What is "general relativity"?

In Newtonian mechanics,

Object falling free: Noninertial system. A state in which the imaginary force called gravity works and is being accelerated

Objects stationary on the ground: Inertial system. A state in which the imaginary force called gravity is balanced and balanced by the reaction received from the ground.

General relativity is the theory developed to hold even in the presence of gravity (acceleration).

Pursuit of identity of gravity

Principle of equivalence

An observer in a closed laboratory cannot distinguish b/w the effects produced by a gravitational field and those produced by an acceleration of the laboratory.

$$m_I a = F$$

$$F = \frac{GMm_G}{r^2} \qquad \Rightarrow \quad g = \frac{GM}{r^2}\left(\frac{m_G}{m_I}\right)$$

$$\boxed{m_I = m_G}$$

The equivalent principle requires that the gravitational mass and the inertial mass are equal, as a basic law of physics.

What is "gravity"?

Einstein brought the idea of "gravity is the power of the apparent power" - that is, the inertia force caused by the acceleration - "and turned everything over.

Unlike "fictional force", "apparent power" (inertial force) exists.

According to Einstein,

Object falling free: Inertial system. Actually this is the state where no power is acting.

Objects stationary on the ground: Noninertial system. It seems to be at rest but actually it is being accelerated by the action from the ground.

Equation of motion

$$\frac{d^2x^i}{d\tau^2} = -\Gamma^i_{jk}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}$$

cf:

$$\frac{d^2x^i}{d\tau^2} = -\frac{\partial\phi}{\partial x^i}$$

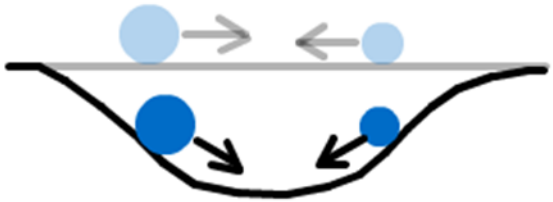
In Newtonian mechanics, we regard the gravitational potential as the source of gravity.

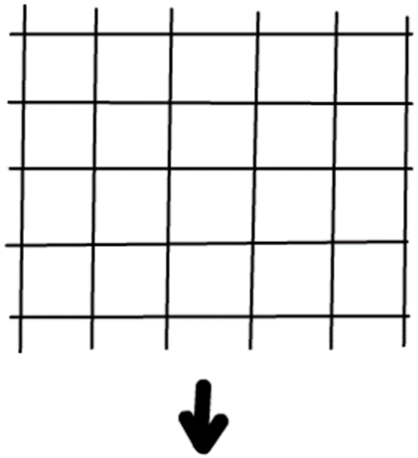
In general relativity, the metric tensor, or geometric quantity, is the source of gravity.

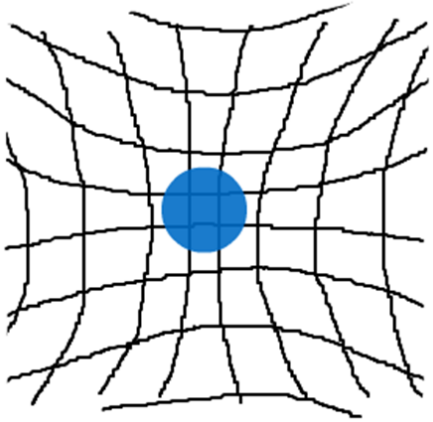
"Spatial distortion Γ (gamma) = acceleration"

Images of gravity

Presence of matter

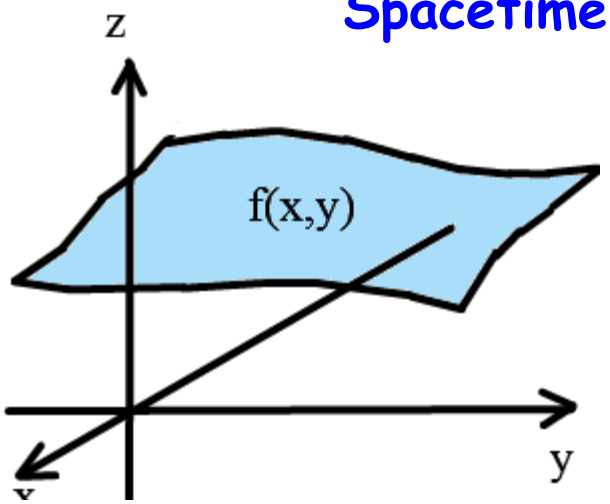






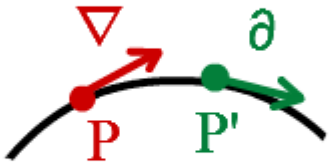
Framework of Relativity

Spacetime distortion



$$\nabla = \partial + \Gamma$$

The difference between the bending of the membrane (∇) seen from you and the bending (∂) of the membrane as seen from me is like $\Gamma \cdots$.



Einstein:
Γ (gamma) can now be calculated without differentiating vectors.

$$\nabla = \partial + \Gamma$$

If $\Gamma = 0$, the surface is flat.

Γ (gamma) : Evaluation quantity
warping of spacetime

What Einstein discovered

Γ (gamma) : Dimension of acceleration

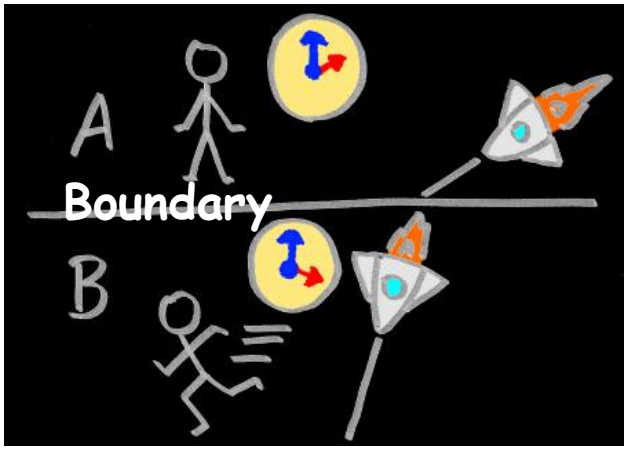
➔ Γ (gamma) : Acceleration of something?
➔ Γ (gamma) : Force of something?

$$\Gamma = a \quad \text{(Gravity)} \quad \Rightarrow \quad \Gamma = F$$

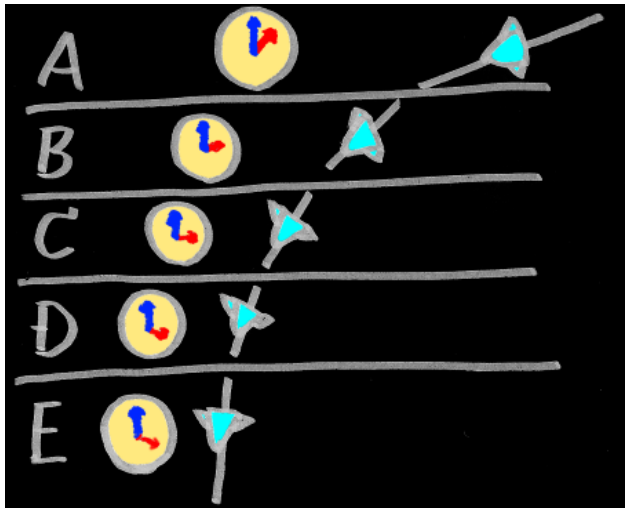
What we feel gravity (F) is actually only space distortion (Γ).

Imagination …

The spacetime changes depending on observers.



What happens if you step in from the spacetime of A to the spacetime of B?

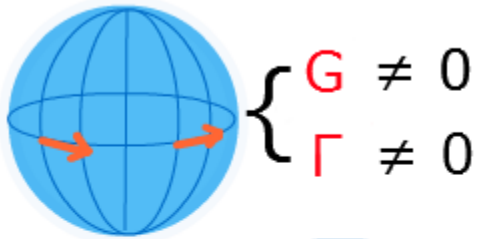


Different spaces of spacetime are made little by little by multiple folds.

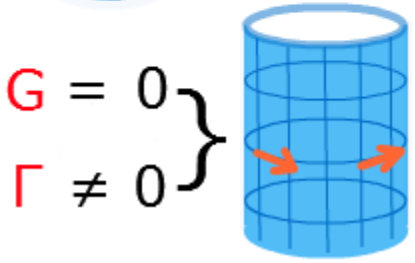
Spacetime boundary
= Difference between two thin spaces
= Differentiation

Gravity equation

$$G_{ij} = \frac{8\pi G}{c^4} T_{ij} \quad \Rightarrow \quad G_{00} = 8\pi G T_{00}$$



If the side of the cylinder is covered with cloth, you can peel it off to make it a perfect plane.



The tool to diagnose bending, Γ (gamma), has the problem that it is judged that there is no distinction of "bending" (Γ (gamma) ≠ 0), bending like the surface of a sphere or bending like the side of a cylinder.

Therefore, Einstein developed to a new diagnostic tool G by further differentiating Γ (gamma) so that only a bend like the surface of a sphere can be diagnosed as "bending".

$$\Rightarrow \quad G_{00} = 2\nabla\Gamma$$

Gravity equation

T_{ij} : **Mechanical energy and momentum tensor**

$T_{00} \approx \rho$: **Spatial density**

$$\Rightarrow \quad 8\pi G T_{00} = 8\pi G\rho$$

$$F = G\frac{Mm}{r^2} \qquad \nabla F = 4\pi G\rho \quad \Leftarrow \quad \nabla^2\phi = 4\pi G\rho$$

$$\Rightarrow \quad 2\nabla F = 8\pi G\rho$$

$$G_{00} = 2\nabla\Gamma$$

$$\Rightarrow \quad 2\nabla F = 2\nabla\Gamma$$

$$\Rightarrow \quad F = \Gamma$$

Γ (gamma) = acceleration = gravity = F

Rough interpretation of gravity equation

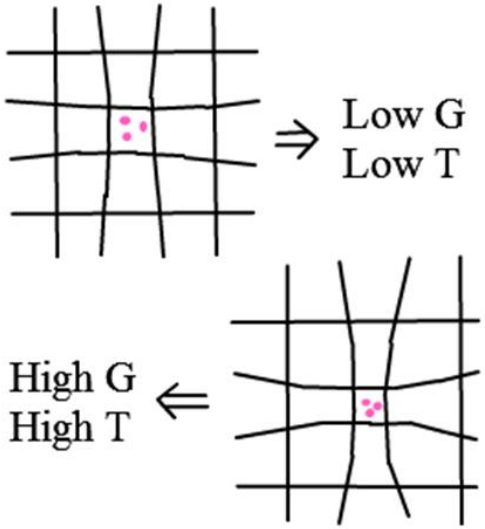
$$G_{ij} = \frac{8\pi G}{c^4} T_{ij}$$

$$\nabla F = 4\pi G\rho$$

$$F = \Gamma$$

The system of two equations

Place the expression representing the distortion "G" of the spacetime on the left side, put the expression representing the density "T" of the space on the right side.
"G" ≡ "T" from the law of universal gravitation (if Γ ≡ F)



Distortion "G" = Density "T"

The image that the density "T" of the space surrounded by the strain increases as the distortion "G" of the space increases.