# Solutions to Midterm (Modern Physics) (5 problems)

04/23/2018 Provided by *Masahito Oh-e*

## Important reminder:

- You are not allowed to open any textbook, copies of my lecture notes and ppt files, but allowed to see **your own notebook or memo**. You can also use **a simple calculator**.
- Do not use the Internet. If anyone is found who is cheating on the exam, she/he will be immediately failed in this course.
- Solve the problems below. Describe the ways of thinking in English: only final solutions are not accepted. Make clear how you reach each solution.
- When 30 minutes pass after the test starts, if you think you have completed the test, you can leave the room by submitting the answer sheets, except the 10 minutes before the exam finishes.
- Several physical constants are listed in the end of the sheet.
- If answers are decimal numbers, calculate them to one places of decimals at least.

**Problem 1.** (15 points)
(a) What is the energy of a single photon of the wavelength $\lambda$? Derive in the unit energy J.
(b) In the case of the wavelength 600 nm, calculate the energy of a single photon.
(c) What is the largest energy spectrum region among 1) Radio wave, 2) Ultraviolet (UV), 3) Microwave, 4) Infrared (IR), 5) X-ray? Describe the reason.

(a) From the relation c=$\nu\lambda$, where c is the speed of light, $\nu$ the frequency of light, and $\lambda$ the wavelength of light, $3.0\times10^8\,[m/s] = \nu\times\lambda\,[m]$.

$$\therefore \nu = \frac{3.0\times10^8}{\lambda}\,[1/s]$$

Therefore, the energy of the photon that have a wavelength $\lambda$ can be derived as follows:

$$E=h\nu=6.6\times10^{-34}\,[J\cdot s]\times\frac{3.0\times10^8}{\lambda}\,[1/s]=\boldsymbol{\frac{1.98\times10^{-25}}{\lambda}\,[J]}$$

(b) With the result of (a), the energy of a single photon with $\lambda$=600 nm = 600 × $10^{-9}$ m can be calculated as

$$E=h\nu=\frac{1.98\times10^{-25}}{\lambda}\,[J]=\frac{1.98\times10^{-25}}{6\times10^{-7}}\,[J]=0.33\times10^{-18}=\boldsymbol{3.3\times10^{-19}\,[J]}$$

(c) From the relation c=$\nu\lambda$, $\nu = \frac{c}{\lambda}$. Therefore, the relation between the energy and wavelength of light is expressed as E=h$\nu$=h$\frac{c}{\lambda}$.

This relation tells us shorter wavelengths give larger energy. Amongst the options, X-ray is the shortest wavelengths. Thus the answer is **5) X-ray**.

**Problem 2.** (20 points)
Choose one of the topics below, or describe both of them together in terms of wave-particle duality. Explain each aspect of duality, raising some of experimental evidence with some keywords below.

You can also use unlisted keywords if you want. **You need to use four keywords at least.**
(a) What is the duality of light? Describe what it is and whatever you know.
(b) What is the duality of electron? Describe what it is and whatever you know.
Keywords: wave-like property, particle-like property, wave-particle duality, interference, diffraction, blackbody radiation, Rayleigh-Jeans formula, Planck radiation formula, photoelectric effect, Compton effect, de Broglie wave, Davisson-Germer experiment, particle diffraction, double slit experiment, electron microscope

See the textbook, lecture notes, ppt files and other references.
The important point is to give an answer to the question as a conclusion, and support the conclusion consistently using several evidence.
For example,
Duality of light expresses nature of light that light exhibits not only wave-like properties but also particle-like properties. Originally, light was considered to be electromagnetic wave because light gives interference and diffraction that are typical wave properties. However, what Planck discovered while deriving Planck radiation formula from correcting Rayleigh-Jeans formula, is that the energy of light is discrete rather than continuous, suggesting that light behaves like particles. Furthermore, Einstein realized photoelectric effect is understood if the energy in light is in small particles. In the photoelectric effect, no photoelectrons are emitted under a critical frequency $\nu_0$, meaning that there is a minimum energy for electrons to be ejected from a metal surface. If frequencies of light that is exposed to a metal surface is under the critical frequency, no photoelectron emission occur by increasing intensity of light. ....................

Scoring policy: since there is a description “you can write whatever you know”, if you correctly explain a keyword, the possible maximum score would be 4 points. That means if you appropriately describe four keywords, the possible score would be 16 points. Obviously, as an answer, you need consistency and how to answer to the question, therefore, those factors are also taken into account.

**Problem 3.** (20 points)
(a) Two spaceships are oppositely approaching at the speed of 0.6c, respectively. Suppose an observer is in one of the spaceship, what is the relative speed of the other spaceship with respect to the observer?
(b) In an inertial frame of reference S, the speed of light that propagates in the positive direction of the X axis is observed to be c. Suppose another frame of reference S’ is moving at a constant speed of V in the positive direction of the X axis of the inertial frame of reference S, What speed of light is observed in the frame of reference S’?

(a) Velocity addition :
Suppose two objects are moving oppositely at the speed of u and v, the velocity addition is expressed by $V = \frac{u+v}{1+\frac{uv}{c^2}}$

$$V = \frac{u+v}{1+\frac{uv}{c^2}} = \frac{0.6\,c+0.6\,c}{1+0.6^2} = \frac{15}{17}\,c$$

Set::write : $\frac{u+v}{1+\frac{uv}{c^2}}$のタグTimesはProtectedです． ≫

0.882353 c

(b) From the observer in the frame of reference S’, the velocity addition ($u_x'$) of c and V is given by

$$u'_x = \frac{c-V}{1-\frac{cV}{c^2}} = \frac{c-V}{\frac{c\,(c-V)}{c^2}} = c$$

Therefore, the speed of light observed in the frame of reference S' is c.

**Problem 4.** (20 points)

(a) Suppose that the minimum wavelength from a X-ray generator is $3.0\times10^{-11}$ $m$. What is the acceleration voltage?

(b) Suppose a monochromatic X-ray impinges on a matter. If the X-ray is scattered by the matter with the reduced energy of the photons, how is the wavelength of the scattered X-ray changed compared with the incidence X-ray?

(a) The minimum wavelength of X-ray can be determined by the energy of the electron that impinges on a target. The energy given by the applied voltage determines the minimum wavelength, $eV = \frac{hc}{\lambda_0}$.

Therefore, $V=\frac{hc}{e\lambda_0}=$**$4.1\times10^4$ V**.

(b) The energy E of X-ray is expressed by $E = h\nu = \frac{hc}{\lambda}$. Therefore, **if the energy E is reduced by the collision, the wavelength $\lambda$ becomes larger compared with the incidence X-ray**.

**Problem 5.** (45 points)

Fill out the blanks below. You still need to describe your ways of thinking properly.

One of a pair of twin (Joe) leaves on a high speed space journey during which he travels at a large fraction of the speed of light, while the other (Debbie) remains on the Earth. The spacecraft goes to a star 2 light-yesars (ly) away from the earth, and returns to the earth. If the speed of the spaceship is v=($\sqrt{3}$/2)c, to Debbie, Joe's life is ( (a) ) % slower than hers. To Joe, the length that is covered by the spacecraft is shortened by 1/2. From the Earth, it takes $\frac{4}{\sqrt{3}}$ years for the spacecraft to reach the star, and to Joe, it takes ( (b) ) years for one way to the star. Therefore, Joe is ( (c) ) years ((d) younger or older) than Debbie, when Joe returns to the Earth.

Suppose that Joe and Debbie send out a radio signal once a day while Joe is away. On the outward trip, Debbie and Joe are being separated at a rate of v=($\sqrt{3}$/2)c. With the help of the Doppler effect, each twin receives signals ( (e) ) days apart. On the return trip, Debbie and Joe are getting closer together at the same rate, and each receives signals more frequently, namely, ( (f) ) days apart.

To Joe, the trip to the star takes ( (g) ) days, and he receives ( (h) ) signals from Debbie. During the return trip, Joe receives ( (i) ) signals from Debbie, for a total of 1685 signals. Joe therefore concludes that Debbie has aged by ( (j) ) years in his absence.

To Debbie, Joe needs $\frac{4}{\sqrt{3}}$ years for the outward trip. Because the star is 2 light years, Debbie continues to receive Joe's signals at the original rate of one every ( (e) ) days for 2 years after Joe has arrived at the star. Hence Debbie receives signals every ( (e) ) days for a total of ( (k) ) days to give a total of ( (l) ) signals. (These are the signals Joe sent out on the outward trip.) Then, for the remaining ( (m) ) days of what is to Debbie a 4.62-y voyage, signals arrive from Joe at the shorter intervals of ( (f) ) days for an additional ( (n) ) signals. Debbie thus receives 844.57 signals in all and concludes that Joe has aged by 2.31 y during the time he was away—

which agrees with Joe's own figure. Joe is indeed (   (c)   ) years ((d) younger or older) than his twin Debbie on his return.

The distance to the star is 2 light-years (ly) and the velocity of the spacecraft is $v = \frac{\sqrt{3}}{2}$c (c: the speed of light). To Debbie, who stays behind, the pace of Joe's life on the spacecraft is slower than hers by a factor of $\sqrt{1-\frac{v^2}{c^2}} = 0.50$, meaning that Joe's life is **(a) 50%** slower than hers. To Joe, the distance to the star is shortened by L=(2 ly)$\times\sqrt{1-\frac{v^2}{c^2}}$=1 ly. The journey to the star takes $L/v =$ **(b)** $\frac{2}{\sqrt{3}}$ years, and his return takes another $\frac{2}{\sqrt{3}}$ years for a total of $\frac{4}{\sqrt{3}}$years. To Debbie, his return takes $\frac{8}{\sqrt{3}}$ years. Therefore, Joe is $\frac{8}{\sqrt{3}}-\frac{4}{\sqrt{3}}=\frac{4}{\sqrt{3}}=$ **(c) 2.31** years **(d) younger** than Debbie.

Since the two are separating on the outward trip,

the time $T'$ to receive the signal becomes $T' = T' + \Delta T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{vT}{c} =$

$$T_0 \frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = T_0 \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} = 1 \times \frac{\sqrt{1+\frac{\sqrt{3}}{2}}}{\sqrt{1-\frac{\sqrt{3}}{2}}} =$$ **(e) 3.73** days. On the retrun trip,

the time interval becomes more frequent, namely, **(f) 1/3.73 = 0.268 = 0.27** days.

To Joe, from (b), the trip to the star takes $\frac{2}{\sqrt{3}}$ years ×365 days/years= **(g) 421.47** days. With (e) and (g), Joe receives 421.47/3.73= **(h) 112.99≈113** signals. On the return trip, he receives 421.47×3.73= **(i) 1572.08**.signals. The total signals are 1685 signals, meaning 1685/365=4.62 years. Therefore Joe and Debbie agree that Debbie has aged by **(j) 4.62** years in Joe's absence.

To Debbie, she receives the signal at the rate of 3.73 days during the period of $\frac{4}{\sqrt{3}}$ years for the outward trip of Joe and another 2 years, since the star is away from the Earth for 2 light-years. So she receives the signals every 3.73 days for a total of $\left(4/\sqrt{3}+2\right)*365=$ **(k) 1572.93≈1573** days, meaning 1573/3.73= **(l) 421.70≈422** signals. Since Debbie spends $\frac{8}{\sqrt{3}}$years=4.62 years= 1686.30≈ 1686 days in his absence, the remaining days are 1686.30-1572.93= **(m) 113.37≈113** days. The number of the signals she receives is 113.37×3.73= **(n) 422.87≈423** signals.

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N[8/√3]
4.6188

1572.9313930168535`
421.697
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