#### *"Modern Physics", M. Oh-e 1* **Periodic table: how electrons are organized.** *06/07/2018*

 $\vee$  Periodic table: an arrangement of the elements according to atomic number in a series of rows such that elements with similar properties form vertical columns.





**Actinides** 

# **Mean field in an atom**

 $\vee$  A system of particles is stable when its total energy is the minimum.

✔ Only one electron can exist in any particular quatntum state in an atom.

Consider each electron as if it exists in a mean field,



### **Quantum numbers**

$$
\psi_{n\ell m}(r,\theta,\phi) = R_{n\ell}(r) Y_{\ell}^{m}(\theta,\phi) \qquad \frac{[n=1,2,3,\cdots]}{[m=0,\pm 1,\pm 2,\pm 3\cdots,\pm \ell]}
$$

Quantum numbers:

- ✔ **Principal quantum number: n=1, 2, 3,…** 
	- : Principal energy level of the electron
- ✔ **Orbital quantum number: ℓ=0, 1, 2, … , (n-1)** 
	- : Values of the angular momentum of the electron
- ✔ **Magnetic quantum number: mℓ=0,** ±**1,** ± **2, … ,** ±**ℓ**

: Possible properties of an electron

in a magnetic field

✔ **Spin quantum number: m<sup>s</sup> = -1/2, +1/2**

: Possible spin vectors or orientations of

an electron in a magnetic field

### **Shells and subshells of electrons**



- $\checkmark$  Shells split into subshells labelled s, p, d and f.
- ✔ Within each sub shell, there are a number of possible Orbitals (2**ℓ**+1). The energy values of the orbitals in a sub shell are normally degenerate.
- $\vee$  These energy levels are then described by the quantum number m which can have values from -**ℓ** to +**ℓ**.
- $\vee$  Each electron has a spin quantum number s.

### **Periodic table with quantum numbers**



 $\vee$  Each electron in an atom is described by a set of four quantum numbers.  $\vee$  n represents the period of the periodic table while also noting that the d and f electrons have n quantum numbers that are 1 and 2 units less than the period in which they are found.

 $\vee$  Next the I quantum number represents various "blocks" within the periodic table similar to the periodic table shown below.

# **Energy term**

C atom: α spin and β spin

C:  $(1s)^{2}(2s)^{2}(2p)^{2}$   $(2p)^{2}$  : Open shell

 $2p_+$ ,  $2p_0$ ,  $2p_-$  : How two electrons are located? Coupling of n electrons:

> $L = \ell_1 + \ell_2 + \cdots + \ell_n, \quad S = s_1 + s_2 + \cdots + s_n$  $\textsf{Total angular momentum:} \qquad J = L + S$  $J = |L - S|, |L - S| + 1, |L - S| + 2, \dots, L + S$ # of J values:  $L \geq S \Rightarrow 2S + 1$   $S \geq L \Rightarrow 2L + 1$ *J*  $2S+1$   $L = 0$ , Term:  ${}^{2S+1}L$ ,  $L=0, 1, 2, 3, ...$ S P D F

### **Russel-Saunders or L-S coupling**

- 1. The orbital angular momenta of the individual electrons add to form a resultant orbital angular momentum L.
- 2. Likewise, the individual spin angular momenta are presumed to couple to produce a resultant spin angular momentum S.
- 3. Then L and S combine to form the total angular momentum.



**How to deduce terms?**

$$
(2p)^{2} \qquad \ell_{1}=1; \ \ell_{2}=1 \qquad m_{1}=1, \ 0, \ -1; \ \ m_{2}=1, \ 0, \ -1
$$
\n
$$
\begin{array}{c|cccc}\n & m_{1} & 1 & 0 & -1 & m_{2} \\
\hline\n & 2 & 1 & 0 & 1 & \\
\hline\n & 1 & 0 & -1 & 0 & \ell_{2}=1 \\
\hline\n & 0 & -1 & -2 & -1\n\end{array}
$$

### **Hund's rule**

- 1. The term with maximum multiplicity lies lowest in energy. (Spin-spin interaction)
- 2. For a given multiplicity, the term with the largest value of L lies lowest in energy. (Orbit-orbit interaction)
- 3. For atoms with less than half-filled shells, the level with the lowest value of J lies lowest in energy. When the shell is more than half full, the opposite rule holds (highest J lies lowest).  $2S+1$ (Spin-orbit interaction) $#1$ Term



### **About Hund's rule #1**

Consider two one-electron orbitals a and b,  $a: \varepsilon_a$  b:  $\varepsilon_b$ (a) (b) Spin: anti-parallel  $\rightarrow$  singlet Spin: parallel  $\rightarrow$  triplet Electron configuration If  $\varepsilon_{\scriptscriptstyle \rm q}$  =  $\varepsilon_{\scriptscriptstyle \rm b}$  : Degenerated states If  $\varepsilon_{\mathfrak{a}}$  <  $\varepsilon_{\mathfrak{b}}$  ${a(1)b(2)+b(1)a(2)}$   ${y = \frac{1}{\sqrt{2}}a(1)b}$  $2$  . The contract of  $\mathcal{L}$ 1  $\left( \frac{1}{1} \right)$   $\left( \frac{1}{2} \right)$  ${}^{1}\psi = \frac{1}{\sqrt{2}}\left\{a(1)b(2)+b(1)a(2)\right\}$   ${}^{3}\psi = \frac{1}{\sqrt{2}}\left\{a(1)b(2)-b(1)a(2)\right\}$  $2$  and  $\sim$   $\sim$   $\sim$ 3 1  $\left( \frac{1}{2} \right)$  $P\psi = \frac{1}{\sqrt{2}} \{a(1)b(2) - b(1)a(2)\}$  $(1) + \hat{h}(2) + \frac{1}{2}, \quad \hat{h}(i) = -\frac{1}{2}\nabla^2 - \frac{1}{2} \quad (i = 1, 2)$ 2  $r<sub>i</sub>$  $\hat{I}$   $\hat{I}$   $\hat{I}$   $\hat{I}$   $\hat{I}$  $, \quad \mu(\iota) = -\frac{\nu}{\iota}$ 1  $\hat{C}$  1  $\hat{C}$  $\hat{H} = \hat{h}(1) + \hat{h}(2) + \frac{1}{2}$ ,  $\hat{h}(i) = -\frac{1}{2}\nabla^2 - \frac{1}{2}$  (*i* = 12  $i = h(1) + h(2) + \dots$ ,  $h(i) = -\frac{1}{2} \nabla^2 - \dots$   $(i = 1, 2)$ *r*  $h(i) = -\frac{1}{2}\nabla^2 - \frac{1}{2}$  $r_{12}$  $H = h(1) + h(2) + \frac{h(i)}{h(i)}$ *i*  ${}^{1}E = h_{aa} + h_{bb} + J_{ab} + K_{ab}$ ,  ${}^{3}E = h_{aa} + h_{bb} + J_{ab} - K_{ab}$ , <u>,</u>  $K_{ab} = \langle a(1)b(2) \frac{1}{2} | a(1)b(2) \rangle = J_{ba}, \quad K_{ab} = \langle a(1)b(2) \frac{1}{2} | b(1)a(2) \rangle = K_{ba}$  $r_{12}$  | |  $a(1)b(2)$  =  $J_{\mu}$ ,  $K_{\mu} = (a(1)b(2)-b(1)a(2))$  $r_{\rm o}$  | |  $J_{a} = (a(1)b(2)-a(1)b(2)) = J_{a}$ ,  $K_{a} = (a(1)b(2)-b(1)a(2)) = K_{a}$  $1 \downarrow \qquad \qquad$  $1 |b(2)\rangle = J_{ba}$ ,  $K_{ab} = \langle a(1)b(2) - b(1)a(2)\rangle = 0$  $1 \left( \bigwedge_{i=1}^n \mathcal{L}_i \right)$  $1 \, |b(2) - a(1) b(2)| =$  $12$   $(12)$ 

### **About Hund's rule #2**

For a given multiplicity, the term with the largest value of L lies lowest in energy.

> High L values: Electrons orbiting Electrons orbiting the same direction the opposite direction to add to L values. to reduce L values.Low L values:



 $\vee$  Electrons meet less often than when they orbit in opposite directions. Hence their repulsion is less on average when L is large.

#### *"Modern Physics", M. Oh-e 13* **About Hund's rule #3** *Heavy atom effect*:*Spin-orbit coupling* Nucleus(*Z*) Field:*Ze*/*r*  $f(r) S L$ **Interaction** Spin Orbital **e**  angular angular Electron spin Magnetic dipole moment momentum momentum  $\log b$  $E = \overrightarrow{\mu} \cdot \overrightarrow{B}$  $E = \overrightarrow{\mu} \cdot \overrightarrow{B}$ *Ze e* like a magnet in an  $\frac{\mu_0}{\mu_3}$ . 0  $\mathbf{B}_n = \frac{2c\mu_0}{\mu_3} \cdot \ell \qquad \mu_{\rm s} = -\frac{c}{\mu_3} \mathbf{S}$  $\mu_n = \frac{2c\mu_0}{4-\lambda_0} \cdot \ell \qquad \mu_s = -1$ applied magnetic field. *s* 4 *r m m* From From  $\pi$ *e* electron orbital 2 spin motion  $\sim$  1  $\sim$  2 *Ze*  $\mu_s = \frac{Ze}{c^2} \hat{\ell} \cdot \hat{s} = \xi(r) \hat{\ell} \cdot \hat{s}$  $\mathbf{H}_{\mathbf{SO}} = -\frac{1}{2}\mathbf{B}_n \cdot \mu_s = \frac{2}{2(1-2)(3)} \ell$  $=-\frac{1}{2}\mathbf{B}_n \cdot \mu_s = \frac{2c}{\sqrt{3}}\hat{c} \cdot \hat{s} = \xi(r)\hat{c} \cdot \hat{s}$  $s = \mathcal{E}(r) \ell \cdot s$  $_{n} \cdot \mu_{s} - \frac{2}{\Omega} \frac{2}{\Omega} \frac{3}{\Omega} \ln 3 - 5$ 2  $^{\prime\prime}$   $^{\prime\prime}$   $^{\prime\prime}$   $8\pi\varepsilon_0 m^2 c^2 r^3$  $\pi \mathcal{E}_{\alpha} m^{\dagger} c^{\dagger} r^{\dagger}$ *m c r*  $0^{\prime\prime\prime}$   $e^{\prime\prime}$ 2 4 *I Z e*  $\mathbf{\hat{H}}_{\rm{so}} = \sum_i \xi_i(r_i)$  $\hat{f}_i(r_i) \hat{\ell}_i \cdot \hat{s}_i$  $=\sum \mathcal{E}_i(r_i) \hat{\ell}_i \ \raisebox{2pt}{.}$  $\langle \psi_{n,\ell,m} | \xi(r) | \psi_{n,\ell,m} \rangle = \frac{e}{2m_e^2 c^2 a_0^3} \frac{1}{n^3 \ell G}$  $(r)|\psi_{n \ell m}\rangle = \mathbf{H}_{\mathbf{SO}} = \sum_i \xi_i(r_i) \ell$  $=\frac{1}{2m_e^2c^2a_0^3}\frac{1}{n^3\ell(\ell+1)\ell^2+1}$  $\equiv$  $\ell, m$   $\triangleright$   $\vee$  / $\vee$   $\vee$   $n, \ell, m$  /  $\subset$   $\sim$  $\bigg\}$ 2 2 3  $2m_e^2c^2a_0^3$   $n^3\ell(\ell+1)\left(\ell+\frac{1}{2}\right)$ , , , , *m c*  $a_0$   $a_1$   $a_2$   $a_3$   $a_4$   $a_1$   $a_2$   $a_3$   $a_4$  $3000 \cdot 1100$  $\ell + \frac{1}{2}$   $0 \times 3$  (f) 1 *i*  $\ell(\ell+1)$   $\ell+\overline{-}$  |  $n \ell(\ell+1) \ell + -1$  $\binom{3}{1}$  $\int$  $2 \mid$

The scalar product **S·L** is negative if the spin and orbital angular momentum are in opposite directions. Since the coefficient of **S·L** is positive, lower J is lower in energy.