1-1 Time Complexity

```
for (int i=0; i<N; i++)
  for (int j=0; j<i*i; j++)
    for (int k=1; k<j; k=k*2)
      print("Hello");
```

$\Omega($ $N^3 \log (N)$ $)$

$O($ $N^3 \log (N)$ $)$

$$f(N) = (\underbrace{1 \log 1 + 4 \log 4 + 9 \log 9 + \ldots. + N^2 \log N^2)}_{N項}$$

$$f(N) \leqq (N * N^2 \log N^2)$$
$$= \Theta (N^3 \log (N))$$

i --> N

j --> N^2

k --> $k*2^m >= j$ (k start at 1)

m >= $\log_2 j = \log j / \log 2$ --> log j

--> $\log j = \log N^2$ --> log N

=> $i*j*k = N^3 \log N$

$$f(N) \geqq (\frac{N}{2} * \left(\frac{N}{2}\right)^2 \log \left(\frac{N}{2}\right)^2)$$
$$= C N^3 \log \left(\frac{N}{2}\right)$$
$$= C N^3 \log (N) - C N^3$$
$$= \Theta (N^3 \log (N))$$

1-2 Time Complexity

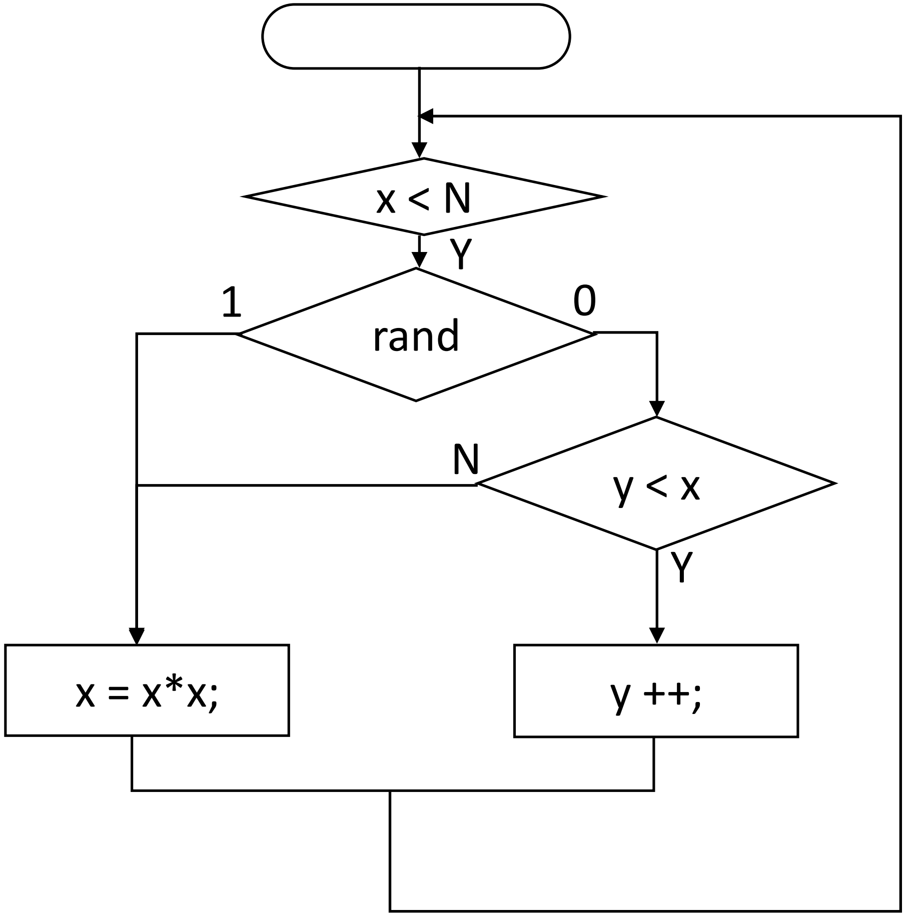
令 j 全部等於1 && --> 得到最小值
令 j 全不等於0 && y>x --> 得到最大值

```
x = 2; y = 2;
while (x<N) {
  j = random 0 or 1;
  if (j == 0 && y<x) {
    y=y+1;
  }else{
    x=x*x;
  }
}
```

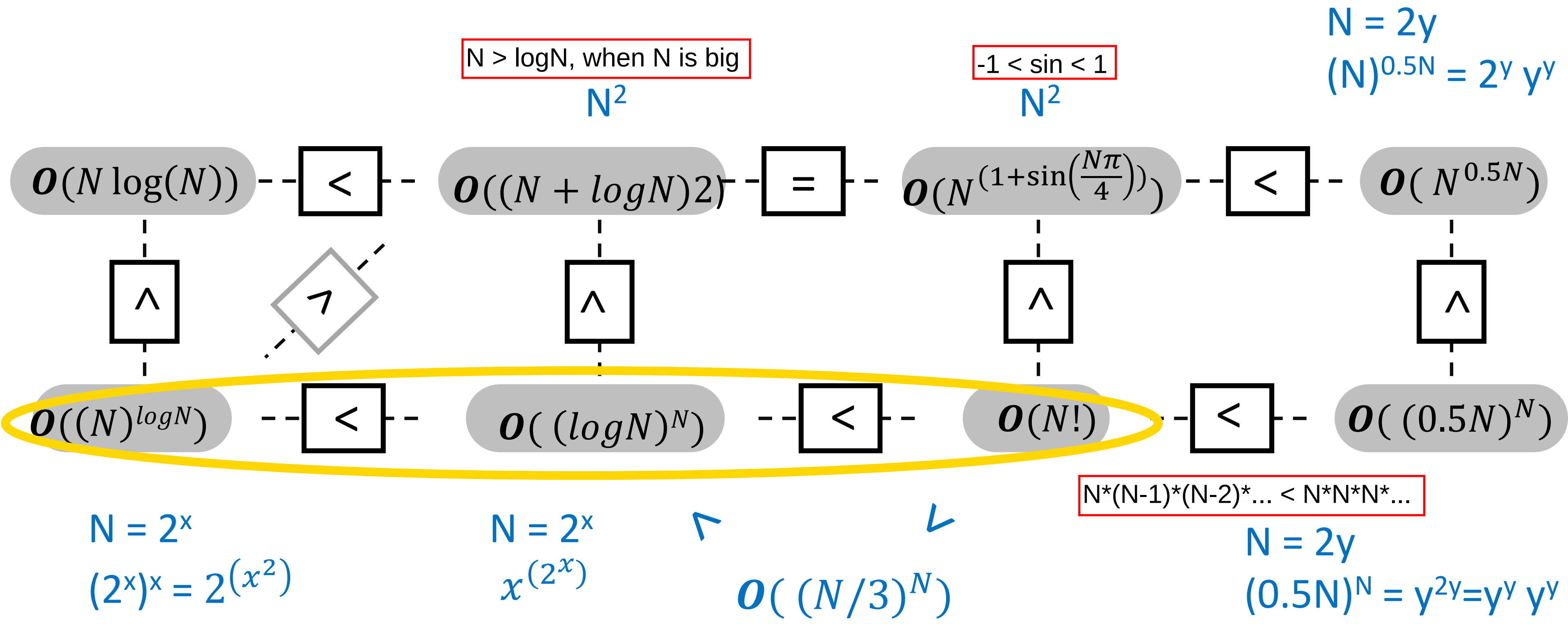
(2^2)^m >= N --> m複雜度為 log log N

Ω(log log (N) **)**

O(N **)**



2 Complexity Hierarchy



N > logN, when N is big

N^2

-1 < sin < 1

N^2

N = 2y

$(N)^{0.5N} = 2^y y^y$

$O(N\log(N))$ < $O((N+logN)2)$ = $O(N^{(1+\sin(\frac{N\pi}{4}))})$ < $O(N^{0.5N})$

$\wedge$ $\wedge$ $\wedge$ $\wedge$ $\wedge$

$O((N)^{logN})$ < $O((logN)^N)$ < $O(N!)$ < $O((0.5N)^N)$

N*(N-1)*(N-2)*... < N*N*N*...

N = 2^x

$(2^x)^x = 2^{(x^2)}$

N = 2^x

$x^{(2^x)}$

$<$ $<$

$O((N/3)^N)$

N = 2y

$(0.5N)^N = y^{2y} = y^y y^y$

3 Recursive MAX

your code goes here

```
int RMAX(int array[], int size)
{
   int m;

   if (size == 1) return array[0];

   int m1 = RMAX (array, size/2);
   int m2 = RMAX (array+size/2, size-size/2);

   if (m1>m2)  m = m1;
   else  m = m2;

   return m;
}
```

Time Complexity

- T(size) = **2 T(size/2) + 1**

  = 4 T(size/4) + 2 + 1
  = 8 T(size/8) + 4 + 2 + 1

$$= \underbrace{\ldots. +4 +2 +1}_{\log(\text{size})}$$

令 size = n, ~ 表"約等於"
$n/(2^m) >= 1$ --> $n >= 2^m$ --> $m \sim \log n$
$1+2+4+\ldots+2^{m-1} = 1*(2^m-1)/(2-1) = (2^m - 1) \sim 2^{\log n}$

$= \Theta(2^{\log(\text{size})})$ = **Θ (size)**

3 Recursive MAX

your code goes here

```
int RMAX(int array[], int size)
{
    int m;
    if (size == 1) return array[0];
    int m1 = array [0];
    for(int i=0; i<size/2; i++)
       if (m1 > array[i]) m1 = array[i];

    int m2 = RMAX (array+size/2, size-size/2);

    m = (m1 > m2)? m1 : m2;
    return m;
}
```

3 Recursive MAX

your code goes here

```
int RMAX(int array[], int size)
{
    int m;
    if (size == 1) return array[0];

    for(int i=0; i<size/2; i++) {
        if (array[i] < array[size-i-1])
            swap(array[i], array[size-i-1]);
    }

    m = RMAX (array, size/2);
    return m;
}
```

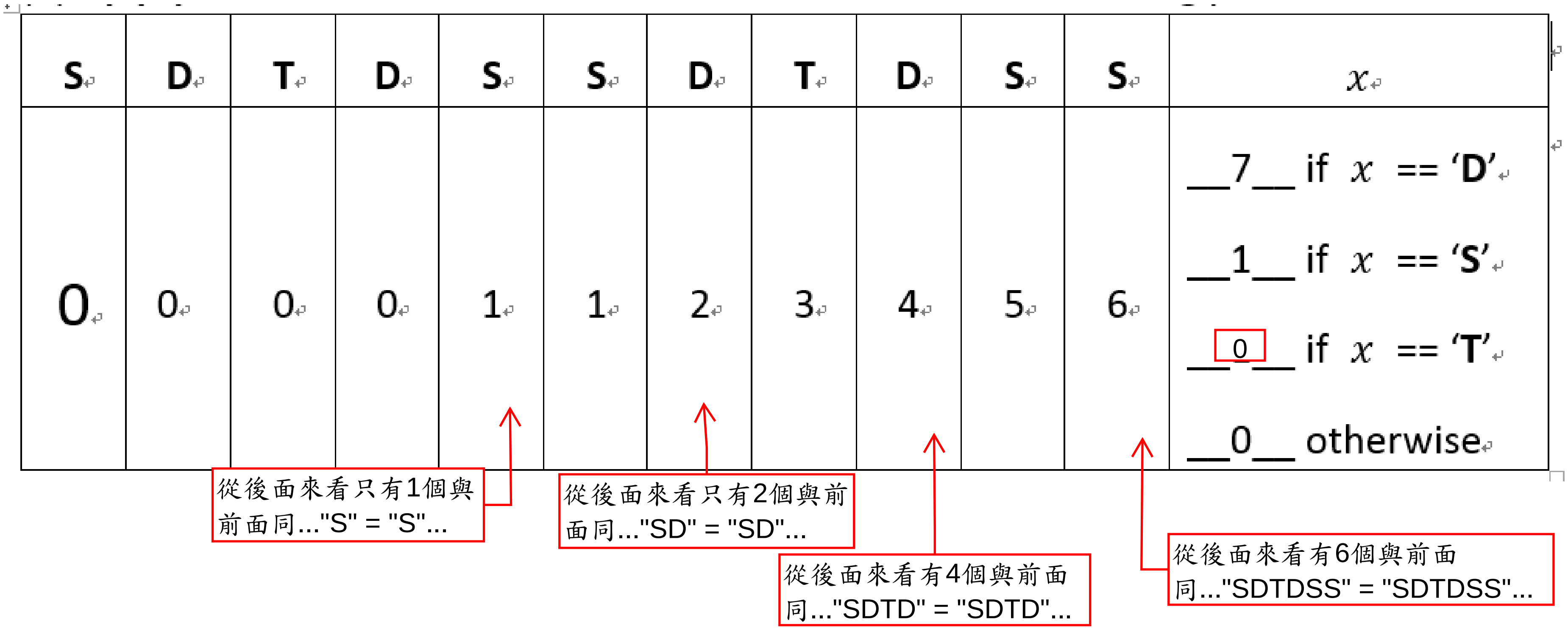
Time Complexity

- T(size) = **T(size/2) + size/2**

  = T(size/4) + size/4 + size/2
  = T(size/8) + size/8 + size/4 + size/2
  = 1 + … + size/8 + size/4 + size/2

  = **Θ (size)**

4-1 KMP



S	D	T	D	S	S	D	T	D	S	S	x
0	0	0	0	1	1	2	3	4	5	6	__7__ if x == 'D' __1__ if x == 'S' __0__ if x == 'T' __0__ otherwise

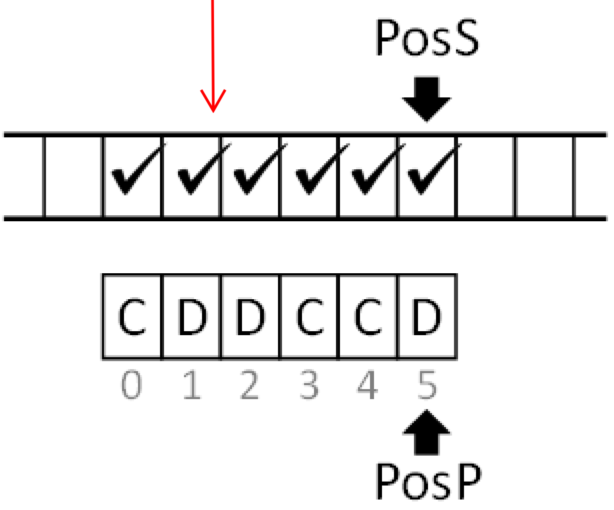
從後面來看只有1個與前面同..."S" = "S"...

從後面來看只有2個與前面同..."SD" = "SD"...

從後面來看有4個與前面同..."SDTD" = "SDTD"...

從後面來看有6個與前面同..."SDTDSS" = "SDTDSS"...

4-2 KMP

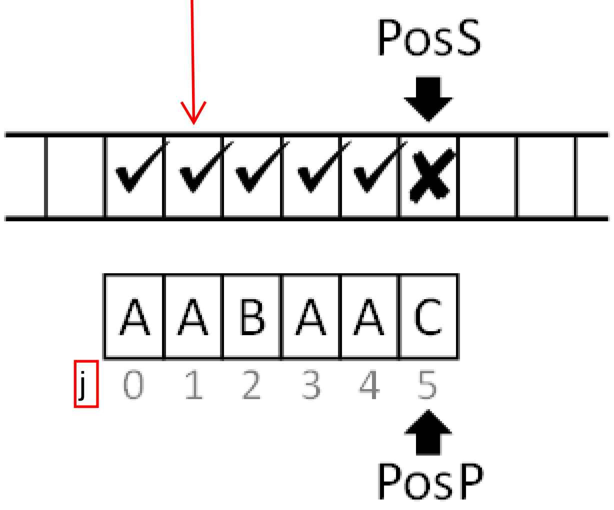


failure function：(-1,-1,-1,0,0,1)

Next action:

PosS ++;

PosP = ____2____;



failure function：(-1,0,-1,0,1,-1)

Next action:

PosS = PosS;

PosP = ____2____; f(4)+1

5 Infix to Postfix

bottom

Token	Stack						Output So Far																
A							A																
||	||						A																
(	||	(					A																
A	||	(					A	A															
+	||	(	+				A	A															
B	||	(	+				A	A	B														
*****	||	(	+	*			A	A	B														
C	||	(	+	*			A	A	B	C													
%	||	(	+	%			A	A	B	C	*												
9	||	(	+	%			A	A	B	C	*	9											
)	||						A	A	B	C	*	9	%	+									
<	||	<					A	A	B	C	*	9	%	+									
D	||	<					A	A	B	C	*	9	%	+	D								
==	||	==					A	A	B	C	*	9	%	+	D	C							
T	||	==					A	A	B	C	*	9	%	+	D	<	T						
&&	||	&&					A	A	B	C	*	9	%	+	D	<	T	==					
B	||	&&					A	A	B	C	*	9	%	+	D	C	T	==	B				
	Final output						A	A	B	C	*	9	%	+	D	C	T	==	B	&&	||		

Priority	**Operator**
High	In-coming (
	*, /, %
	+, -
	<, <=, >=, >
	==, !=
	&&
	||
Low	In-stack (

6 Asymptotic Notations

$F(N) + G(N) = O(N!)$

→ exist natural numbers c and N_0 such that
$F(N) + G(N) \leqq c\,N!$ for $N \geqq N_0$

$F(N)*G(N) \leqq (F(N) + G(N))^2/4$

$\leqq c^2/4\,(N!)^2$ for $N \geqq N_0$

$(N!)^2 < (2N)!$

Let $c' = c^2/4$ $N_0' = N_0$

$F(N)*G(N) \leqq c'\,(2N)! = O((2N)!)$ for $N \geqq N_0'$ QED

7-1 Three Basic Structures

- Sequential
- Selection (if-else)
- Iteration (loop)

7-2 Structured vs non-Structured

- Structured program cannot always achieve better speed than a non-structured one

- Structured programs are always compiled into machine language programs, which are non-structured programs

7-3 Access levels of objects

- Changing the access levels of objects cannot affect the function of a program
  - Access levels are NOT designed for realizing functions
  - Access levels are NOT designed for protecting 智慧財產權

Ch1_33

- Access levels are for maintaining a clean object interface
  - Preventing object users from accidently messing up the internal values of the object
  - Preventing object users from relying on the internal values of the object

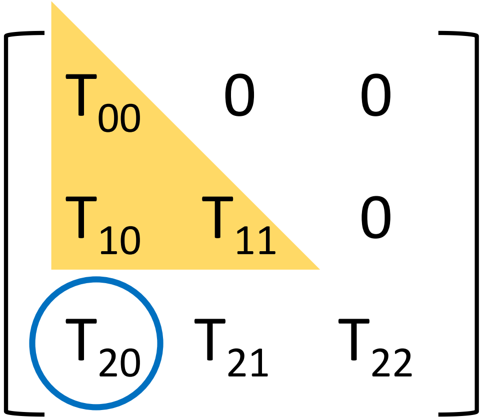
8 Lower-Triangular Matrix

$$\begin{bmatrix} T_{00} & 0 & 0 \\ T_{10} & T_{11} & 0 \\ T_{20} & T_{21} & T_{22} \end{bmatrix}_{\boxed{2*2}}$$

T_{00}	T_{10}	T_{11}	T_{20}	T_{21}	T_{22}
0	1	2	3	4	5

- Number of integers for storing directly using a raw array
- 1 + 2 + 3 +… + N = (1+N)N/2

8 Lower-Triangular Matrix



$$\begin{bmatrix} T_{00} & 0 & 0 \\ T_{10} & T_{11} & 0 \\ T_{20} & T_{21} & T_{22} \end{bmatrix}$$

位置

T_{00}	T_{10}	T_{11}	T_{20}	T_{21}	T_{22}
0	1	2	3	4	5

- Offset of T_{ij}

= (1+i) i /2 + j

- T_{20}

= (1+2)*2/2 + 0 = 3

Ch2_37~

Lower-Triangular Matrix

$$\begin{bmatrix} T_{00} & 0 & 0 \\ T_{10} & T_{11} & 0 \\ T_{20} & T_{21} & T_{22} \end{bmatrix}$$

T_{00}	T_{10}	T_{11}	T_{20}	T_{21}	T_{22}
0	1	2	3	4	5

$$\text{\# of integers} = \overset{\text{probability}}{\boldsymbol{(1\text{-}R)}} * (1+N)N/2\ \boldsymbol{*}\ \overset{\text{row, col, value}}{\boldsymbol{3}} + 2$$

If the matrix is stored in a sparse matrix