Twin paradox

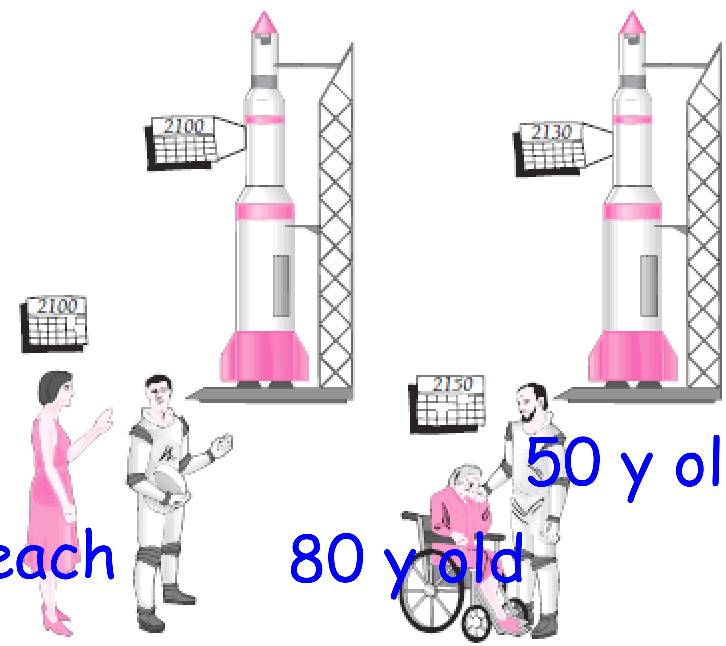
✔ One of a pair of twins (Joe) leaves on a high speed space journey during which he travels at a large fraction of the speed of light, while the other (Debbie) remains on the Earth.

✔ Because of time dilation, time is running more slowly in the spacecraft as seen by the earthbound twin (Debbie) and the traveling twin (Joe) will find that Debbie will be older upon return from the journey.

What is the paradox?

From the viewpoint of Joe, Debbie is in motion relative to him.

Opposite?



A case study in details

If the speed of spacecraft v=0.6c, Joe's life (t') is slower:

$$t = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad t' = t\sqrt{1-\frac{9}{25}} = \frac{4}{5}t$$

Debbie feels his life is

80% slower than Debbie's (t).

In 10 years on the earth for Debbie, Joe travels 6 ly because v=0.6c.

To Joe, the distance L he covers is shortened:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = (6\ light\ years)\sqrt{1-\frac{9}{25}} = 4.8\ light\ years$$

Therefore, to Joe, the journey takes L/v=8 years, and his return takes another 8 years for a total of 16 years.

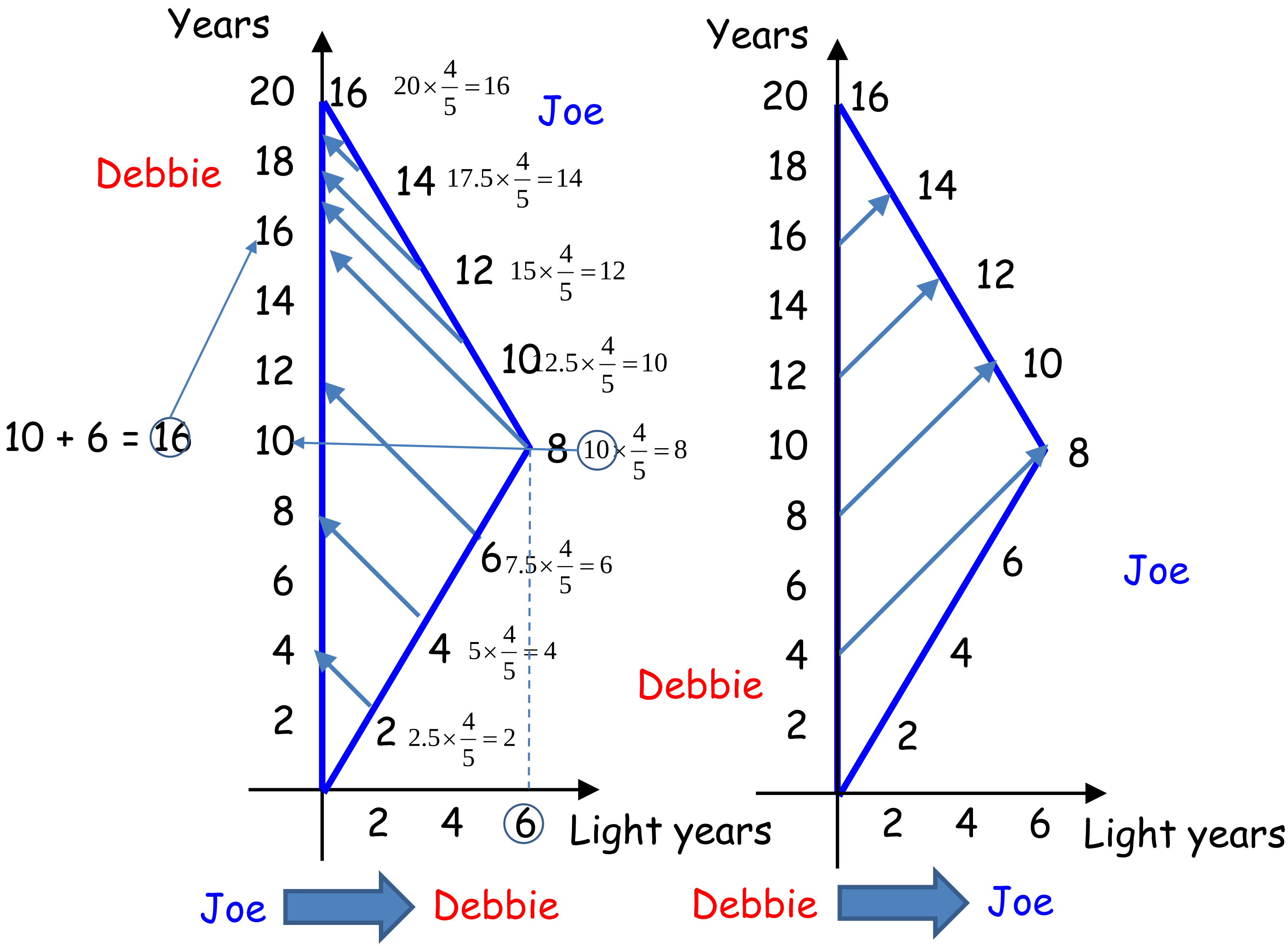
Paradox ?

The two persons are not equivalent: No symmetry

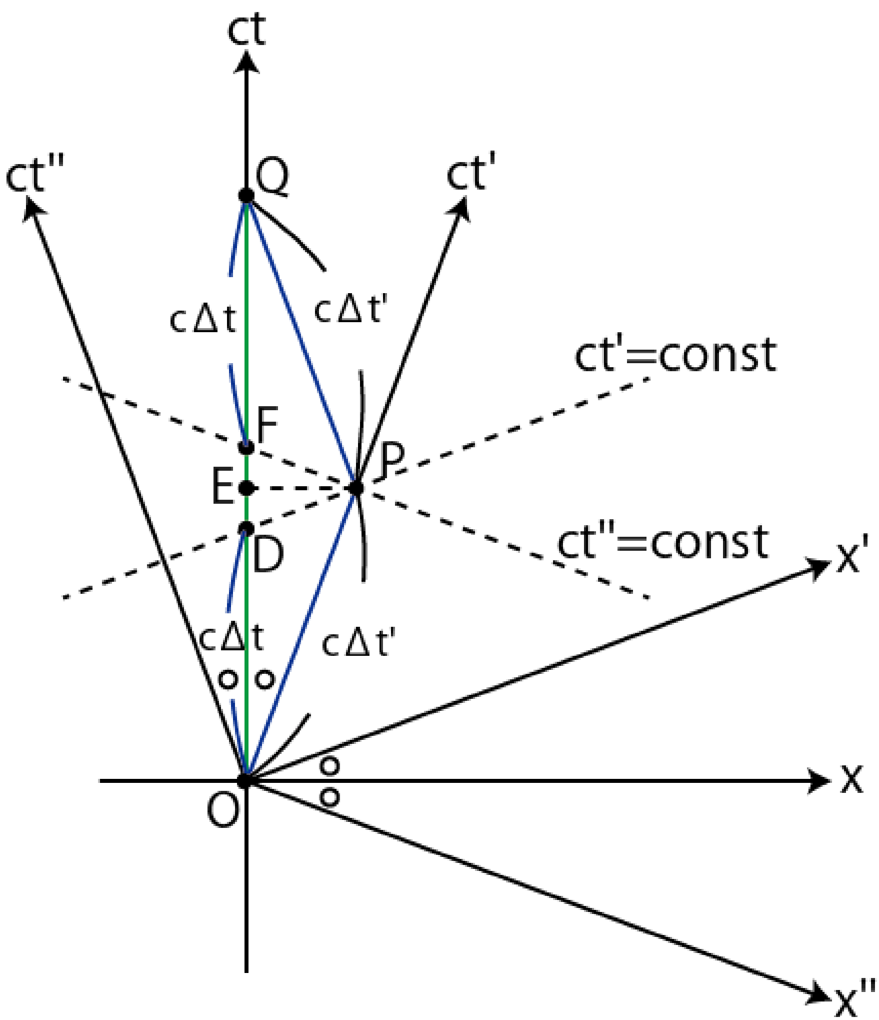
Joe changed direction to head for the earth.
➔Joe changed from one inertial frame to a different one.

Debbie remained in the same inertia frame during Joe's whole voyage.

How to understand the paradox.



Equally transmitted signals



Compare OP and OD,

$$\Delta t' = \Delta t\sqrt{1-\frac{v^2}{c^2}} < \Delta t$$

The same is true between PQ and FQ.

Plus, time elapses for DEF when the direction is reversed at P.