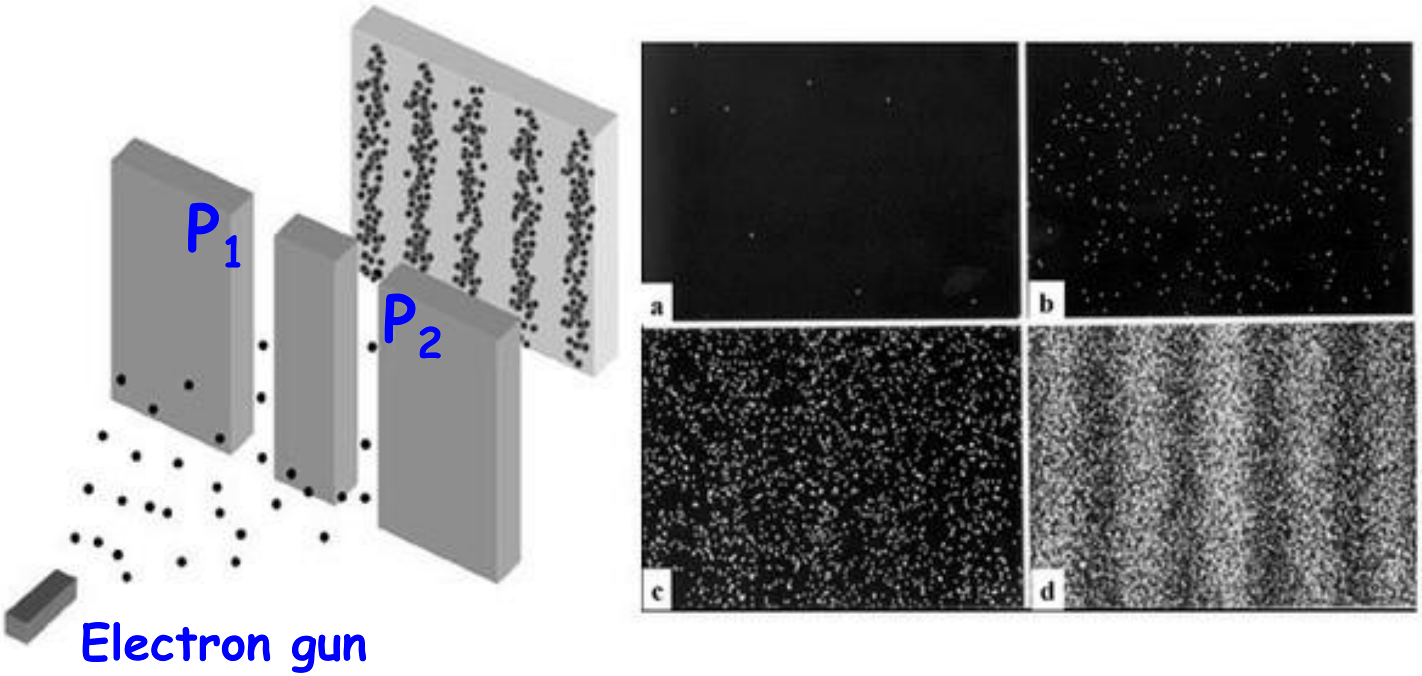
Duality of electrons



Probability that electrons reach position x ≠ $P_1 + P_2$

$$\Psi = \psi_1 + \psi_2$$

$$|\Psi|^2 = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_1 \psi_2^*$$

de Broglie wave

Momentum of a photon:

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \qquad c = \lambda\nu$$

Photon wavelength:

$$\lambda = \frac{h}{p}$$

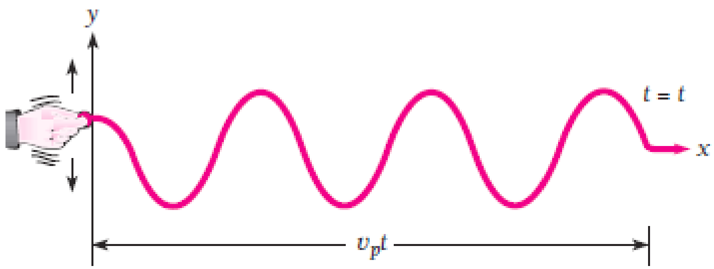
de Broglie wavelength:

$$\Rightarrow \qquad \lambda = \frac{h}{\gamma m_0 v} = \frac{h}{mv} \qquad m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

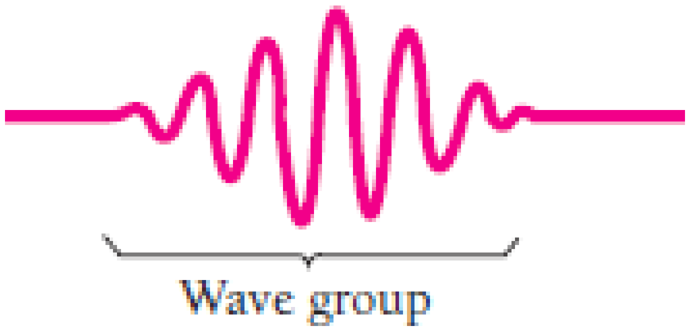
Probability density:

$$P = \left|\psi\left(\vec{r}, t\right)\right|^2$$

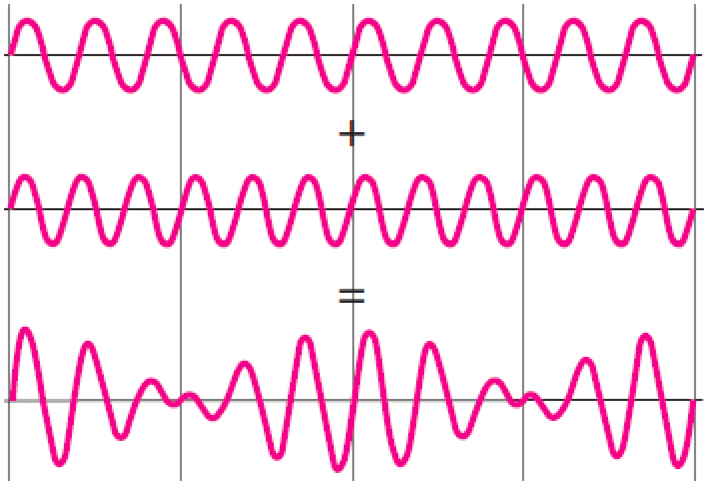
Infinite waves and wave packets



✔ De Broglie waves cannot be represented by infinite waves.



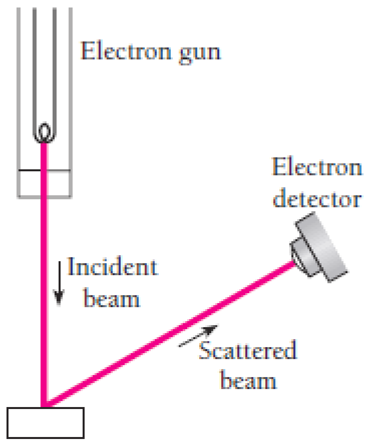
✔ Amplitudes of wave packets depend on likelihood of detecting a body.

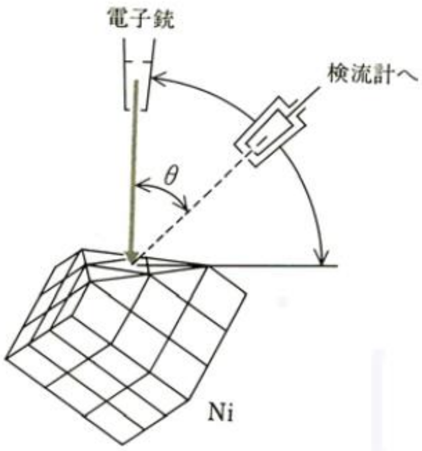


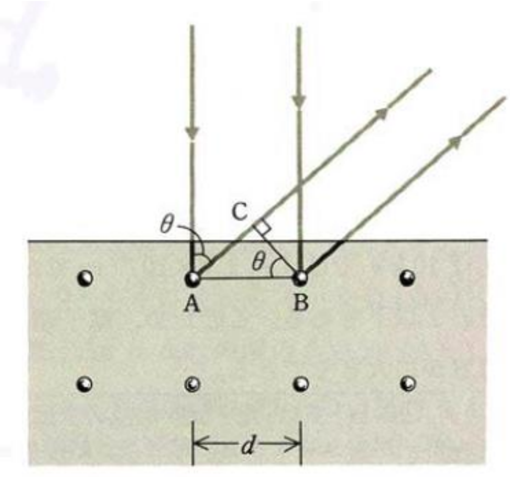
✔ A typical example of how wave packets come into being.

Electron diffraction

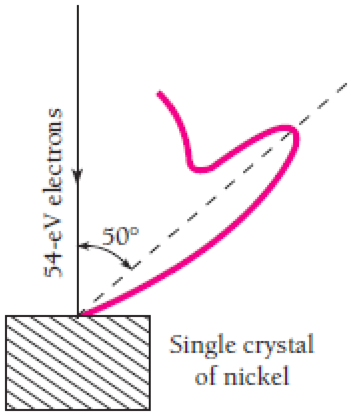
An experiment that confirms the existence of de Broglie waves





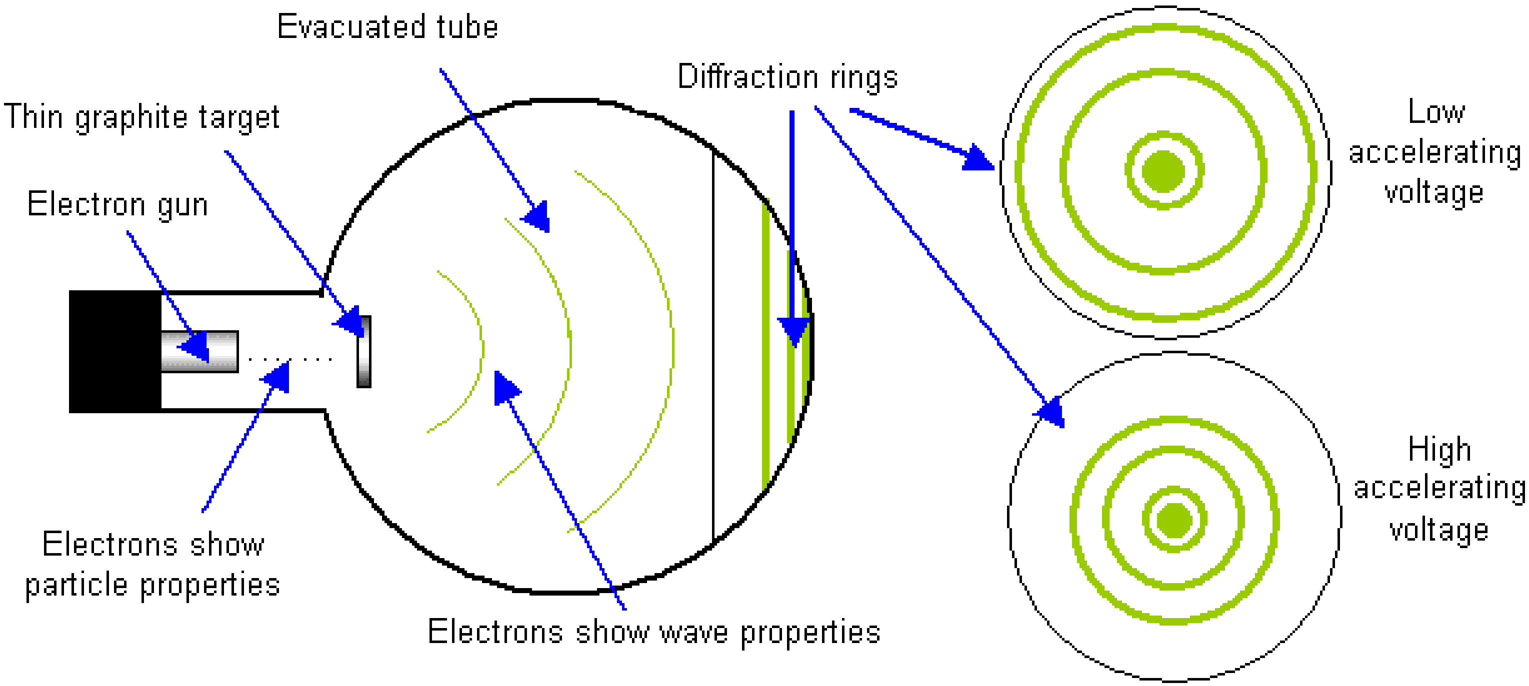


$$n\lambda = d \sin\theta$$



$$eV = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \qquad \lambda = \frac{h}{\sqrt{2meV}}$$

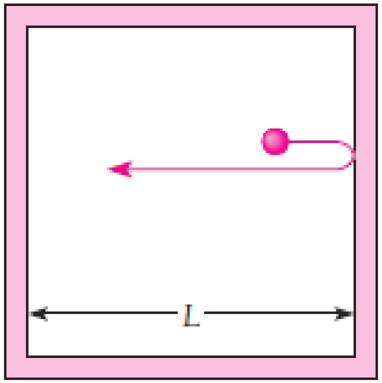
Quiz

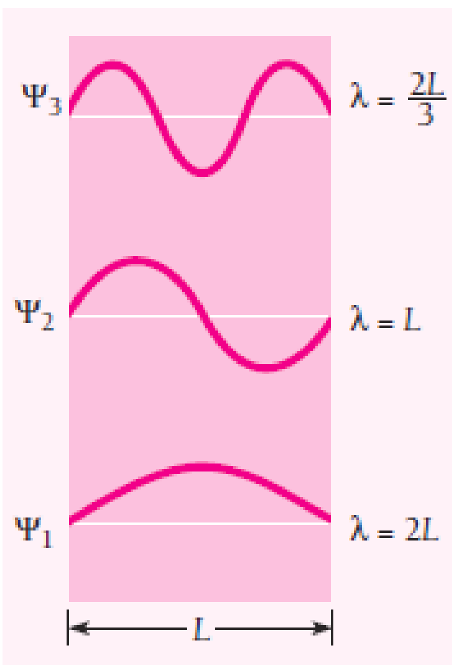


Why?

Hint: $$\lambda = \frac{h}{p}$$

Particle in a box





Assumption: no dissipation of energy
no relativistic considerations

de Broglie wavelength of trapped particles: $\lambda_n = \dfrac{2L}{n} \quad n = 1,2,3\cdots$

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Energy level E_n is expressed by,

$$E_n = \frac{n^2h^2}{8mL^2} \quad n = 1, 2, 3, \cdots$$

n: quantum number

Any particle confined to a certain region

Quantized energies for particle in a box:

$$E_n = \frac{n^2 h^2}{8mL^2} \quad n = 1, 2, 3, \cdots$$

n: quantum number

✔ A trapped particle cannot have an arbitrary energy, as a free particle can.

✔ A trapped particle cannot have zero energy.

$$\lambda = \frac{h}{mv}, \quad v = 0 \quad \Rightarrow \quad \lambda \to \infty$$

No meaning!

✔ Planck's constant h=6.63x10^{-34}J·s is so small,
→ Quantization of energy is conspicuous
only when m and L are so small .

Two contrast examples

An electron in a box 0.10 nm across.

$$m = 9.1\times10^{-31}\,kg,\quad L = 0.10\ nm = 1.0\times10^{-10}\,m$$

$$E_n = \frac{n^2(6.63\times10^{-34})^2}{8*(9.1\times10^{-31}\,kg)(1.0\times10^{-10}\,m)^2} = 6.0\times10^{-18}n^2\ J$$

$$= 38n^2\ eV \qquad\qquad (n = 1, 2, 3, \cdots)$$

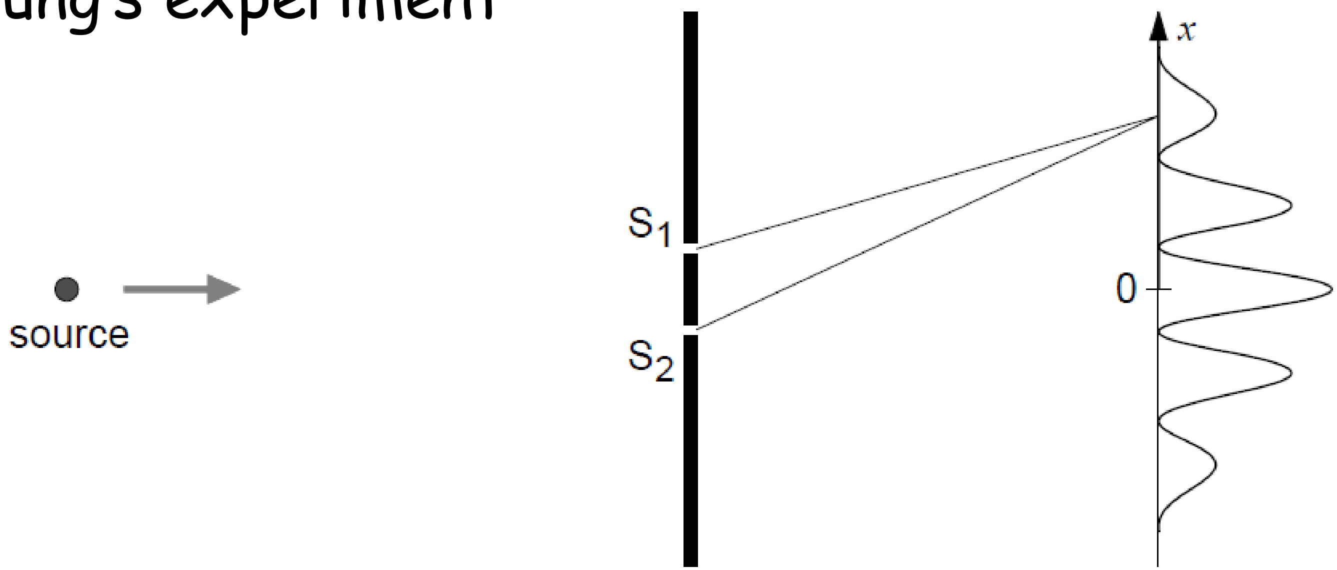
A 10-g marble in a box 10 cm across.

$$m = 1.0\times10^{-2}\,kg,\quad L = 10\ cm = 1.0\times10^{-1}\,m$$

$$E_n = \frac{n^2(6.63\times10^{-34})^2}{8*(1.0\times10^{-2}\,kg)(1.0\times10^{-1}\,m)^2} = 5.5\times10^{-64}n^2\ J \qquad\qquad (n = 1, 2, 3, \cdots)$$

Uncertainty principle

Young's experiment

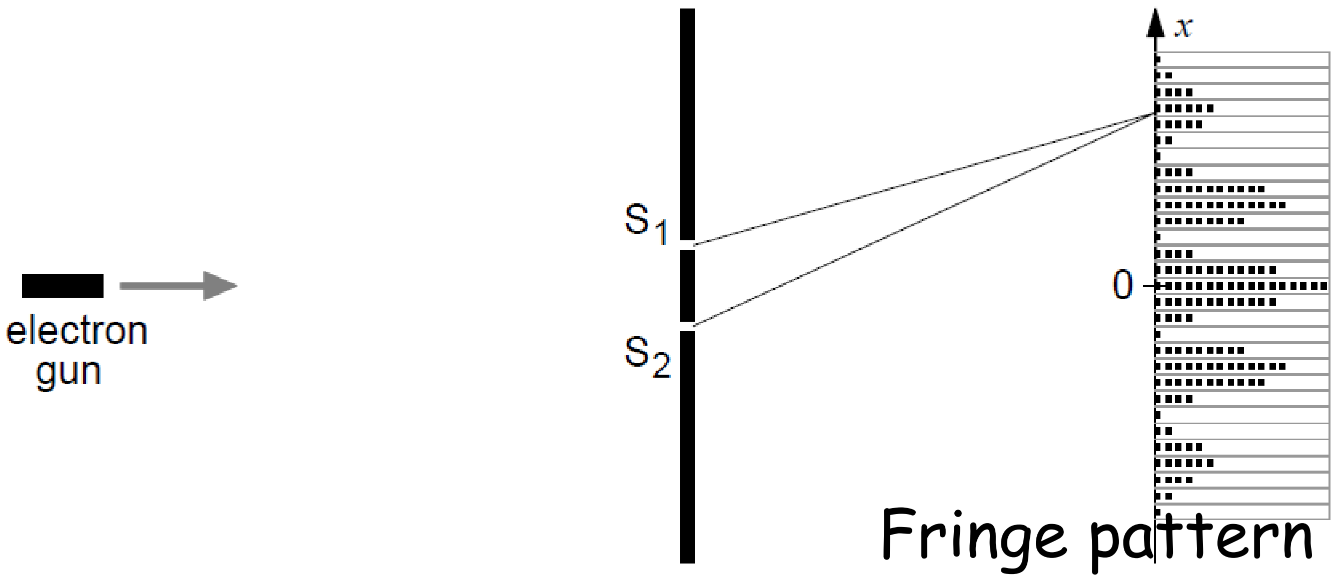


$$\Delta D = \left| \left[L^2 + \left(x + \frac{d}{2} \right)^2 \right]^{1/2} - \left[L^2 + \left(x - \frac{d}{2} \right)^2 \right]^{1/2} \right|$$

$$x = \frac{nL\lambda}{d} \quad (L >> d, x)$$

Instead of the light source and the screen , we use an electron gun and a photographic plate or electron counter.

Double slit experiment by electrons



Fringe pattern
in contrasting density

Even in the case that a single electron is emitted one by one, a fringe pattern can be observable.

Which slit an electron passes though does not mean anything in the quantum world.

Classical and quantum physics

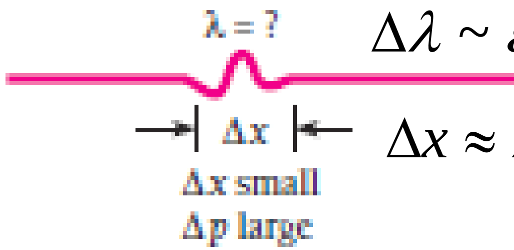
Classical physics:

The state of a particle:
position and velocity (position and momentum)
cf. equation of motion

Quantum physics:

The state of an electron:
Exact propagation path of electrons cannot be determined.
Only probability of position and momentum
(Quantum state)

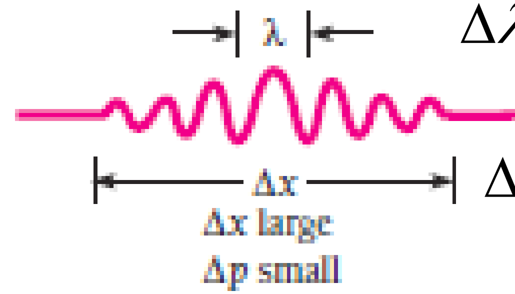
What is uncertainty?



$\Delta\lambda \sim \varepsilon\lambda$

$\Delta x \approx \lambda$

$\Delta x \Delta\lambda \sim \varepsilon\lambda^2$



$\Delta\lambda \sim \varepsilon\lambda / N$

$\Delta x \approx N\lambda$

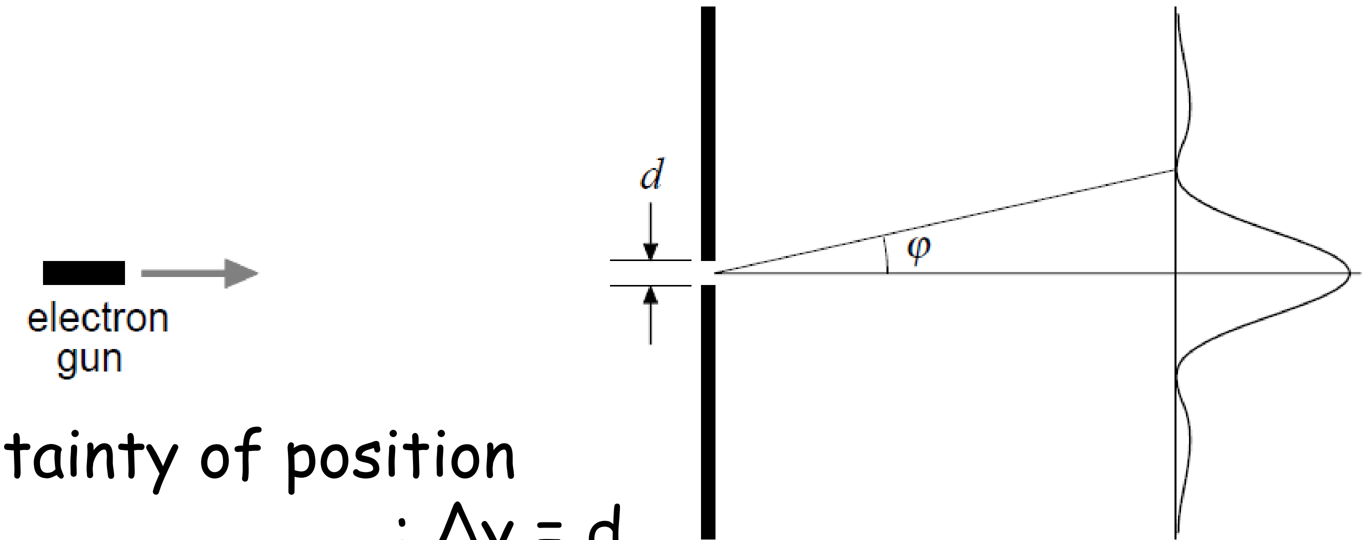
$\Delta x \Delta\lambda \sim \varepsilon\lambda^2$

✔ Narrower wave groups → more precise position $\lambda = \frac{h}{p}$

✔ Wider wave groups → more accurate wavelength
→ more accurate momentum

It is impossible to know both the exact position and exact momentum of an object at the same time.

Uncertainty of position and momentum



Uncertainty of position
: $\Delta y = d$

Suppose an electron is detected by a counter
in the direction φ, the momentum y is described as $p_y = \frac{h}{\lambda}\sin\varphi$

Probability distribution is describe by the diffraction of light.

$$\sin\varphi = \frac{\lambda}{d}$$

Uncertainty of momentum
in the y direction:

$$\Delta p_y = \frac{h}{\lambda}\frac{\lambda}{d} = \frac{h}{d}$$

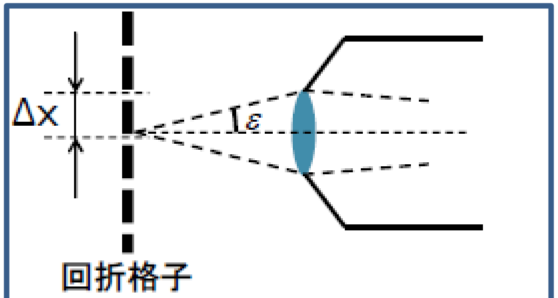
Uncertainty relation: $\Delta y \cdot \Delta p_y \geq h$

Heisenberg's thought experiment

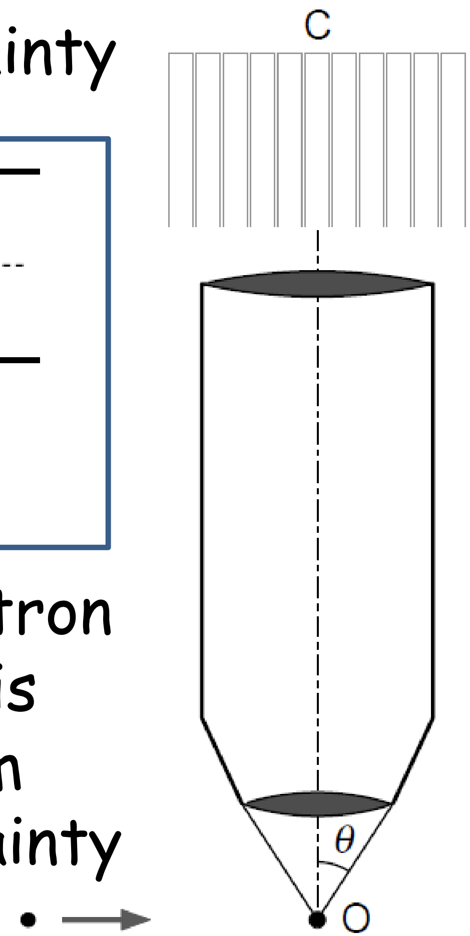
Position x can be determined by uncertainty

$$\Delta x \approx \frac{\lambda}{\sin\theta}$$

(Abbe's resolution)



$$\Delta x \sin\varepsilon = n\lambda$$



Suppose that scattered light by an electron is detected by the counter C. The light is scattered within ±θ with the momentum P=h/λ. So in the x direction, the uncertainty of p is

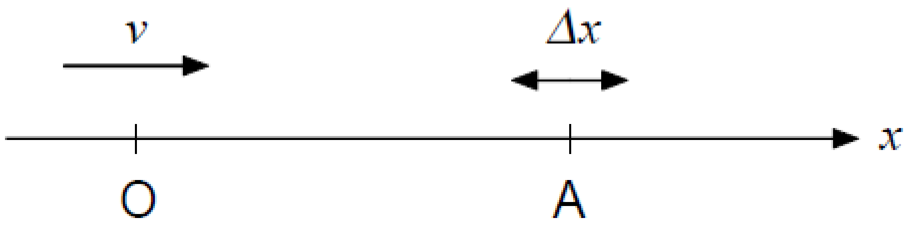
$$\Delta p_x \approx \frac{h}{\lambda}\sin\theta$$

$$\Delta x \cdot \Delta p_x \geq h$$

Uncertainty of time and energy

Time when a particle passes at *A* has uncertainty Δt,

$$\Delta t = \frac{\Delta x}{v}$$



Energy (only the kinetic energy) is $E = \frac{p^2}{2m}$

$$\Delta E = \frac{p}{m}\Delta p$$

$$\Delta E \Delta t = \frac{p}{m}\Delta p \cdot \frac{\Delta x}{v} = \frac{p}{mv}\Delta p \Delta x = \Delta x \Delta p$$

$$\Delta E \cdot \Delta t \geq h$$

Comments

$$\Delta x \cdot \Delta p_x \geq h \qquad \Delta E \cdot \Delta t \geq h$$

✔ This is not a statement about the inaccuracy of measurement instruments, nor a reflection on the quality of experimental methods.

✔ It arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and technique, the uncertainty is inherent in the nature of things.