# Data Structures Final Examination

## 3:30pm-5:20pm (110 minutes), Tuesday, Jan. 12, 2016

#________ ID ________________________ Name ________________________

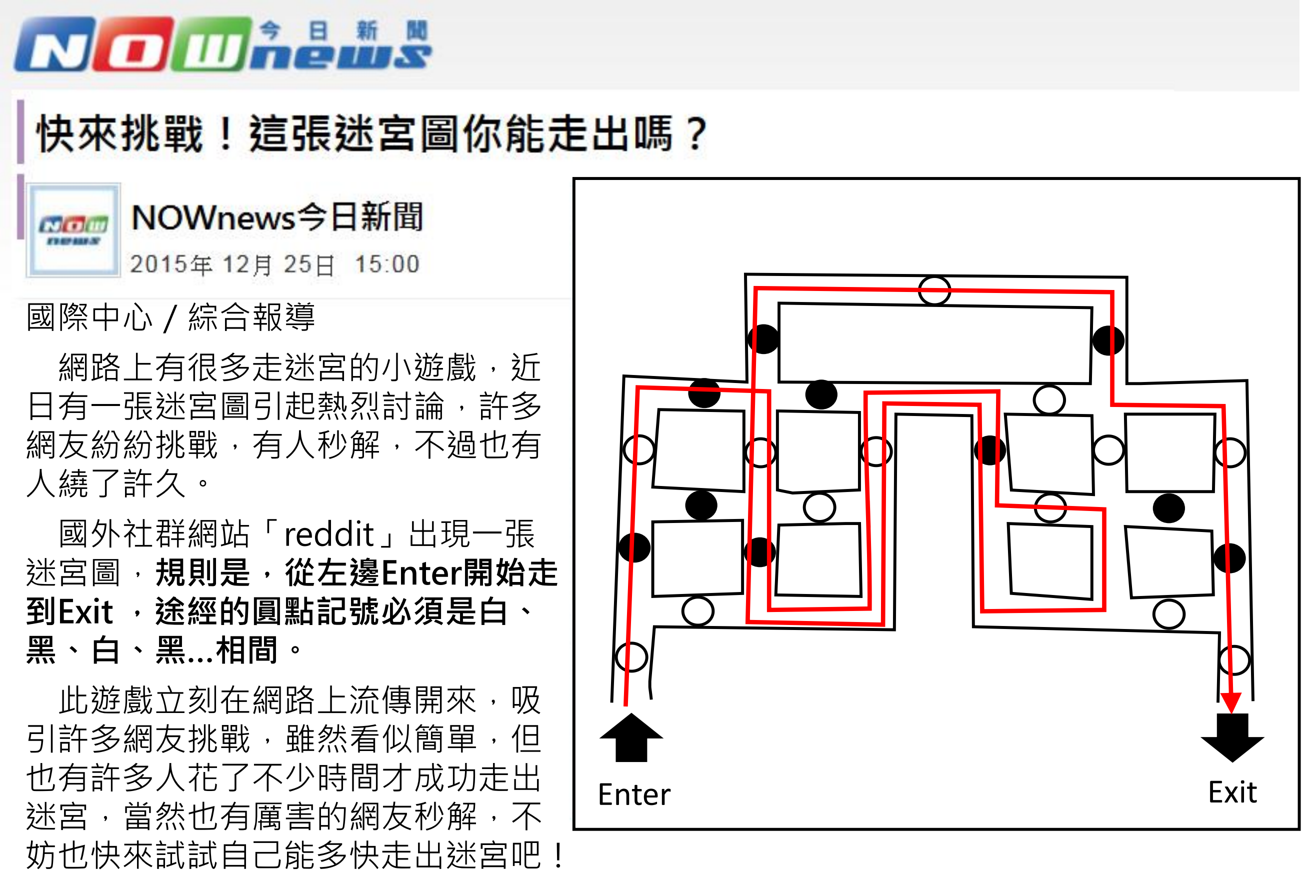
✧ 共 12 題，130 分。**請看清楚題目再作答**。

✧ 第 5~12 題請直接回答在試題卷。1~4 請用答案卷(如回答在試題卷，請於答案卷註明)。

✧ 題目有難有易 (1-B, 2B 相對較難)，請分配時間，作答不用按照順序。

✧ 若依照題目條件限制，有多個答案，請回答其中任一個。如果某題沒有答案，請回答“本題無解”。

1. Please read a recent news:



**快來挑戰！這張迷宮圖你能走出嗎？**

NOWnews今日新聞

2015年 12月 25日 15:00

國際中心 / 綜合報導

網路上有很多走迷宮的小遊戲，近日有一張迷宮圖引起熱烈討論，許多網友紛紛挑戰，有人秒解，不過也有人繞了許久。

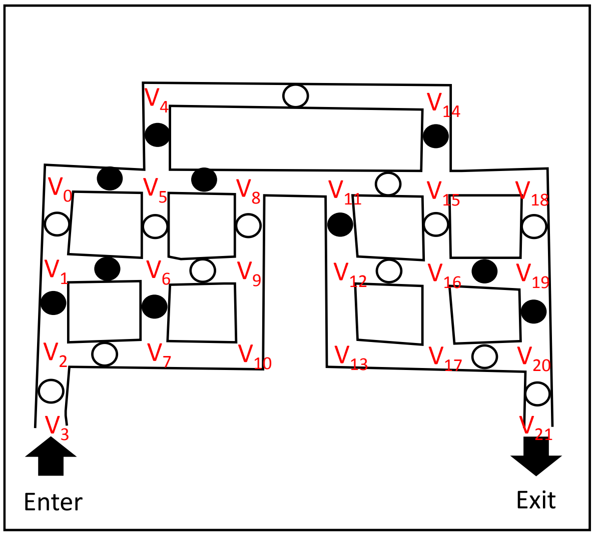
國外社群網站「reddit」出現一張迷宮圖，**規則是，從左邊Enter開始走到Exit ，途經的圓點記號必須是白、黑、白、黑...相間。**

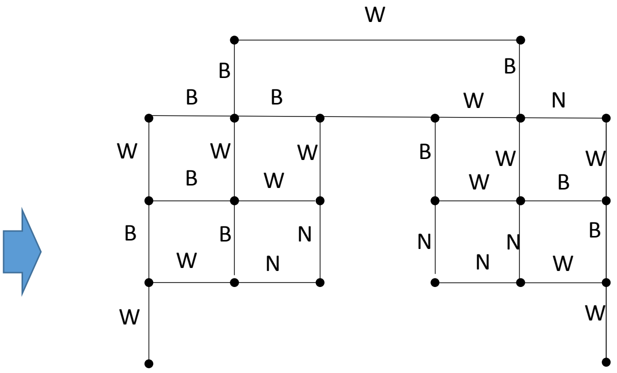
此遊戲立刻在網路上流傳開來，吸引許多網友挑戰，雖然看似簡單，但也有許多人花了不少時間才成功走出迷宮，當然也有厲害的網友秒解，不妨也快來試試自己能多快走出迷宮吧！

(The above red path is the solution of this maze. For your reference only. This question does not indeed require you to solve the maze. Please note that a same place can be visited twice. Therefore, a basic DFS algorithm (even if it takes white and black marks into consideration) cannot solve the maze because a basic DFS visits each place no more than one time.

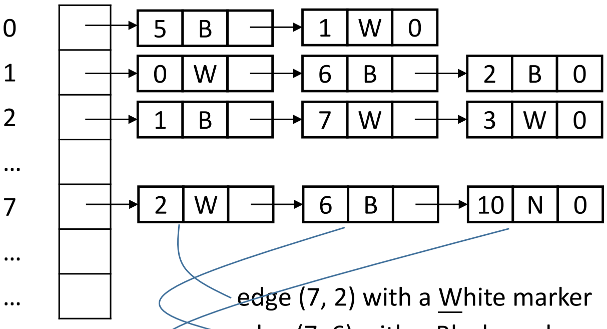
We would like to develop a program that can solve this type of mazes. Please answer the following questions:

A. [5%] Please show an **adjacency list type** data structure that can represent this type of maze.





Let each crossroad be a vertex, each road segment be an edge, and each mark be the edge cost. According to this rule, we can have an adjacency list representation:

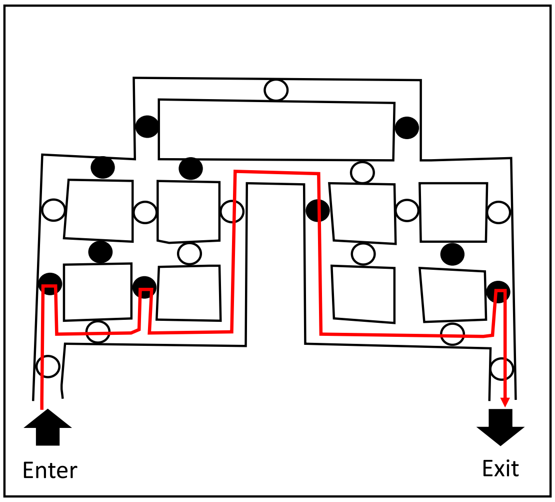


edge (7, 2) with a White marker

edge (7, 6) with a Black marker

edge (7, 10) with No mark

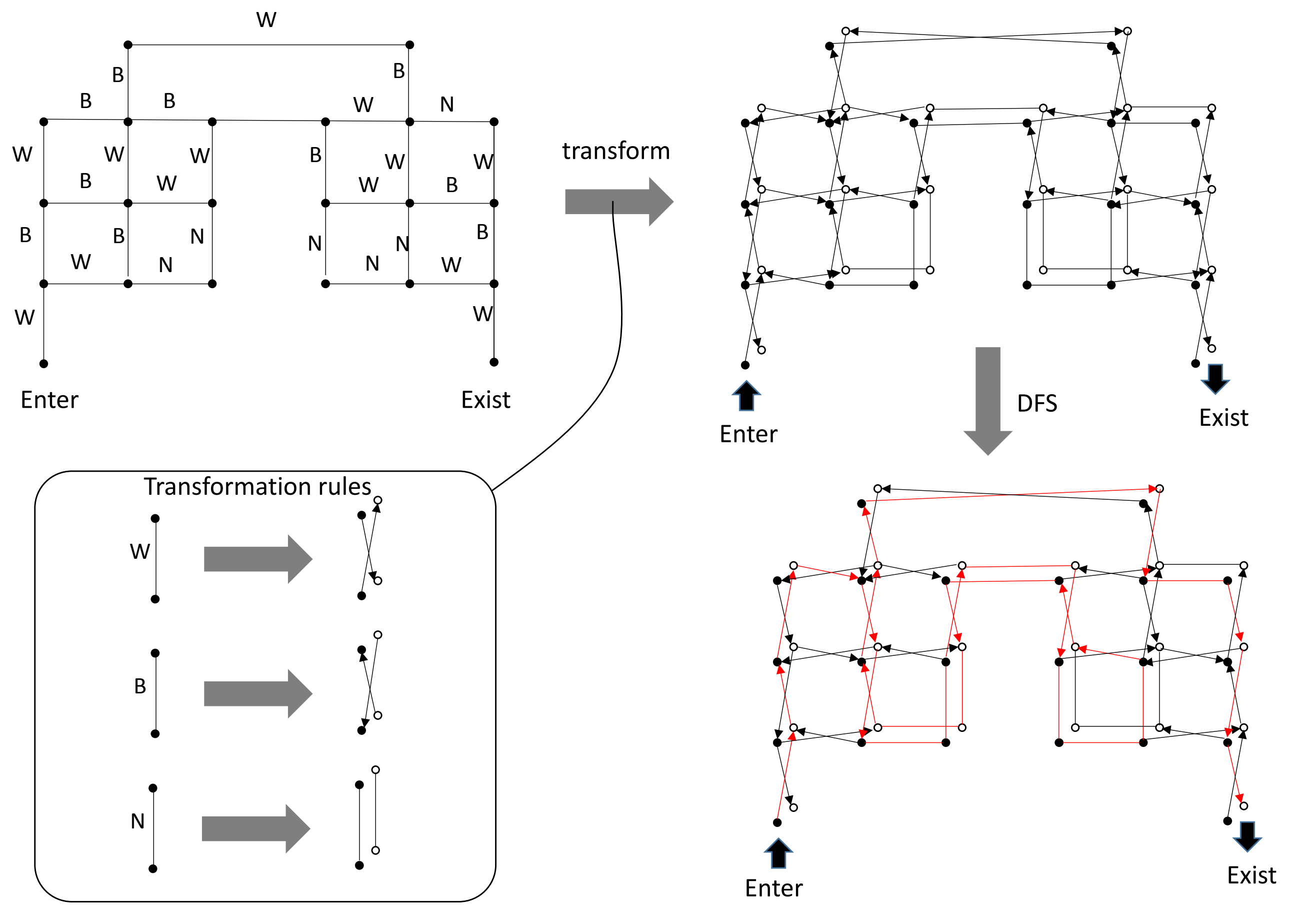
Please note that the following red path is not a valid solution. Therefore, if we consider white and black marks as vertices when constructing the graph, we can run into this kind of issue.



B. [10%] Please describe the algorithm of your program that can solve this type of maze.

**Hint:** One recommended strategy is transforming this problem into our known problem. You can describe a method that transforms any maze with black and white marks into an equivalent (等效的/等價的) maze (e.g., a directed graph) without such marks. If finding a path in the latter maze is a known problem, your work is done.

A possible algorithm is as follows:



C. [5%] Let **n** and **e** be the number of vertices and edges of the input maze. Please analyze the time complexity of your algorithm.

Transformation: $\theta(n+e)$

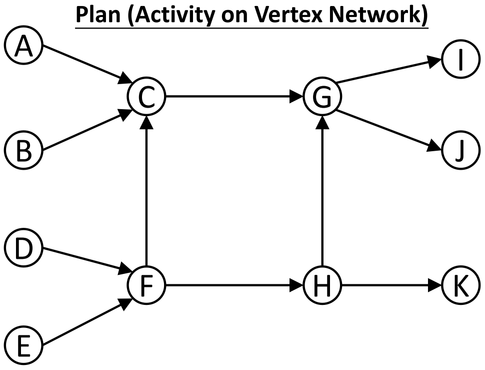
DFS: $\theta(n+e)$

Overall: $\theta(n+e)$

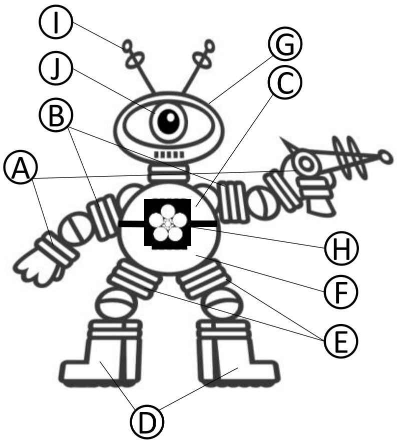
2. 校長 would like NTHU students to build a robot for the upcoming NTHU-NCTU competition! The project plan is set up as follows.

## Tsing-Hua Robot Project

Plan (Activity on Vertex Network)



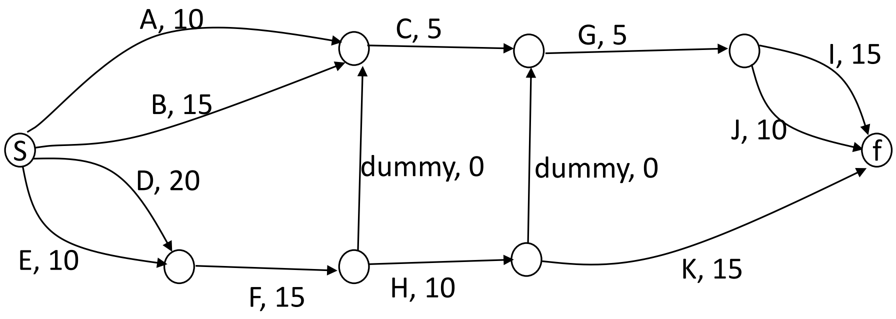
| Task | Required Time (Days) | Task | Required Time (Days) |
|---|---|---|---|
| A | 10 | G | 5 |
| B | 15 | H | 10 |
| C | 5 | I | 15 |
| D | 20 | J | 10 |
| E | 10 | K | 15 |
| F | 15 | | |



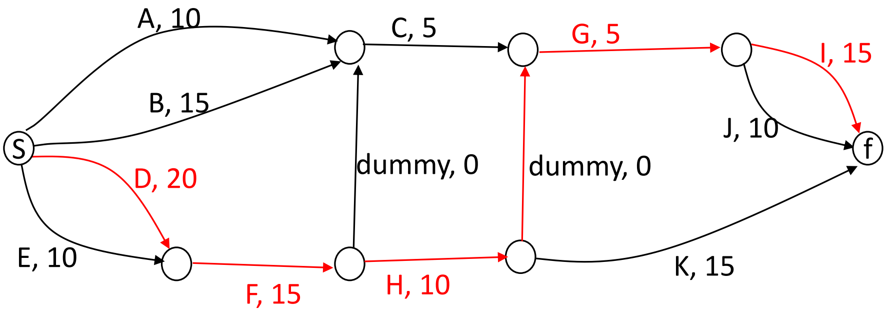
A. [5%] Please derive a **topological order** of the above activity on vertex (AoV) network. Please answer the one with **the minimum dictionary order**. For example, if both "ABCD" and "ADBC" are valid topological orders, please answer the former one.

A B D E F C H G I J K

B. [5%] Please convert the above AoV network to an **activity on edge (AoE) network**.
Hint: You need to add a start and a finish vertex. Furthermore, you can add edges with zero cost to handle the case that multiple activities depend on one activity (e.g., C and H both depend on F).



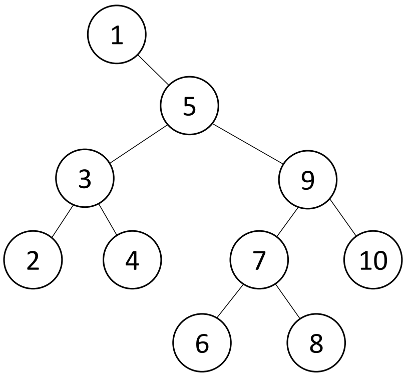
C. [5%] Please derive the **critical activities** and the **earliest finish time** of the project.



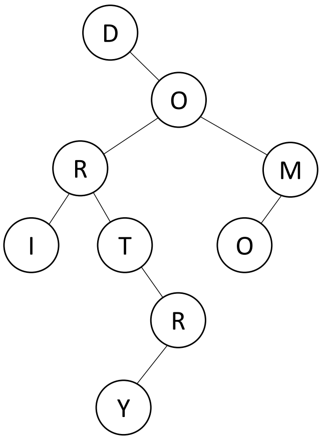
Critical activities: D, F, H, G, I

Earliest finish time = path cost of D, F, H, G, I = 65

3. [6%] Please insert ten keys **1, 5, 9, 7, 3, 2, 6, 4, 8, 10** into an empty **binary search tree** **(BST)** and plot the final tree. We want each left child to hold a smaller number than its parent in the BST.



4. [6%] Please plot a 9-node binary tree whose **level-order** sequence is "**D O R M I T O R Y"** and whose **in-order** sequence is "**D I R T Y R O O M**". (Just kidding! no offense.)
Hint: Specifically, the sequences are "**D $O_1$ $R_1$ M I T $O_2$ $R_2$ Y**" and "**D I $R_1$ T Y $R_2$ $O_1$ $O_2$ M**"



5. A **Bloom filter** uses the following **three hash functions** to index a **10-entry table**.

h1(k) = k % 10

h2(k) = (2*k) % 10

h3(k) = (k*k) % 10

A. [5%] Please show the table after three keys, **3, 5, 8**, are added to the Bloom filter.

h1(3) = 3, h2(3) = 6, h3(3) = 9 → these positions are set for the key 3

h1(5) = 5, h2(5) = 0, h3(5) = 5 → these positions are set for the key 5

h1(8) = 8, h2(8) = 6, h3(8) = 4 → these positions are set for the key 8

so the table becomes:

| Table index: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

B. [5%] Given the above condition, please list all the **false-positive** keys from **1 to 9**.

From 1 to 9, the only false-positive key is 4. h1(4) = 4, h2(4) = 8, h3(4) = 6 → these three positions are all set in the Bloom filter, but 4 should not belong to the Bloom filter. The Bloom filter answers a wrong "yes" (i.e., false positive).

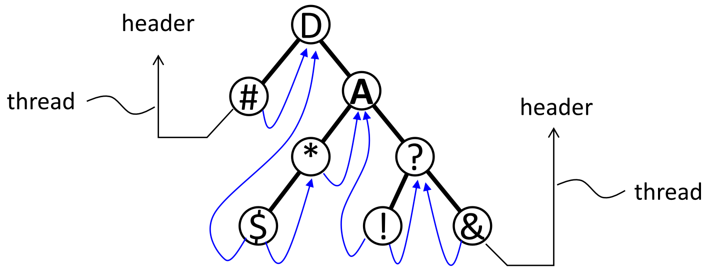
6. [10%] Please complete the following **infix-to-postfix** conversion process.

| Input | 2 | / | 5 | * | ( | 3 | + | 9 | ) | - | 7 | (End) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Stack | | / | / | * | (<br>* | (<br>* | +<br>(<br>* | +<br>(<br>* | * | - | | |
| Output | 2 | | 5 | / | | 3 | | 9 | + | * | 7 | - |

Time

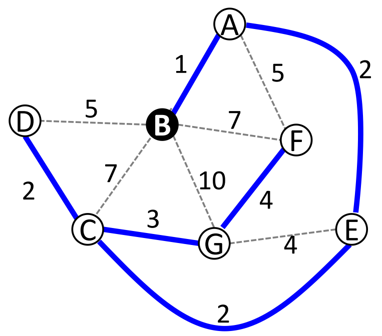
Overall postfix results

7. [6%] Please add threads (to form a **threaded-binary tree**) to the following binary tree

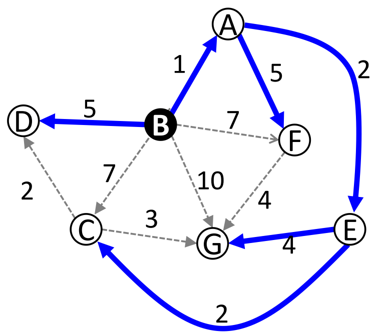


8. [10%] Please plot the **spanning tree** obtained using **Prim's** algorithm and **Dijkstra's** algorithm using B as the **starting vertex**.

**Prim's (B)**

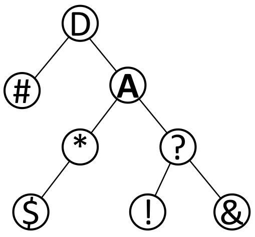


**Dijkstra's (B)**

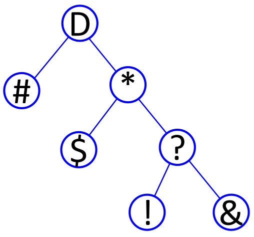


9. Please answer the following questions about a BST and a heap.

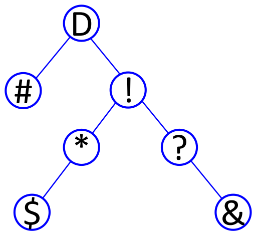
A. [4%] Please show the BST after we remove the key "A".



Remove "A" from the BST



or

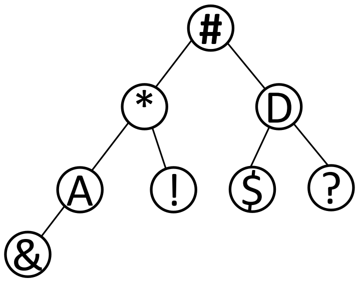


B. [4%] Please list the order among all the eight keys (including A) in the BST. Either ascending or descending order is good (由大至小或由小至大均可)

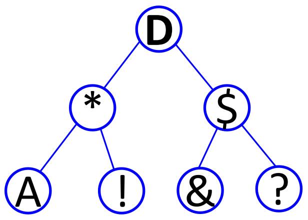
# D $ * A ! ? & or

& ? ! A * $ D #

C. [4%] Please show the heap after we remove the key, "#". The order among the keys are the same as that of the above BST.

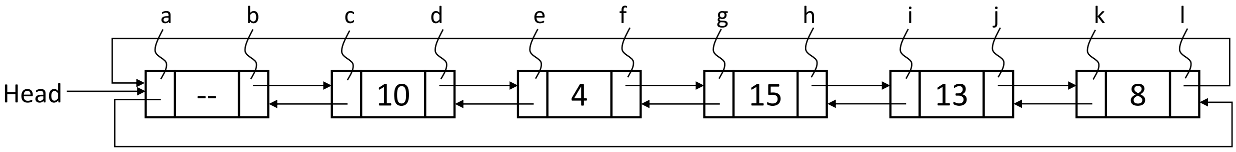


Remove "#" from the heap



10. Please perform the first swapping step of **Quick Sort** on a **doubly linked list with a header**. We want **ascending order** (由小排到大).

15 and 8 need swapping.

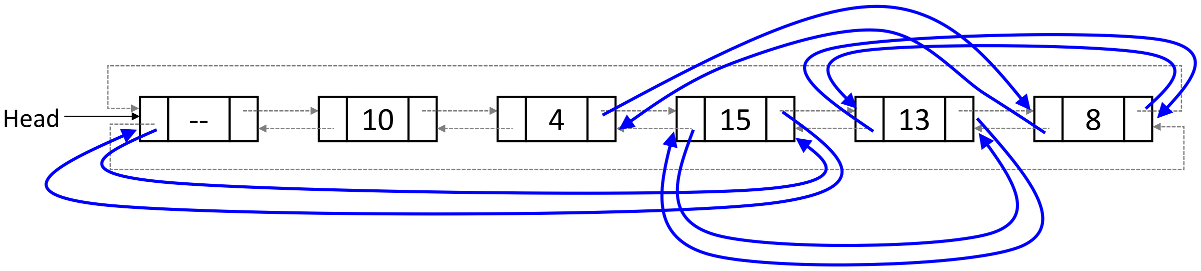


A. [5%] Quick Sort first picks 10 as the pivot, then Quick Sort swaps two keys by change links (instead of only moving keys).

Please tell which links (a, b, ..., k, l) need to be change? a, f, g, h, i, j, k, l

B. [5%] Please **redraw those changed links** (arrows) to reflect the above swapping step.

(請畫出被改動的 links 的箭頭來反映上述 swapping step)**:**



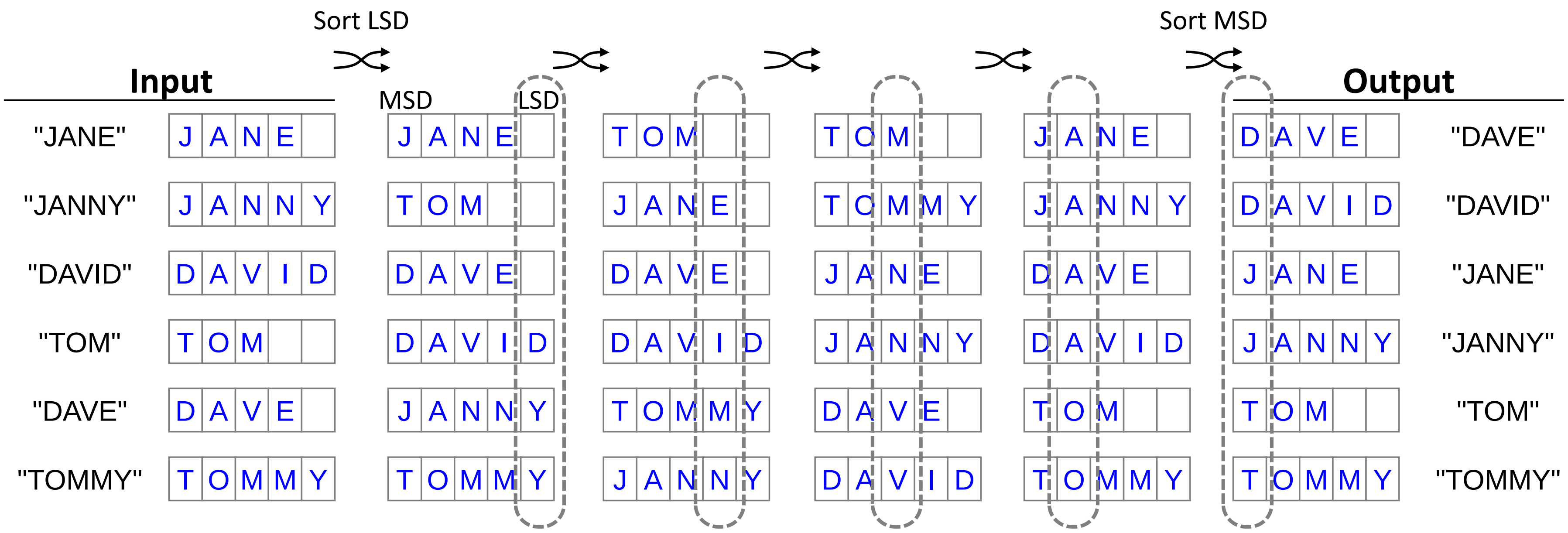
C. [5%] Please show pseudocode for swapping any two nodes in a doubly linked list with a header (by changing links instead of just moving keys):

```
void swap (NODE * a, NODE * b)
// a and b are pointers (a != b) pointing to the two nodes we want to swap.
{
    if (b->right == a) { swap(a, b); }

    a->left->right = b;     b->right->left = a;

    if (a->right == b) {
        a->right = b->right;    b->left = a->left;
        a->left = b;       b->right = a;
    } else {
        a->right->left = b;    b->left->right = a;
        swap(a->left, b->left);      swap(a->right, b->right);
    }
}
```

11. [10%] Please complete the following **LSD-first Radix Sort** process that sort the following names according to the **dictionary order**.

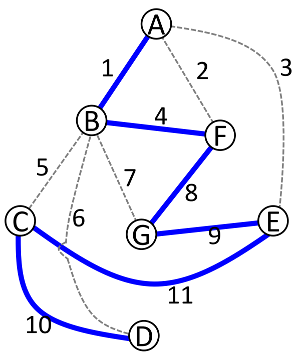


| Input | | | | | | Output | |
|---|---|---|---|---|---|---|---|
| "JANE" | JANE | JANE | TOM | TOM | JANE | DAVE | "DAVE" |
| "JANNY" | JANNY | TOM | JANE | TOMMY | JANNY | DAVID | "DAVID" |
| "DAVID" | DAVID | DAVE | DAVE | JANE | DAVE | JANE | "JANE" |
| "TOM" | TOM | DAVID | DAVID | JANNY | DAVID | JANNY | "JANNY" |
| "DAVE" | DAVE | JANNY | TOMMY | DAVE | TOM | TOM | "TOM" |
| "TOMMY" | TOMMY | TOMMY | JANNY | DAVID | TOMMY | TOMMY | "TOMMY" |

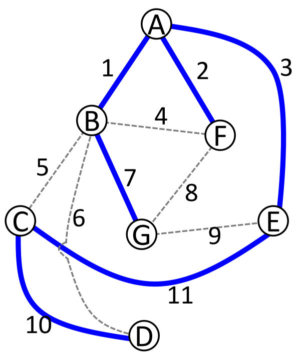
12. [10%] Please show the spanning tree of the following graphs using depth-first search (DFS) and breadth-first search (BFS) traversal given a starting vertex.

- DFS(A) means vertex A is the starting vertex of DFS.
- Please <u>prioritize edges with a smaller number</u> during traversal.

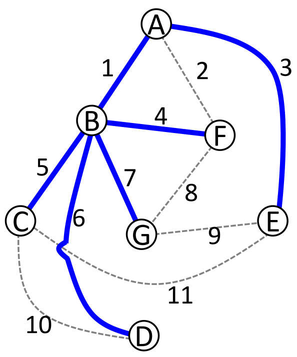
DFS **(A)**



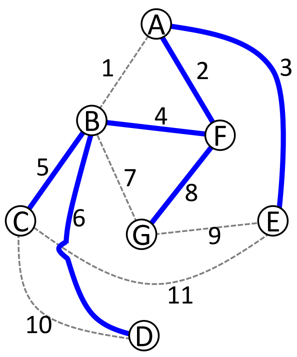
DFS **(G)**



BFS **(B)**



BFS **(F)**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