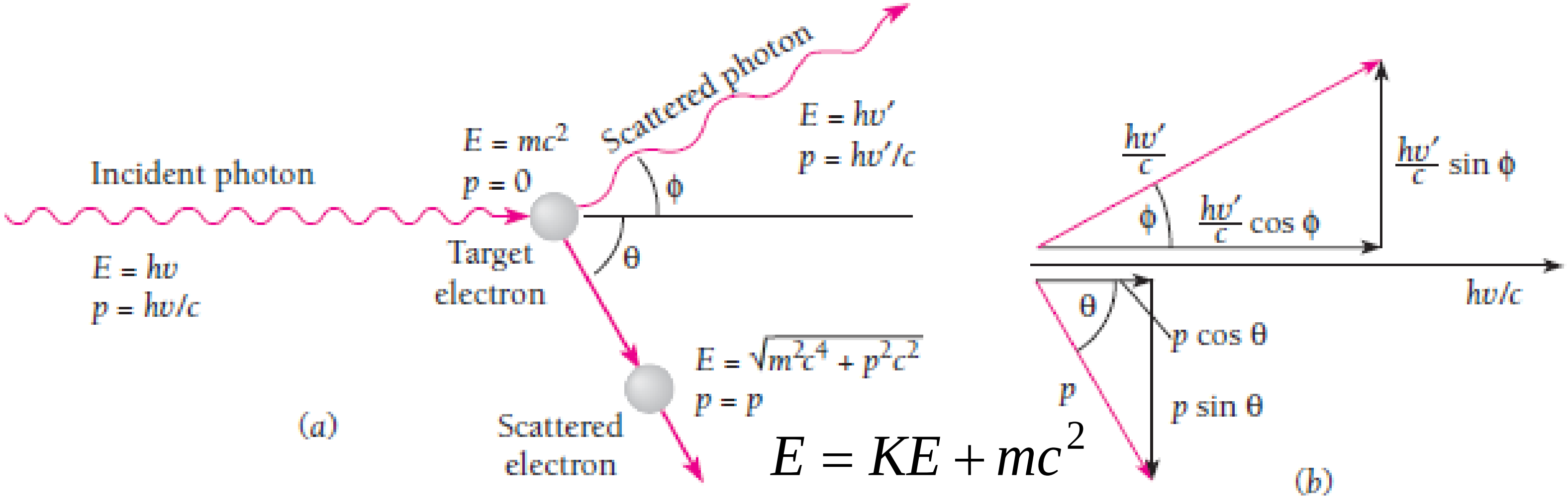
E_e

Compton found that if you treat the photons as if they were particles of zero mass, with energy $E = hc/\lambda$ and momentum $p = \lambda/h$.

➔ The collision behaves just as if it were two billiard balls colliding !

Photon behaves like a particle with energy & momentum as given above!

The Compton Effect



$$E = KE + mc^2$$

$$\frac{ch}{\lambda} + mc^2 = \frac{ch}{\lambda'} + \sqrt{m^2c^4 + p_e^2c^2}$$

Energy conservation

$$h\nu - h\nu' = KE$$

Gain in electron energy

$$\left.\begin{aligned} \frac{h}{\lambda} &= \frac{h}{\lambda'}\cos\phi + p_e\cos\theta \\ \frac{h}{\lambda'}\sin\phi &= p_e\sin\theta \end{aligned}\right\}$$

Momentum conservation

$$\Rightarrow \quad \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Photon momentum

IN FREE SPACE:

$$E = cp \quad \Rightarrow \quad p = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar k$$

IN OPTICAL MATERIALS:

$$E = v_p p \quad \Rightarrow \quad p = \frac{E}{v_p} = \frac{\hbar\omega}{v_p} = \hbar k_{vac} n$$