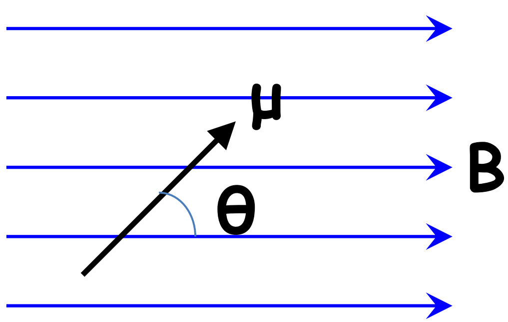
Zeeman effect

Magnetic moment in magnetic fields:



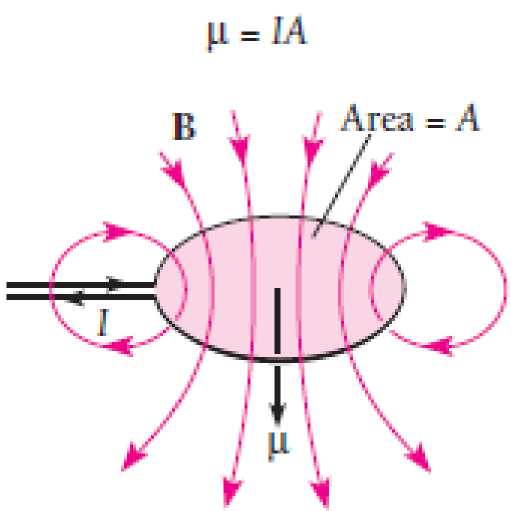
Torque: $\tau = \mu B \sin\theta$

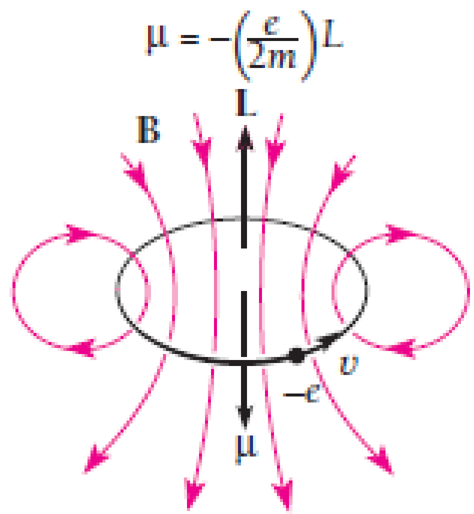
Potential energy:

$$U_m = \int_{\pi/2}^{\theta} \tau\, d\theta = \mu B \int_{\pi/2}^{\theta} \sin\theta\, d\theta = -\mu B \cos\theta$$

Only changes in potential energy are ever experimentally observed, the choice of a reference is arbitrary.

A current loop behaves a magnetic moment:





$$\mu = IA = -ef\pi r^2 \qquad \mu \propto L$$

$$L = mvr = 2\pi m f r^2 \quad (v = 2\pi f r)$$

$$\Rightarrow \quad \mu = -\left(\frac{e}{2m}\right)L$$

$$U_m = \left(\frac{e}{2m}\right)LB\cos\theta$$

How to extract the information on the angular momentum from a wave function.

$$\hat{L}_z = x\hat{p}_y - y\hat{p}_x = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = -i\hbar\frac{\partial}{\partial \phi}$$

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

$$\psi(r,\theta,\phi) = Ae^{im\phi}$$

$$\Rightarrow \quad \hat{L}_z\psi = m\hbar\psi$$

The z component of the angular momentum of electrons traveling around the nucleus can only take an integer multiple of ħ.

"Magnetic quantum number m"

$$\mu_z = -\frac{e}{2m_e}L_z = -\frac{e}{2m_e}m\hbar = \mu_B m$$

Bohr magneton:

$$\mu_B = \frac{e\hbar}{2m} = 9.274\times10^{-24}\,J/T = 5.788\times10^{-5}\,eV/T$$

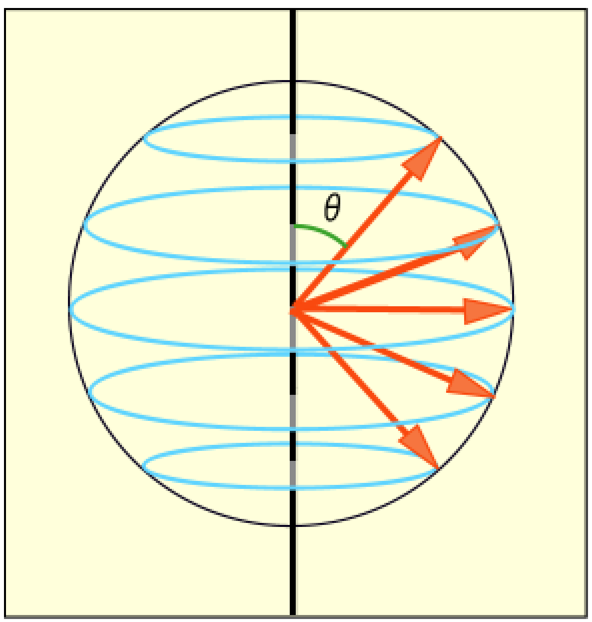
$\hat{L}_x, \hat{L}_y$ **??**

The wave functions of the atom are the eigen functions of L_z, but not those of L_x and L_y.

Total angular momentum of atoms

$$\hat{L}^2 = \hat{L}\cdot\hat{L} = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$\hat{L}^2 Y(\theta,\phi) = \ell(\ell+1)\hbar^2 Y(\theta,\phi) \qquad |L| = \sqrt{\ell(\ell+1)}\hbar$$

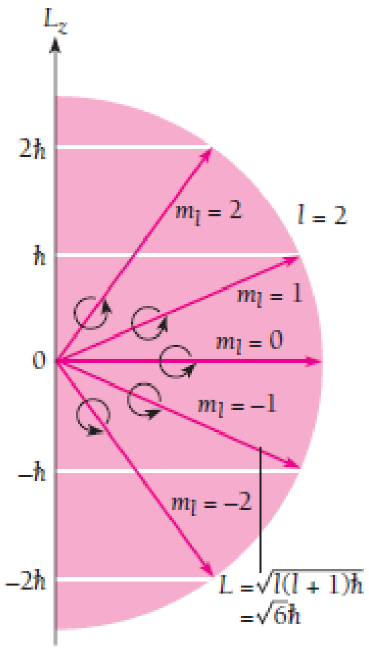


The z component of the angular momentum is discrete, and the vector never points directly up and down.

Splitting of spectra lines by a magnetic field

$$U_m = -\mu \cdot \mathbf{B} = -\mu_z B = \left(\frac{e}{2m_e}\right) LB\cos\theta \qquad \cos\theta = \frac{m_\ell}{\sqrt{\ell(\ell+1)}} \qquad L = \sqrt{\ell(\ell+1)}\hbar$$

$$\Rightarrow \quad U_m = m_\ell \left(\frac{e\hbar}{2m_e}\right) B$$



Bohr magneton:

$$\mu_B = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} J/T = 5.788 \times 10^{-5} eV/T$$

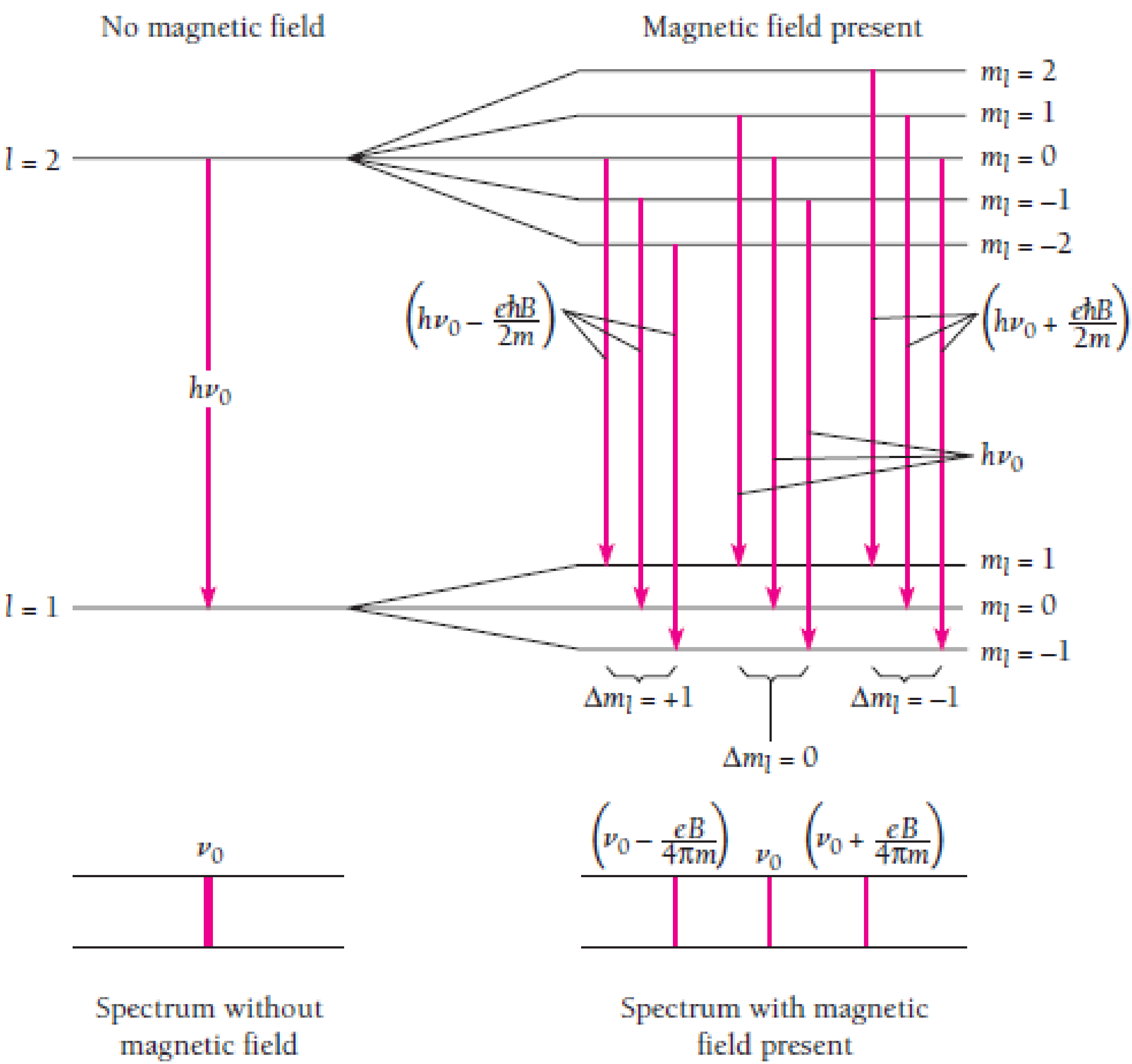
The energy of a particular atomic state depends on m_ℓ as well as that of n.

In a magnetic field,

Splitting of individual spectra lines into separate lines

Zeeman effect

Zeeman effect



$$E = E_0 - \mu \cdot \mathbf{B} = E_0 - \mu_z B$$

$$= E_0 + \frac{e\hbar}{2m_e} m_\ell B$$

$$= E_0 + \left(\frac{e}{4\pi m_e} B\right) h m_\ell$$

$$= h\nu_0 + h\left(\frac{eB}{4\pi m_e}\right) m_\ell$$