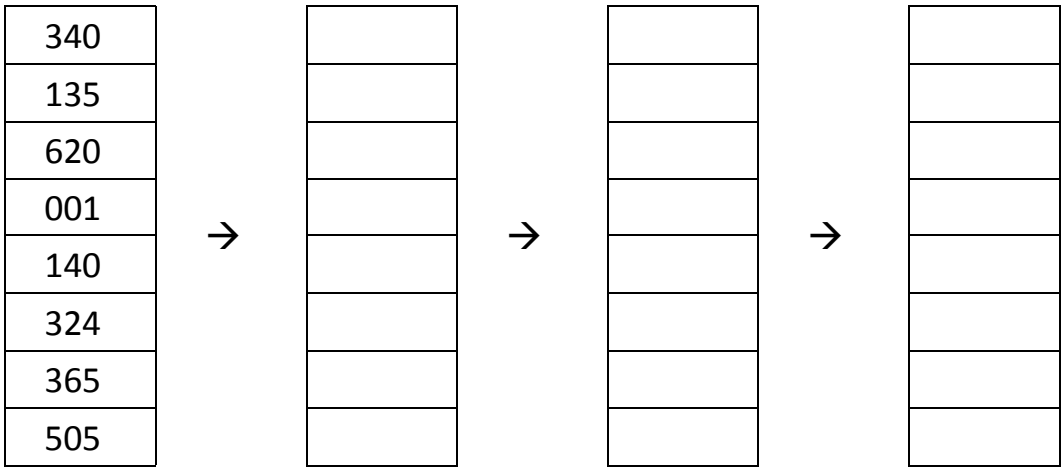
Data Structures Final Examination
3:30pm-5:20pm (110 minutes), Monday, June 22, 2015

ID _____ Name _____

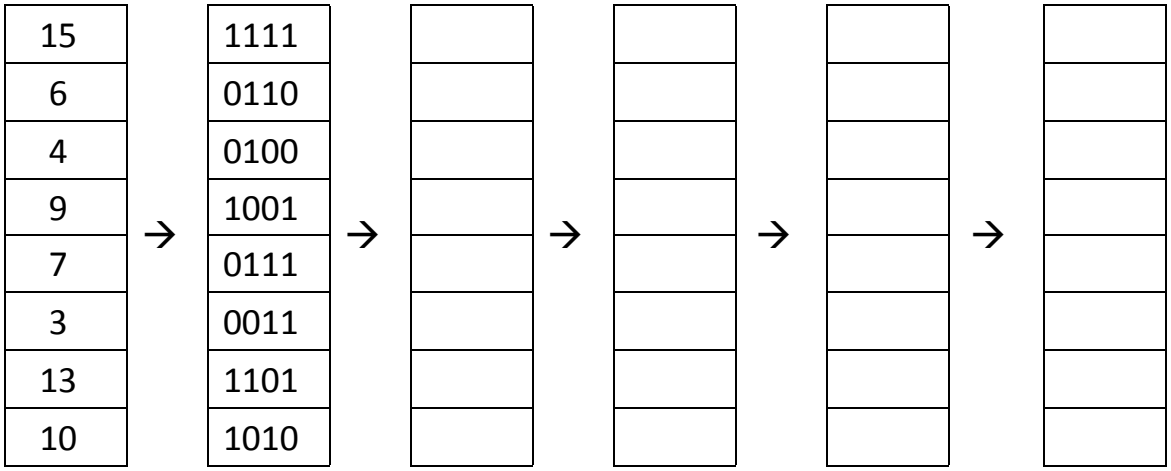
- ✧ Please answer questions 1 ~ 6B on the Question Sheet. For other questions, please answer on the Answer Sheet in any order.
- ✧ There are 6 pages, 10 questions, a total of 115 points.

1.

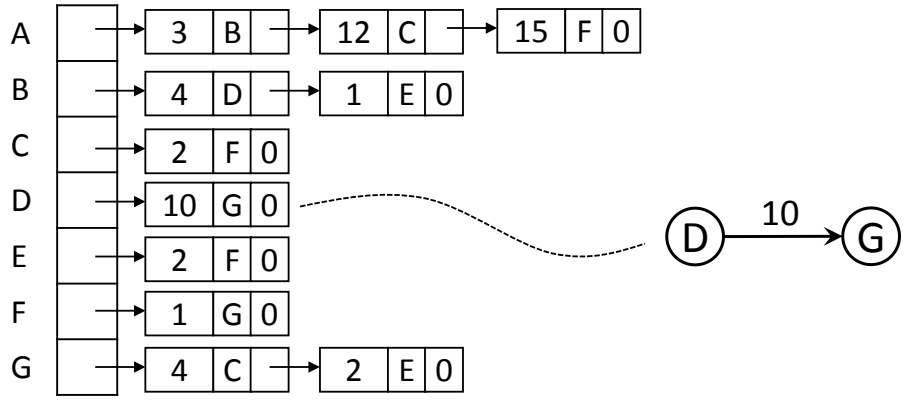
A. Please perform least significant digit (LSD)-first radix sort (radix = 10) over the following numbers in **non-decreasing** order. (5%)



B. Please perform LSD-first radix sort (**radix = 2**) over the following numbers in **non-decreasing** order. (5%)



2. Given a weighted directed graph whose adjacency list representation is as follows



A. Please complete the following table to perform single-source, all-destinations Dijkstra's algorithm starting from vertex A. Please use parenthesis to denote a dist[] value that is not touched and use "—" to denote that a vertex is already selected. (5%)

Iteration	dist[]						Selected Destination	Path Cost
	B	C	D	E	F	G		
1	3	12	∞	∞	15	∞	B	3
2	—	(12)						
3	—							
4	—							
5	—							
6	—							

B. Please complete the following graph-traversal sequences of the graph. If there are multiple valid traversals, just list one of them. Mark an "X" in a field where the traversal cannot continue. (5%)

Depth-first traversal: A, C, F, ___, ___, ___, ___

Breadth-first traversal: A, B, F, ___, ___, ___, ___

Topological traversal: A, B, D, ___, ___, ___, ___

3. Please fill in the following tables to perform Quick Sort. Table 1 shows **Basic Quick Sort** that always takes the **left-most** key of a list/sublist as the pivot (5%). Table 2 shows **Ideal Quick Sort** in which the selected pivot always ideally splits a list/sublist into equal halves, (i.e., 3, 1, and 5 are sequentially selected as pivots) (5%). Please note that common practices always use a swap to move the pivot to the left-most position if the pivot is elsewhere (e.g., the first swap in Table 2).

Table 1 Basic Quick Sort							
Pivot	**Keys**						
1	1	5	4	2	3	0	6
	1	0	4	2	3	5	6

Table 2 "Ideal" Quick Sort							
Pivot	**Keys**						
3	1	5	4	2	3	0	6
	3	5	4	2	1	0	6

4. Please consider inserting the keys 46, 24, 31, 10, 14, 16, 17, 88 into a hash table. The hash function h(k) = (k mod 23).

   A. Please show the result if we use an **eight-bucket table**, **single-slot buckets**, **three least-significant bits of h(k)**, (i.e., h(k) mod 8), and **linear probing**. (5%)

0	
1	
2	
3	
4	
5	
6	
7	

   B. Please show the result if we use an **eight-bucket table**, **three least-significant bits of h(k)**, and **chaining**. (5%)

0	
1	
2	
3	
4	
5	
6	
7	

C. Please show the result if **chaining** and **directory-less dynamic hashing** is used and the number of buckets increases from eight to nine after 88 is inserted. (5%)

0	
1	
2	
3	
4	
5	
6	
7	
8	

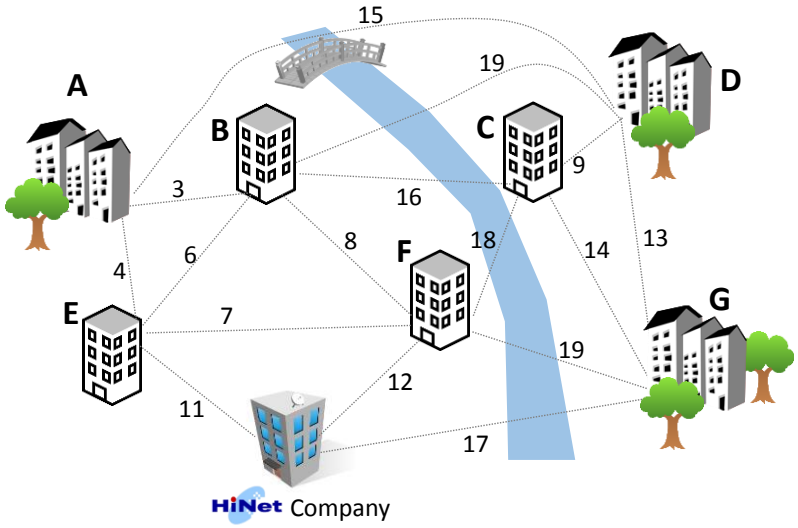
5. We want to perform heap sort to sort an array "5, 19, 25, 15, 20, 16, 10, 30" into non-decreasing order. The first phase of heap sort is to in-place heapify the array as a **max heap**.

A. Please list the array contents after heapification completes if we use a **binary heap**, in which every parent have two children. (5%)

B. Please list the array contents after heapification completes if we use a **ternary heap**, in which every parent have three children. (5%)

C. Please list the array contents after the sorting algorithm pops the top element from the max heap and places the element at the end of the array (considering a **binary heap**) (5%)

6. HiNet company wants to use a fiber network to connect eight buildings, A to G, and the HiNet building, together. The candidate fiber routes and the corresponding cost (in the unit of 100,000 NT$) are shown as follows.



A. Please mark the fiber network that has the minimum total cost. (5%)

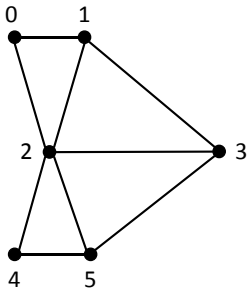
B. Please describe an algorithm that can find the network with **the second minimum** total cost (5%).

7. omited

8.

A. Please design an algorithm (using pseudo code) that takes an undirected graph in adjacency matrix representation as input and determines whether an Eulerian path exists. An Eulerian path is a path in a graph which visits each edge exactly once. A graph has an Eulerian path if and only if the number of vertices that have odd degree is either zero or two. (5%)

B. Please use an adjacency matrix to represent the following undirected graph. (5%)

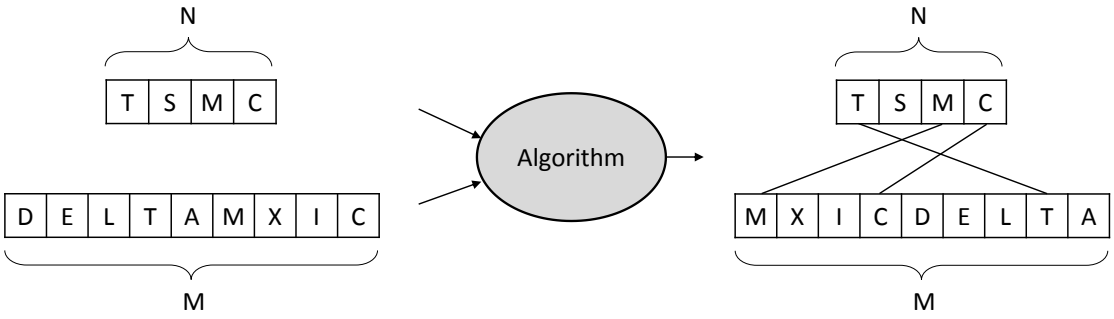


9. Please perform decision tree based algorithm analyses.

A. Please prove that any comparison-based algorithm requires log(N!) comparisons in the worst case to sort an N-element list. (5%)

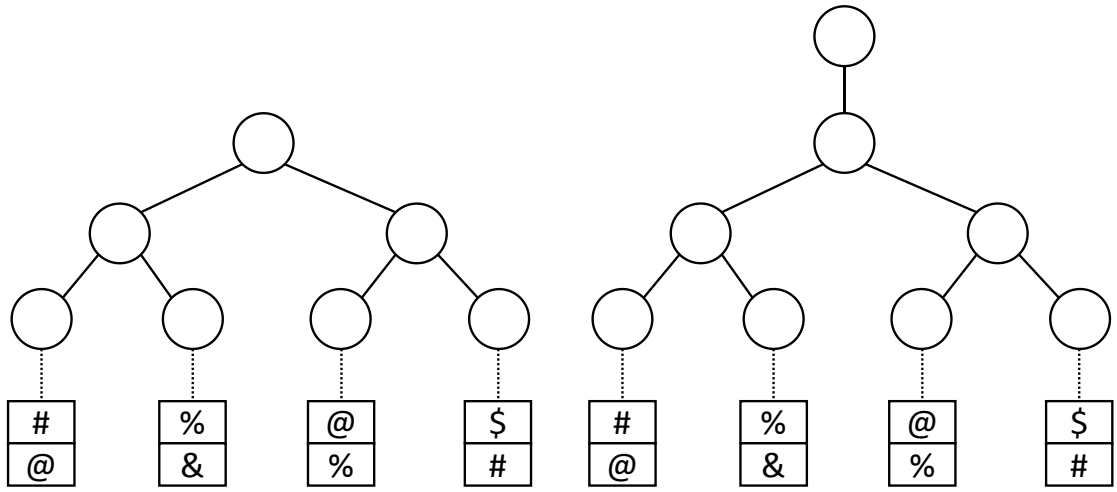
B. Please derive the lower bound of the worst-case number of comparisons any

comparison-based algorithm requires to pair two lists, one with N distinct keys and the other with M distinct keys (N<M). The following graph shows exampling inputs and outputs of such a pairing algorithm. (5%)



10.

A. Please complete the following winner and loser trees (5%).



B. Please plot the result of sequentially inserting eight letters, “J U N E 2 0 1 5”, into an empty, **standard binary search tree**. (0 < 1 < 2 < 5 < E < J < N < U) (5%)

C. Please plot the result of sequentially inserting eight letters, “J U N E 2 0 1 5”, into an empty, **red-black tree**. (0 < 1 < 2 < 5 < E < J < N < U) (Hint: Check whether two consecutive red nodes appear after insertion; if so, check whether the uncle node is red or black; perform rotation or color changes accordingly; always color the root node black.) (5%)

Thank you for your participation during the class.
Best wishes in your future studies!