

Goal

1. Having interests in the introductory video for relativity.
2. Why is relativity needed?
 - No absolute space & time.
 - Any physical law should be the same from any observers.
(Mechanics & electromagnetics)
3. Understanding what are produced from relativity.
 - Mixing time & space, matter & energy, electricity & magnetism
 - Lorentz transformation
 - Time dilation, Length contraction ... etc.

Scenario

- What are impressions of the video?
- There are no absolute space & time. *let me think "space" first.*
 Time and space do not exist independently, and only the concept that time and space mix does exist. \Rightarrow See the other prepared doc. (yellow)
- All motions are relative; we cannot tell which objects are moving and which objects are standing still.
Meaning. there are no absolute space (coordinates).
For details. \Rightarrow see the other prepared doc. (yellow)
 Geocentric theory, Sun-centered theory ...
- How about time?
 Time = Length in space per unit time \Rightarrow Time is also not absolute.
Not absolute *(Neither is time.)*
- Newtonian mechanics is based on the assumption of the existence of the universal frame of reference in terms of space and time.

- The core of special theory of relativity

: The speed of light in free space has the same value in all inertial frame of reference.

How Einstein reached this postulate?

No absolute space & time

⇒ All physical laws should be the same from any observer.
(inertial frame of reference)

⇒ Maxwell eqs. $\rho = 0 \quad i = 0$

$\nabla \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$

$\nabla \times \vec{B} - \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} = 0$

$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

$\nabla \times (\nabla \times \vec{E}) + \frac{\partial}{\partial t} (\nabla \times \vec{B}) = 0$
 $\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$

⇒ $\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}, \quad \nabla^2 \vec{B} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

Two possibility

cf. $\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$

⇒ $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ * ① If there are no absolute (everything is relative) space and time,

or ② In what frame of reference is the speed of light observed? the value of c should be the same.

⇒ Suppose the speed of light is measured in an universal frame of reference.

⇒ In Newtonian mechanics, Galilean transform... can be applied to any frame of reference with holding equations of motions good.

⇒ But applying Galilean transform... to Maxwell eqs. gives rise to the discrepancy of the speed of light.

⇒ Maxwell eqs. are true only in an absolute space?

⇒ "Ether" was came up with,

- Invariance of equation motion by Galilean transform.

one dimension,

$$x' = x - vt$$

$$t' = t$$

$$m \frac{d^2x}{dt^2} = F \Rightarrow m \frac{d^2x'}{dt^2} = F$$

$$\Rightarrow \frac{dx'}{dt} = \frac{dx}{dt} - v$$

$$\Rightarrow \frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2}$$

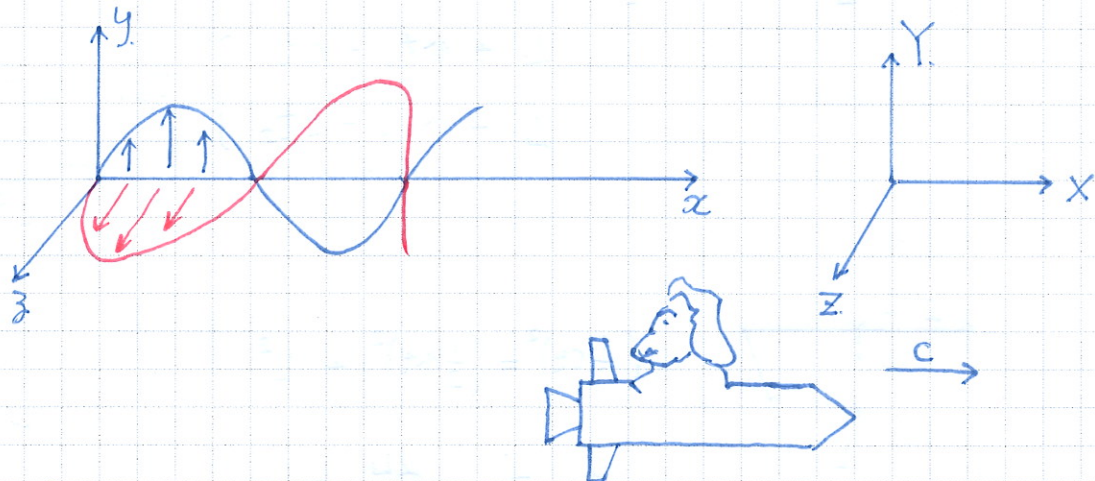
- If one flies at the speed of light, can he/she see the waves?

$$E_x = E_z = 0, \quad E_y = E_0 \sin k(x-ct)$$

$$B_x = B_y = 0, \quad B_z = \frac{E_0}{c} \sin k(x-ct)$$

These are solutions of

$$\begin{cases} \text{div } \vec{B} = 0 & \text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \text{div } \vec{E} = 0 & \text{rot } \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \end{cases}$$



Observer: $X = x - ct, \quad T = t$

$$E_x = E_z = 0, \quad E_Y = E_0 \sin kX$$

$$B_x = B_Y = 0, \quad B_Z = \frac{E_0}{c} \sin kX$$

} These do not satisfy Maxwell eqs.

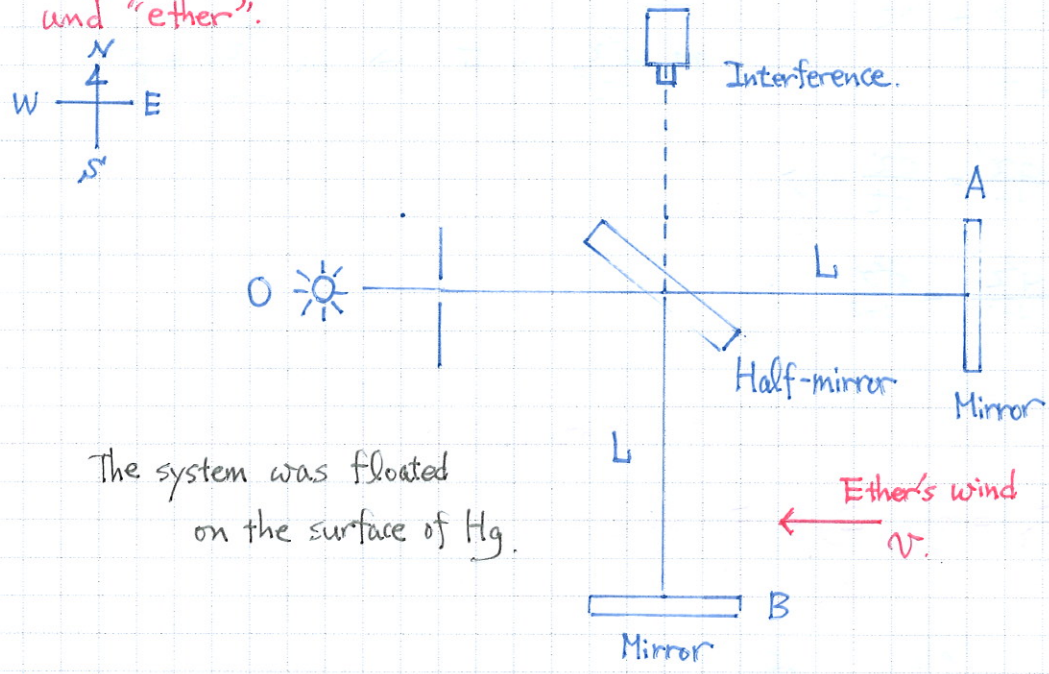
$$\text{rot } \vec{E} \neq -\frac{\partial \vec{B}}{\partial T} \leftarrow \begin{cases} \partial_X E_Y = k E_0 \cos kX \\ \frac{\partial \vec{B}}{\partial T} = 0 \end{cases}$$

For more general formalism, see the viewgraphs.

- ① Maxwell eqs. should hold in any frame of reference.
- ② There is an universal frame of reference where Maxwell eqs. hold.

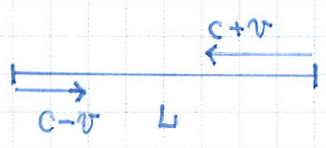
- Michelson & Morley's experiment (1887)

The experiment was done by assuming there is an universal frame of reference und "ether".

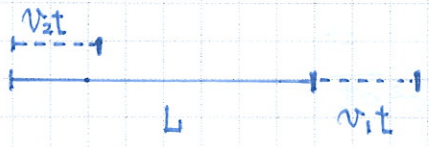


The system was floated on the surface of Hg.

E-W



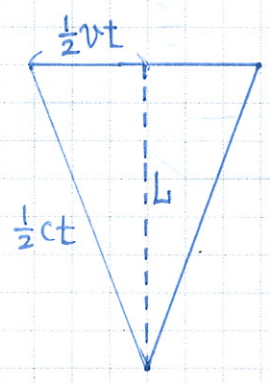
from the viewpoint of the experiment apparatus.



from the viewpoint of "ether"

$$\left. \begin{aligned} L + vt_1 &= ct_1 \\ L - vt_2 &= ct_2 \end{aligned} \right\} t_{EW} = t_1 + t_2 = \frac{2cL}{c^2 - v^2}$$

N-S



$$\left(\frac{1}{2}ct\right)^2 = \left(\frac{1}{2}vt\right)^2 + L^2$$

$$t_{NS} = \frac{2L}{\sqrt{c^2 - v^2}}$$

$$t_{NS} = \frac{2L}{\sqrt{c^2 - v^2}} \approx \frac{2L}{c} \left(1 + \frac{1}{2} \left(\frac{v}{c} \right)^2 + \dots \right)$$

$$t_{EW} = \frac{2cL}{c^2 - v^2} \approx \frac{2L}{c} \left(1 + \left(\frac{v}{c} \right)^2 + \dots \right)$$

$$\Rightarrow \Delta t = t_{EW} - t_{NS} = \frac{2L}{c} \times \frac{1}{2} \left(\frac{v}{c} \right)^2$$

$$c: 3 \times 10^8 \text{ m/s} = 3 \times 10^5 \text{ km/s}$$

$$\text{Daily rotation: } \sim 0.46 \text{ km/s}$$

$$\text{Revolution: } \sim 30 \text{ km/s}$$

at most

$$\frac{v}{c} \approx 10^{-4}$$

$$L = 3 \text{ m (first exp.)}$$

optical path diff.: $10^{-16} \times c \approx 3 \times 10^{-8} \text{ m}$

$$\Rightarrow \Delta t = \frac{2 \times 3}{3.0 \times 10^8} \times \frac{1}{2} \times (10^{-4})^2 \approx 10^{-16} \text{ s}$$

Not direct measurement of time but interference

$$\phi = 2\pi \nu \Delta t = \frac{2\pi L}{\lambda} \left(\frac{v}{c} \right)^2 \approx 10^{-1}$$

$$2\pi \nu \frac{2L}{c} \times \frac{1}{2} \left(\frac{v}{c} \right)^2 = \frac{2\pi L}{\lambda} \left(\frac{v}{c} \right)^2$$

$$c = \lambda \nu$$

$$\Delta \phi = \phi - \phi' = \frac{4\pi L}{\lambda} \left(\frac{v}{c} \right)^2$$

$$\phi': t_{NS} - t_{EW} = -\frac{2L}{c} \times \frac{1}{2} \left(\frac{v}{c} \right)^2 = \ominus \frac{2\pi L}{\lambda} \left(\frac{v}{c} \right)^2$$

exchange b/w NS & EW.

✓ Light source Na.D.

$$\frac{3 \times 10^{-8}}{6 \times 10^{-7}} \approx \frac{1}{20}$$

⇒ Conclusion: no change in interference

✓ 90° rotation.

$$\frac{1}{20} \times 2 = \frac{1}{10} \text{ shift (expected)}$$

$\frac{1}{10}$ length of wavelength.

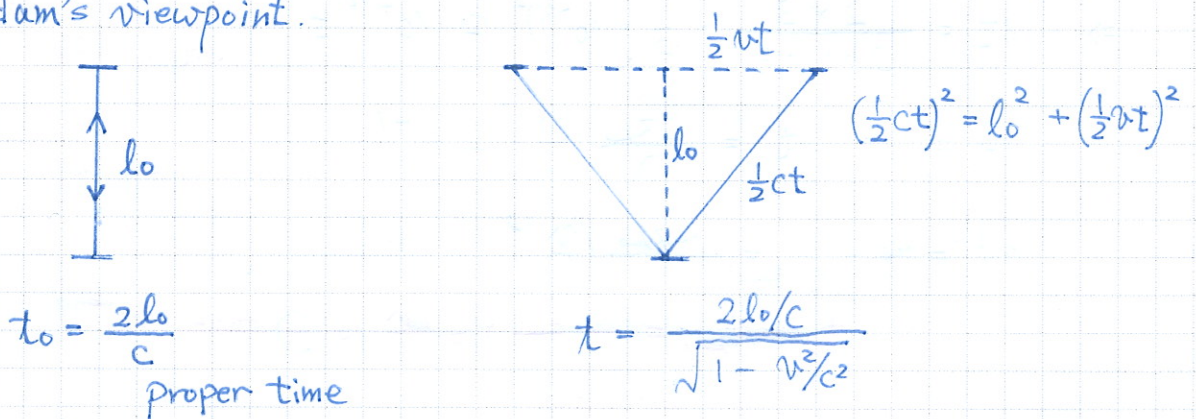
- Time dilation. (Consequences of the light speed invariance)

A moving clock ticks more slowly than a clock at rest.

If someone (Adam) in a moving spacecraft finds that the time interval b/w two events in the spacecraft is t_0 , we (Sarah) on the ground would find that the same interval has the longer duration t .

The quantity t_0 , which is determined by events that occur **at the same place** in an observer's frame of reference, is called the **proper time** of the interval b/w the events.

Adam's viewpoint.

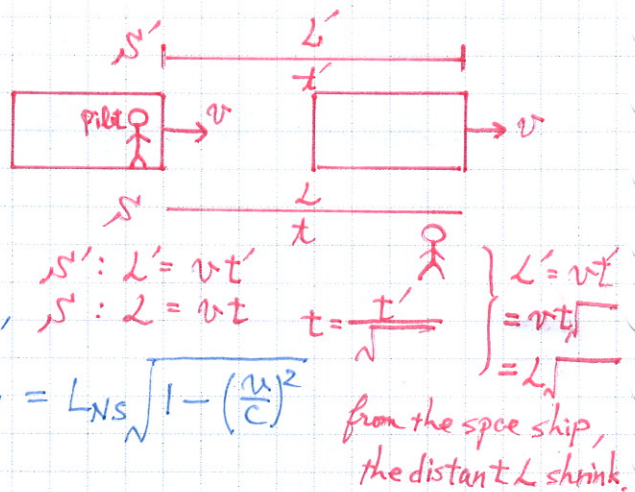


$t > t_0$

- Lorentz contraction

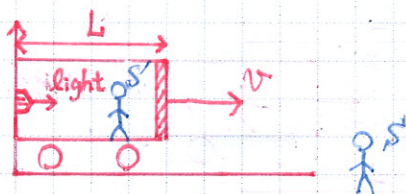
Faster means shorter.

In Michelson & Morley's experiment,



$t_{NS} = t_{EW} \sqrt{1 - (v/c)^2} \Rightarrow L_{EW} = L_{NS} \sqrt{1 - (v/c)^2}$

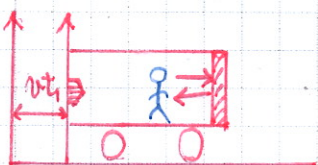
How to understand?



Inside the car, (S')

$\Delta t' = \frac{2L'}{c}$

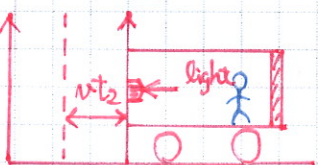
$\Delta t' = \frac{2L'}{c}$



$ct_1 = L + vt_1 \rightarrow t_1 = \frac{L}{c - v}$

From the observer S' outside the car,

$\Delta t = t_1 + t_2 = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2cL}{c^2 - v^2} = \frac{L}{L'} \cdot \frac{\Delta t'}{1 - v^2/c^2}$



$ct_2 = L - vt_2 \rightarrow t_2 = \frac{L}{c + v}$

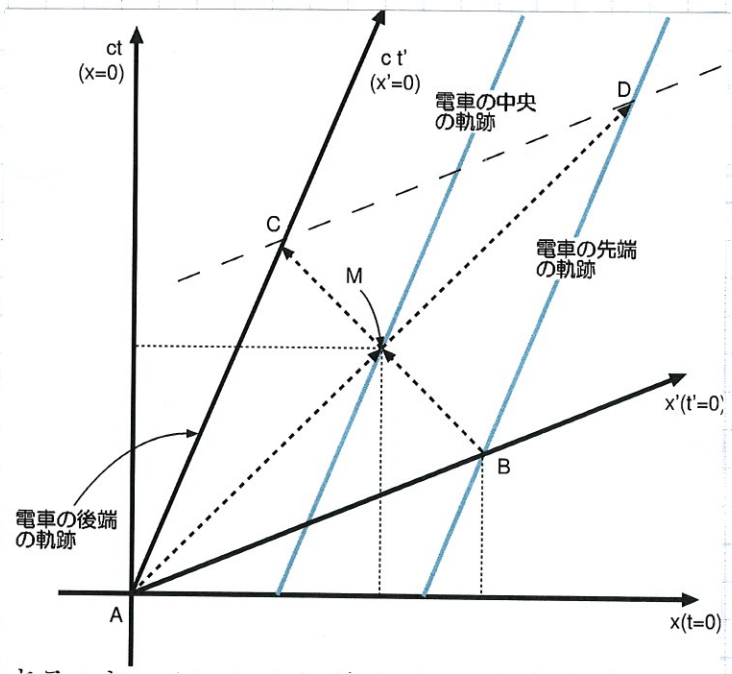
$\Delta t' = \Delta t \sqrt{1 - v^2/c^2}$
 $L = L' \sqrt{1 - v^2/c^2}$

- What can be deduced from the invariance of the speed of light.
- Lorentzean transformation -

Maxwell eqs. do not hold in Galilean transformation

- ⇒ or.
1. Maxwell eqs. are available only in an universal frame of reference.
 2. Applying Galilean transformation is no good.

⇒ Explanation: "Relative simultaneity" (by the viewgraphs.)



ct' axis: $ct = \frac{c}{v}x \left(t = \frac{x}{v} \right) \Rightarrow x - vt = 0$

$x - \frac{v}{c}(ct) = 0 \iff ct - \frac{v}{c}x = 0$

Symmetry operation with the 45° line

⇒ x' axis: $ct - \frac{v}{c}x = 0$

ct' axis $\rightarrow x' = 0 \iff x - \frac{v}{c}(ct) = 0$

x' axis $\rightarrow ct' = 0 \iff ct - \frac{v}{c}x = 0$

$$\left. \begin{aligned} x' &= A \left(x - \frac{v}{c}(ct) \right) \\ ct' &= B \left(ct - \frac{v}{c}x \right) \end{aligned} \right\} \text{General expressions.}$$

Since $x' = ct'$

$$A \left(x - \frac{v}{c}(ct) \right) = B \left(ct - \frac{v}{c}x \right)$$

with $x = ct$.

$$A \left(ct - \frac{v}{c}ct \right) = B \left(ct - \frac{v}{c}ct \right)$$

$$\Rightarrow A = B$$

$$\Rightarrow x' = A \left(x - \frac{v}{c}ct \right)$$

$$ct' = A \left(ct - \frac{v}{c}x \right)$$

If $A = 1$. \Rightarrow Galilean transform

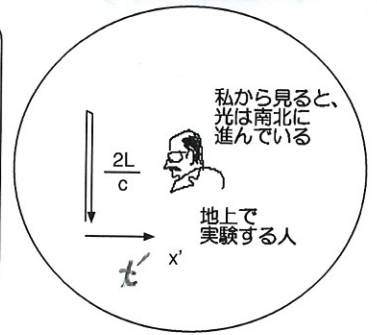
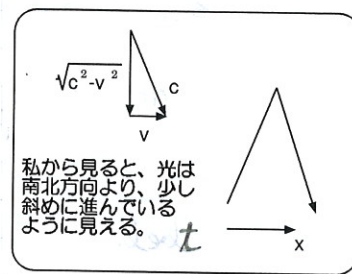
To determine A , consider the time dilation

(Rip Van Winkle effect)

$$t' = \frac{2L}{c} \quad (\text{on earth})$$

$$t = \frac{2L}{\sqrt{c^2 - v^2}} \quad (\text{from outside earth})$$

$$t > t' \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

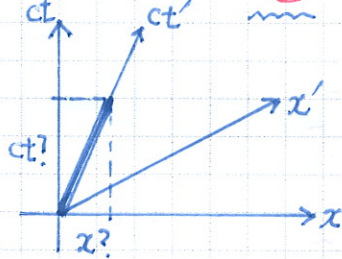


地球全体がこう運動

Coord. on earth.

Start $x' = 0, t' = 0$

Stop $x' = 0, t' = \frac{2L}{c}$



$(x', ct') \rightarrow (x, ct)$

Determine from the coordinate.

Coord. in universe.

$x = 0, t = 0$

$$x = A'v \times \frac{2L}{c}, t = A' \times \frac{2L}{c}$$

$$ct' \text{ axis: } x = v \cdot t = A'v \times \frac{2L}{c}$$

Think this way:

$$A' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Plug in

$$x = A' \left(x' + \frac{v}{c}ct' \right)$$

$$ct = A' \left(ct' + \frac{v}{c}x' \right)$$

Lorentz inverse transform

- Velocity addition

Consider the point $M(x(t), y(t), z(t))$ in the S system,

$$(v_x(t), v_y(t), v_z(t)) = \left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt}, \frac{dz(t)}{dt} \right)$$

In the S' system,

$$(v'_x(t), v'_y(t), v'_z(t)) = \left(\frac{dx'(t)}{dt'}, \frac{dy'(t)}{dt'}, \frac{dz'(t)}{dt'} \right)$$

$$ct'(t) = \gamma(ct - \beta x(t)) \Rightarrow t'(t) = \gamma \left(t - \frac{\beta}{c} x(t) \right)$$

$$x'(t) = \gamma(x(t) - \beta ct), \quad y'(t) = y(t), \quad z'(t) = z(t)$$

$$v'_x(t) = \frac{dx'(t)}{dt'} = \frac{dx'(t)}{dt} / \frac{dt'(t)}{dt} = \frac{\gamma \left(\frac{dx(t)}{dt} - \beta c \right)}{\gamma \left(1 - \frac{\beta}{c} \cdot \frac{dx(t)}{dt} \right)} = \frac{v_x(t) - V}{1 - \frac{v_x(t)V}{c^2}}$$

$$v'_y(t) = \frac{dy'(t)}{dt'} = \frac{dy'(t)}{dt} / \frac{dt'(t)}{dt} = \frac{\frac{dy(t)}{dt}}{\gamma \left(1 - \frac{\beta}{c} \cdot \frac{dx(t)}{dt} \right)} = \frac{v_y(t)}{1 - \frac{v_x(t)V}{c^2}}$$

$$v'_z(t) = \frac{dz'(t)}{dt'} = \frac{dz'(t)}{dt} / \frac{dt'(t)}{dt} = \frac{\frac{dz(t)}{dt}}{\gamma \left(1 - \frac{\beta}{c} \cdot \frac{dx(t)}{dt} \right)} = \frac{v_z(t)}{1 - \frac{v_x(t)V}{c^2}}$$

If $v_x(t) = c$,

$$v'_x(t) = \frac{v_x - V}{1 - \frac{v_x(t)V}{c^2}} = \frac{c - V}{1 - \frac{cV}{c^2}} = \frac{c - V}{\frac{c(c - V)}{c^2}} = c$$

An object: $v'_x(t')$ in S' \Rightarrow $v_x(t)$ in S ?

$$v_x(t) = \frac{v'_x(t') + V}{1 + \frac{v'_x(t')V}{c^2}} \quad -V \text{ (S' from S)}$$

$$v_x(t) \leftrightarrow v'_x(t'), \quad V \leftrightarrow -V$$

$$v'_x(t'), \quad V < c$$

$$\Rightarrow (c - v'_x(t'))(c - V) > 0 \quad c^2 + v'_x(t')V > c(v'_x(t') + V)$$

$$c > \frac{c^2(v'_x(t') + V)}{c^2 + v'_x(t')V} = \frac{v'_x(t') + V}{1 + \frac{v'_x(t')V}{c^2}} = v_x(t)$$

$$\begin{aligned}
 a_x'(t) &= \frac{dv_x'(t)}{dt'} = \frac{dv_x'(t)}{dt} / \frac{dt'(t)}{dt} = \frac{d}{dt} \left(\frac{v_x(t) - V}{1 - \frac{v_x(t)V}{c^2}} \right) / \gamma \left(1 - \frac{v_x(t)V}{c^2} \right)^{-10} \\
 &= \frac{1}{\gamma} \left(\frac{a_x(t)}{1 - \frac{v_x(t)V}{c^2}} - \frac{v_x(t) - V}{\left(1 - \frac{v_x(t)V}{c^2}\right)^2} \cdot \left(-\frac{a_x(t)V}{c^2} \right) \right) / \left(1 - \frac{v_x(t)V}{c^2} \right) \\
 &= \frac{1}{\gamma} \left(\frac{1}{\left(1 - \frac{v_x(t)V}{c^2}\right)^2} + \frac{v_x(t) - V}{\left(1 - \frac{v_x(t)V}{c^2}\right)^3} \cdot \frac{V}{c^2} \right) a_x(t) \\
 &= \frac{1}{\gamma} \left(\frac{1 - \frac{V^2}{c^2}}{\left(1 - \frac{v_x(t)V}{c^2}\right)^3} \right) a_x(t) = \frac{1}{\gamma^3 \left(1 - \frac{v_x(t)V}{c^2}\right)^3} a_x(t)
 \end{aligned}$$

$$\begin{aligned}
 a_y'(t) &= \frac{dv_y'(t)}{dt'} = \frac{dv_y'(t)}{dt} / \frac{dt'(t)}{dt} = \frac{d}{dt} \frac{v_y(t)}{\gamma \left(1 - \frac{v_x(t)V}{c^2}\right)} / \gamma \left(1 - \frac{v_x(t)V}{c^2}\right) \\
 &= \frac{1}{\gamma^2} \left(\frac{a_y(t)}{1 - \frac{v_x(t)V}{c^2}} - \frac{v_y(t)}{\left(1 - \frac{v_x(t)V}{c^2}\right)^2} \cdot \left(-\frac{a_x(t)V}{c^2} \right) \right) / \left(1 - \frac{v_x(t)V}{c^2} \right) \\
 &= \frac{1}{\gamma^2} \left(\frac{1}{\left(1 - \frac{v_x(t)V}{c^2}\right)^2} a_y(t) + \frac{v_y(t)}{\left(1 - \frac{v_x(t)V}{c^2}\right)^3} \cdot \frac{V}{c^2} a_x(t) \right)
 \end{aligned}$$

- Doppler effect.

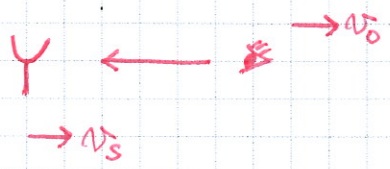
1. Sound

$$f_o = \frac{V - v_o}{V - v_s} f_s$$

$$\frac{V - v_o}{f_o} = \frac{V - v_s}{f_s}$$

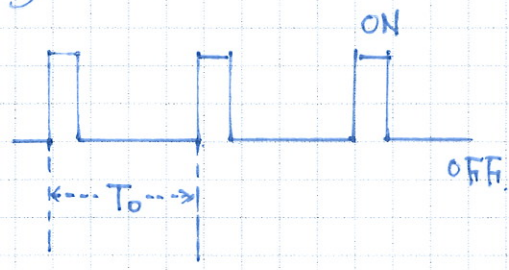
$$\lambda = \frac{v}{f}$$

λ : const.



Galilean transformation.

2. Light.

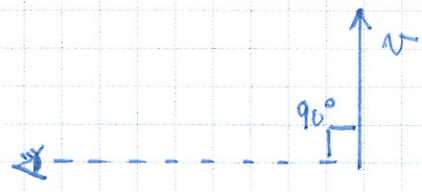


$$\frac{1}{T_0} = \nu_0$$

2-1. Transverse direction

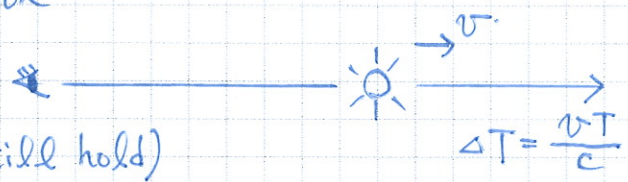
$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \nu = \nu_0 \sqrt{1 - v^2/c^2}$$



2-2. Longitudinal direction

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}} \quad (\text{still hold})$$



$$T' = T + \Delta T = T + \frac{vT}{c}$$

$$= T_0 \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} = T_0 \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}$$

$$\Rightarrow \nu = \nu_0 \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}$$

№	Имя	Фамилия	Год рождения	Уровень	Результат	Комментарий
1	Иванов	Иван	1990	Средний	85	
2	Петров	Петр	1985	Высокий	92	
3	Сидоров	Сидор	1995	Низкий	78	
4	Климов	Климов	1980	Средний	80	
5	Смирнов	Смирнов	1990	Высокий	90	
6	Мухоморов	Мухоморов	1985	Средний	82	
7	Васильев	Васильев	1995	Низкий	75	
8	Попов	Попов	1980	Высокий	88	
9	Лебедев	Лебедев	1990	Средний	80	
10	Зайцев	Зайцев	1985	Высокий	90	
11	Соколов	Соколов	1995	Низкий	75	
12	Кузнецов	Кузнецов	1980	Средний	82	
13	Борисов	Борисов	1990	Высокий	88	
14	Миронов	Миронов	1985	Средний	80	
15	Иванов	Иванов	1995	Низкий	78	
16	Петров	Петров	1980	Высокий	90	
17	Сидоров	Сидоров	1990	Средний	82	
18	Климов	Климов	1985	Высокий	88	
19	Смирнов	Смирнов	1995	Низкий	75	
20	Мухоморов	Мухоморов	1980	Средний	80	
21	Васильев	Васильев	1990	Высокий	88	
22	Попов	Попов	1985	Средний	80	
23	Лебедев	Лебедев	1995	Низкий	75	
24	Зайцев	Зайцев	1980	Высокий	88	
25	Соколов	Соколов	1990	Средний	82	
26	Кузнецов	Кузнецов	1985	Высокий	88	
27	Борисов	Борисов	1995	Низкий	75	
28	Миронов	Миронов	1980	Средний	80	
29	Иванов	Иванов	1990	Высокий	88	
30	Петров	Петров	1985	Средний	80	
31	Сидоров	Сидоров	1995	Низкий	75	
32	Климов	Климов	1980	Высокий	88	
33	Смирнов	Смирнов	1990	Средний	82	
34	Мухоморов	Мухоморов	1985	Высокий	88	
35	Васильев	Васильев	1995	Низкий	75	
36	Попов	Попов	1980	Средний	80	
37	Лебедев	Лебедев	1990	Высокий	88	
38	Зайцев	Зайцев	1985	Средний	80	
39	Соколов	Соколов	1995	Низкий	75	
40	Кузнецов	Кузнецов	1980	Высокий	88	
41	Борисов	Борисов	1990	Средний	82	
42	Миронов	Миронов	1985	Высокий	88	
43	Иванов	Иванов	1995	Низкий	75	
44	Петров	Петров	1980	Средний	80	
45	Сидоров	Сидоров	1990	Высокий	88	
46	Климов	Климов	1985	Средний	80	
47	Смирнов	Смирнов	1995	Низкий	75	
48	Мухоморов	Мухоморов	1980	Средний	80	
49	Васильев	Васильев	1990	Высокий	88	
50	Попов	Попов	1985	Средний	80	

- Relativistic momentum

$$v = (v_x, v_y, v_z)$$

$$u = \left(\frac{c}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_x}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_y}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_z}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

Four-vector

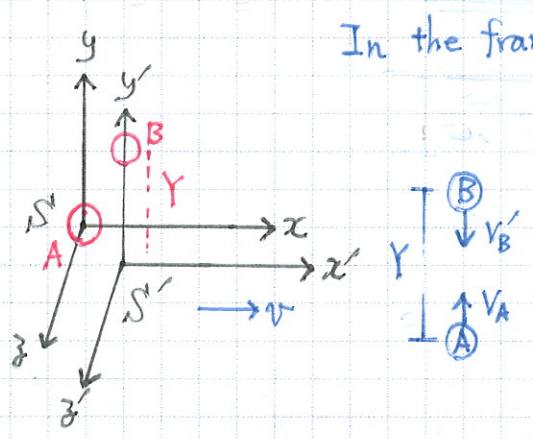
$$x^{i'} = \Lambda^{i'}_j x_j \quad \Lambda^{i'}_j: \text{Lorentz transformation}$$

$$\frac{dx^{i'}}{d\tau} = \Lambda^{i'}_j \frac{dx_j}{d\tau} \quad u^{i'} = \Lambda^{i'}_j u_j \quad \text{Velocity}$$

$$m u^{i'} = \Lambda^{i'}_j m u^j \quad P^{i'} = \Lambda^{i'}_j P^j \quad \text{Momentum}$$

In the textbook,

Consider an elastic collision,



In the frame S' : Particle A

S' : B

In each frame of S and S' ,

$$T_0 = \frac{Y}{v_A} \quad T_0 = \frac{Y}{v_B'} \quad \begin{matrix} \text{(in } S) \\ \text{(in } S') \end{matrix}$$

From the frame of S' , v_B is found from

$$v_B = \frac{Y}{T}$$

T is the time required for B to make its round trip as measured in S' .

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}} \quad \text{(Time dilation)}$$

$$\Rightarrow v_B = \frac{Y \sqrt{1 - v^2/c^2}}{T_0}$$

In the frame S' , $P = mv$

$$P_A = m_A v_A = m_A \left(\frac{Y}{T_0} \right)$$

$$P_B = m_B v_B = m_B \sqrt{1 - v^2/c^2} \left(\frac{Y}{T_0} \right)$$

If $m_A = m_B$,

P is not conserved.

$$\text{But, } m_B = \frac{m_A}{\sqrt{1 - v^2/c^2}}$$

Define relativistic momentum

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv$$

Relativistic second law:

$$F = \frac{dp}{dt} = \frac{d}{dt}(\gamma mv)$$

- Mass and energy

How $E_0 = mc^2$ comes out?

$$W(\text{Work}) = F \cdot s$$

If no other forces act on an object and the object starts from rest, all the work done on it becomes kinetic energy KE

$$\begin{aligned} KE &= \int_0^s F ds = \int_0^s \frac{d(\gamma mv)}{dt} ds = \int_0^v v d\left(\frac{mv}{\sqrt{1 - v^2/c^2}}\right) \\ &= \frac{mv^2}{\sqrt{1 - v^2/c^2}} - m \int_0^v \frac{v dv}{\sqrt{1 - v^2/c^2}} \quad \left(\int x dy = xy - \int y dx\right) \\ &= \frac{mv^2}{\sqrt{1 - v^2/c^2}} + \left[mc^2 \sqrt{1 - v^2/c^2} \right]_0^v \\ &= \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 \\ &= \gamma mc^2 - mc^2 = (\gamma - 1) mc^2 \end{aligned}$$

By interpreting $\gamma mc^2 = E$ (total energy)

If $KE=0$, $E = mc^2$ (Rest energy E_0)

$$\Rightarrow E = E_0 + KE \quad E_0 = mc^2$$

$$KE = \gamma mc^2 - mc^2$$

$$\begin{aligned} &= \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 \approx \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) mc^2 - mc^2 \\ &= \frac{1}{2} mv^2 \quad (v \ll c) \end{aligned}$$

- Energy and momentum

Energy and momentum conservation in a isolated system.

Relation b/w energy, momentum, rest energy
(invariant quantity)

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \Rightarrow E^2 = \frac{m^2c^4}{1 - v^2/c^2}$$

$$P = \frac{mv}{\sqrt{1 - v^2/c^2}} \Rightarrow P^2c^2 = \frac{m^2v^2c^2}{1 - v^2/c^2}$$

$$\Rightarrow E^2 - P^2c^2 = \frac{m^2c^4 - m^2v^2c^2}{1 - v^2/c^2} = (mc^2)^2$$

mc^2 : invariant

$$\Rightarrow E^2 = (mc^2)^2 + P^2c^2$$

$$\Rightarrow E^2 - P^2c^2$$

: the same value in all frames of references

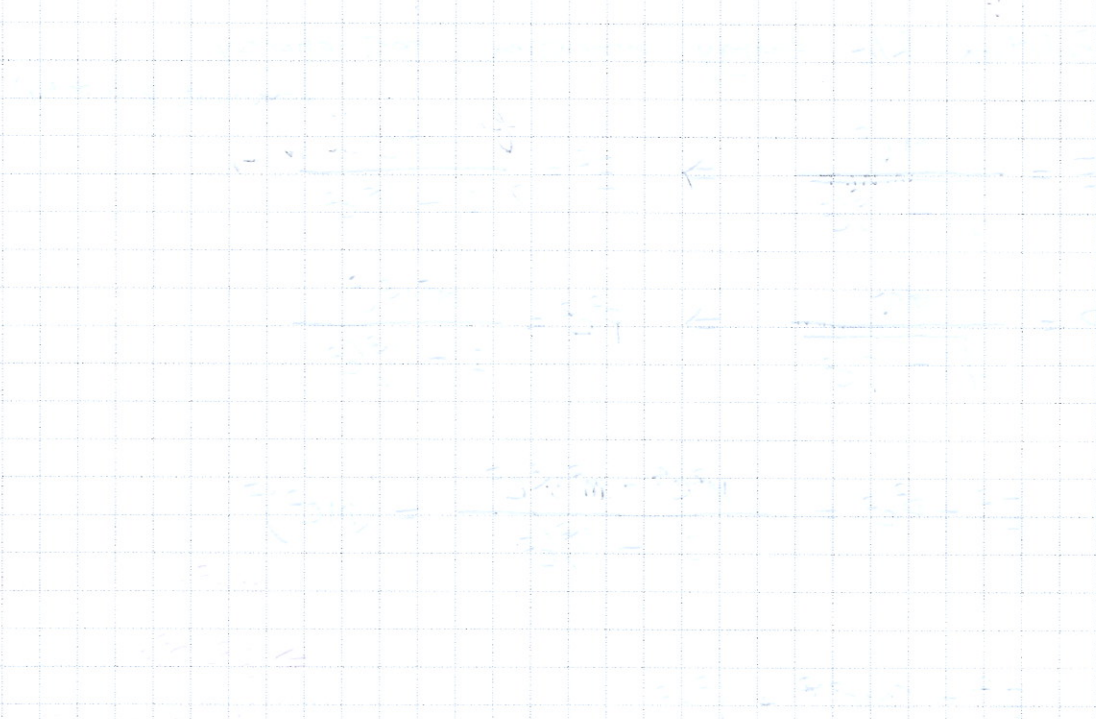
If $m=0$ (massless),

$$E = 0, P = 0 \quad (\text{if } v \ll c)$$

$$E = \frac{0}{0}, P = \frac{0}{0} \quad (\text{if } v = c)$$

(indeterminate)

$$E = Pc \quad (\text{Photon})$$



Logic.

- ✓ In the special theory of relativity, we deal with only the special cases where the observers are at rest or moving at a constant speed.
- ✓ These observers are in inertial frames of reference.
- ✓ The special relativity is the theory, in which space and time are unified.
- ✓ "Relative" means to change depending on what you think as a reference.
- ✓ "Absolute" means what does not change regardless of what is a reference.
- ✓ Since the speed of a ball changes depending on observers, the movement of the ball is relative.
- ✓ A smart person here may have a question, "it is ok to think that any motion is relative. But the laws of physics do not change depending on what we think as a reference?"
- ✓ One answer to this question is Galilei's principle of relativity. Galilei's principle of relativity is the principle that the law of motion of an object is the same whether the observer's frame of reference is stationary or moving, if it is constant velocity.
- ✓ However, Galilei's principle of relativity cannot be applied to the laws of electromagnetism.

- ✓ As its name suggests, electromagnetism is one of the physical fields dealing with phenomena related to electricity and magnetism. It is a field that is theoretically highly prepared. Einstein felt doubt Galilei's principle of relativity could not be applied to such an orderly system, and he extended Galilei's principle of relativity.
- ✓ All laws of physics are identical in any inertial frame of reference
 - ⇒ The speed of light has the same value in all inertial frame of reference.
- ✓ The value of light speed is actually derived from Maxwell eqs. Therefore, assuming the special relativity principle, the principle of light velocity invariance is required and become an appropriate assumption.

- General Relativity.

- ✓ General theory of relativity is the theory that is developed from the extension of special relativity.
- ✓ The meaning "special": a certain inertia frame of reference moves by linear uniform motion.
- ✓ However, almost all objects, astronomical bodies and objects, are not linear uniform motions but accelerated motions.
- ⇒ "General" relativity. Relativity under accelerated motions.
- ✓ Einstein started with the question: what is acceleration and what is gravity?
- ✓ Einstein discovered "Principle of equivalence": an observer in a closed laboratory, cannot distinguish between the effects produced by a gravitational field and those produced by an acceleration of the laboratory.
- ✓ Accelerations counteract the effects of gravity, and this leads to the conclusion that accelerations are equivalent to gravity.

Equivalence: giving the effects of gravity.

Not the identity of gravity.

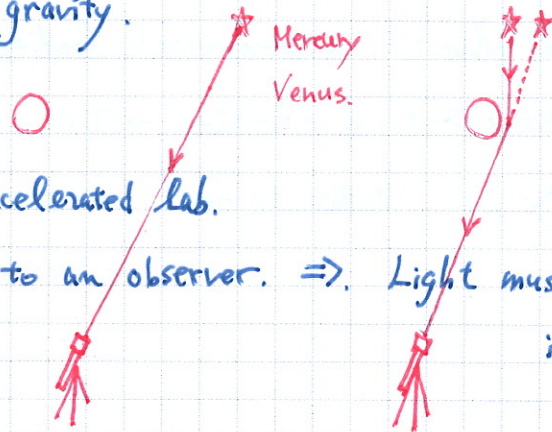
- ✓ What Einstein thought:
Suppose you put a ball on the surface of very elastic and springy films. Like trampolines, the surface of the trampolines would warp.
- ⇒ Gravity is a warping of spacetime.
- ⇒ Presence of mass produces a warping of spacetime that leads to gravity.
- ✓ Applying the equivalence among objects, mass and energy to this consideration, degree of a warping of spacetime gives the energy of mass in the spacetime.
- ✓ Larger mass of objects creates a warping of spacetime around the objects, and in return the warping of spacetime gives the presence of mass.

✓ This suggests that spacetime is not simply a container of objects, but mass and energy themselves.

✓ A clock with gravity ticks more slowly than a clock without gravity.

Presence of mass \rightarrow Warping of spacetime \rightarrow Warping of time

✓ Light is subject to gravity.



✓ A light beam in an accelerated lab.

is deflected relative to an observer. \Rightarrow Light must be similarly deflected in a gravitational field.

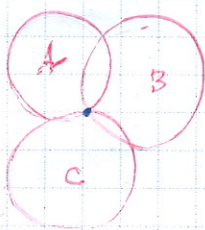
✓ GPS (Global Positioning System).

Relativity plays a key role in a multi-billion dollar growth industry centered around GPS.

The GPS is a network of about 30 satellites orbiting the earth in an altitude of 20,000km.

At least four GPS satellites are 'visible' at any time.

Trilateration



How far away you are from satellite A.

B.

C.

\Rightarrow Your location is where three circles intersect.

✓ GPS and relativity.

GPS satellites have atomic clocks on board.

General and special relativity predict that differences appear between these clocks and an identical clock on the earth.

General relativity: the clocks on board the satellites run faster.

Special relativity: moving clocks on the satellites run slower.

\Rightarrow The whole GPS network has to make allowances for these effects.