訊號與系統
Signals and Systems
EECS2020
Final Exam

16:30 – 18:45, 06/24/2021

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National Tsing Hua University

Spring 2021

- Remember to write down your name, student ID number, date, and page number (page number/total page number) at the **top-right corner** of **each page** of your solution paper,

  e.g., Calvin Li, 1080610111, 06/24/2021, 2/7

- **10 problems** in total (100 points)

- Total pages of this exam paper: **19** (including 7 pages of the FT, LT, and z-T pair tables)

- If any definition is not clear, please feel free to make your own assumptions or ask the TAs and instructor. If you make your own assumptions, you will need to clarify what your assumptions are, nonetheless.

- Tables of Fourier transform, Laplace transform, and z-transform pairs are provided at the end of the exam paper.

Problem 1 (12%)

Consider a continuous-time LTI system whose impulse response $h(t)$ is real and even and its frequency response is band limited. Tell each of the following statements is **true** or **false**.

F (a) The system is causal.

F (b) The system is invertible.

T (c) The frequency response is even.

T (d) The frequency response is real.

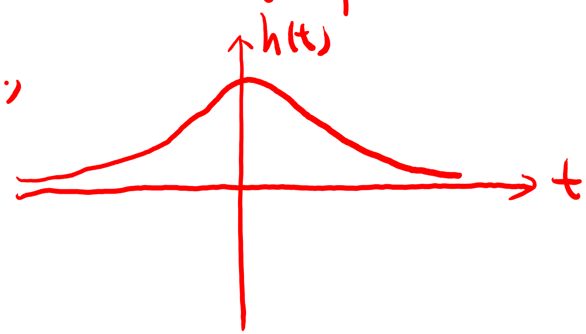
T (e) The continuous-time Fourier transform of $\frac{dh(t)}{dt}$ is imaginary.

F (f) The continuous-time Fourier transform of $t \cdot h(t)$ is real.

$h(t)$: real & even ⇒ freq response $\mathcal{F}\{h(t)\}$: real & even

$H(j\omega)$: band limited freq response ⇒ IIR system, not invertible because of no inverse system $\cancel{\frac{1}{H(j\omega)}}$

e.g.,



(a) $h(t) \neq 0,\ t < 0$ ⇒ Not causal

(b) Not invertible ($\because \frac{1}{H(j\omega)}$ does not exist)

(c) $H(j\omega)$: even

(d) $H(j\omega)$: odd.

(e) $\frac{dh(t)}{dt}$: odd & real ⇒ $\mathcal{F}\{\frac{dh(t)}{dt}\}$: imaginary

(f) $th(t)$: real, $\mathcal{F}\{th(t)\}$: conjugate symmetry, Not real

Problem 2 (8%)

Let $x(t)$ be a real-value continuous-time signal and its maximum frequency (in Fourier transform) is 100 Hz. Determine the minimum sampling rate for each of the following signals, which results in no aliasing in frequency domain. (You do not need to show the calculation procedure. Just write down your answer)

(1) $x(t) + x(t\text{-}1)$ $> 200\text{Hz}$

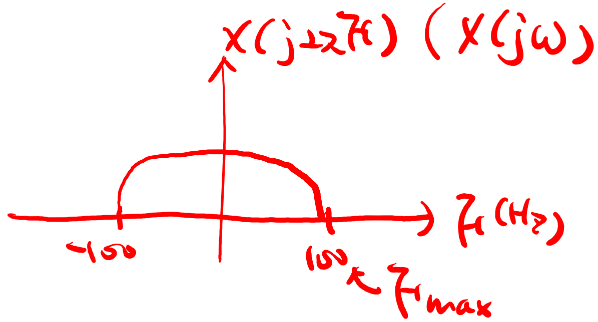
(2) $\dfrac{dx(t)}{dt}$ $> 200\text{Hz}$

(3) $x^2(t)$ $> 400\text{Hz}$

(4) $x(t)\cos(400\pi t)$ $> 600\text{Hz}$.

Sampling theorem

$f_s > 2 \cdot f_{max}$



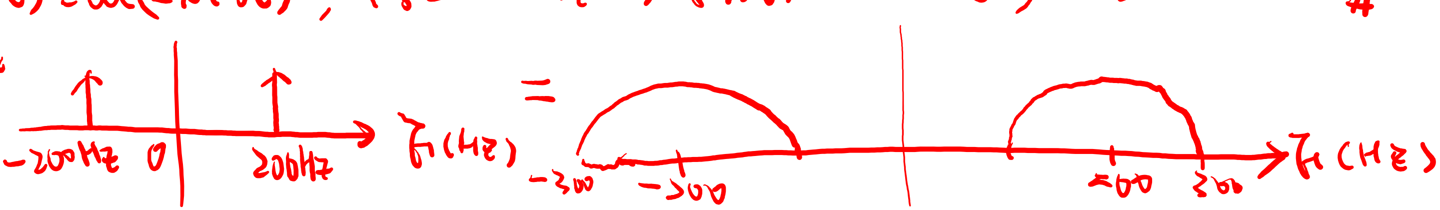
(1) $X(j\omega) + e^{-j\omega \cdot 1} X(j\omega) \Rightarrow f_{max} = 100\text{Hz}$, $f_s > 200\text{Hz}$ #

(2) $j\omega \cdot X(j\omega) \Rightarrow f_{max} = 100\text{ Hz}$, $f_s > 200\text{Hz}$ #

(3) $X(j\omega) * X(j\omega) \Rightarrow f_{max} = 200\text{Hz}$, $f_s > 400\text{ Hz}$ #

(4) $\cos(400\pi t) = \cos(2\pi f_0 t)$, $f_0 = 200\text{Hz} \Rightarrow f_{max} = 300\text{Hz}$, $f_s > 600\text{ Hz}$ #

$X(j\omega) *$



Problem 3 (5%)

Find the Fourier transform of $x(t) = \frac{1}{1+jt}$. (*Hint*: Duality property.)

Remember to write down your derivation.

From the table, $e^{-at}u(t),\ \mathrm{Re}\{a\}>0 \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{a+j\omega}$.

$\Rightarrow e^{-t}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{1+j\omega}$ —(*)

from (*) $e^{-t}u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+j\omega} e^{+j\omega t}\, d\omega$.

replace t by -t & x 2π

$\Rightarrow 2\pi \cdot e^{t}u(-t) = \int_{-\infty}^{\infty} \frac{1}{1+j\omega} e^{-j\omega t}\, d\omega$

interchange the name of t & ω

$\Rightarrow 2\pi e^{\omega}u(-\omega) = \int_{-\infty}^{\infty} \underbrace{\frac{1}{1+jt}}_{x(t)} e^{-j\omega t}\, dt$.

$\Rightarrow \mathcal{F}\{x(t)\} = \mathcal{F}\left\{\frac{1}{1+jt}\right\} = 2\pi e^{\omega}u(-\omega)$ #

Problem 4 (10%)

Derive the transfer functions of the following impulse responses. Remember to write down your derivation and include the region of convergence (ROC) in your solution.

(1) $h(t) = -te^{-0.5t}u(-t)$ (Given transform pairs 6&7 of Table 9.2 in page 17) (5%)

(2) $h[n] = -(n+1)0.1^n u[-n-1]$(Given transform pairs 5&6 of Table 10.2 in page 19) (5%)

(1) From transform pair 7, Table 9.2.

$$-e^{-at}u(-t) \xleftrightarrow{\mathcal{L}} (s+a)^{-1},\quad Re\{s\} < -a.$$

$a = 0.5$

$$\Rightarrow -e^{-0.5t}u(-t) \xleftrightarrow{\mathcal{L}} (s+0.5)^{-1},\quad Re\{s\} < -0.5 \quad -(*)$$

differentiation in s-domain property

$$x(t) \xleftrightarrow{\mathcal{L}} X(s),\quad ROC: R$$

$$t\,x(t) \xleftrightarrow{\mathcal{L}} -\frac{dX(s)}{ds}$$

$$(*) \Rightarrow -te^{-0.5t}u(-t) \xleftrightarrow{\mathcal{L}} (s+0.5)^{-2},\quad Re\{s\} < 0.5 \ \#$$

$$\xleftrightarrow{\mathcal{L}} \frac{1}{(s+0.5)^2},\quad Re\{s\} < 0.5 \ \#$$

(2) From transform pair 6, Table 10.2

$$-a^n u[-n-1] \xleftrightarrow{z} (1-az^{-1})^{-1}, \quad |z|<a. \quad \text{—(*)}$$

$$h[n] = -(n+1)(0.1)^n u[-n-1]$$

$$= \underbrace{-n\cdot(0.1)^n u[-n-1]}_{(1)} \; \underbrace{-(0.1)^n u[-n-1]}_{(2)}.$$

$$a = 0.1 \quad (*) \Rightarrow -(0.1)^n u[-n-1] \xleftrightarrow{z} \underbrace{(1-0.1z^{-1})^{-1}, \; |z|<0.1}_{\mathcal{Z}\{(2)\}} \quad \text{—(**)}$$

differentiation in the z-domain property

$$x[n] \xleftrightarrow{z} X(z), \quad \text{ROC: } R$$

$$n\,x[n] \xleftrightarrow{z} -z\frac{dX(z)}{dz}, \quad \text{ROC: } R.$$

$$(**) \Rightarrow -n(0.1)^n u[-n-1] \xleftrightarrow{z} (-z)(-1)(1-0.1z^{-1})^{-2}(0.1)(-1)z^{-2}$$

$$\xleftrightarrow{z} \underbrace{\frac{0.1z^{-1}}{(1-0.1z^{-1})^2}, \; |z|<0.1}_{\mathcal{Z}\{(1)\}} \quad \text{—(***)}$$

$$\mathcal{Z}\{h[n]\} = \underbrace{\frac{0.1z^{-1}}{(1-0.1z^{-1})^2} + \frac{1}{1-0.1z^{-1}}, \; |z|<0.1}$$ #

Problem 5 (10%)

A CT LTI system is described by the following transfer function.

$$H(s) = \frac{1}{s^2 + 7s + 6}$$

(1) Plot the pole-zero plot. (1%)

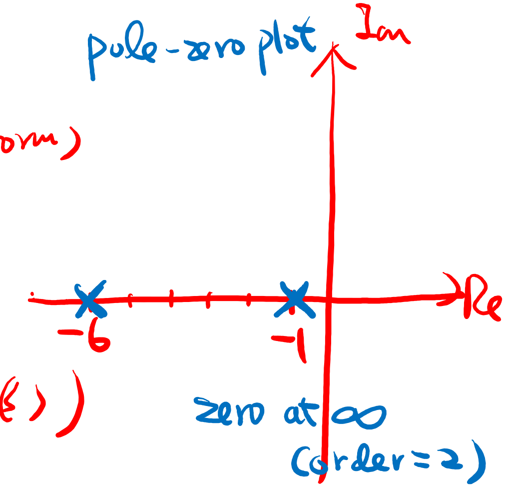
(2) Discuss the causality and stability of different ROCs. (3%)

(3) Find the impulse responses of different ROCs. (6%)

(1) $H(s) = \frac{N(s)}{D(s)} = \frac{1}{s^2+7s+6} = \frac{1}{(s+6)(s+1)}$ (pole - zero form)

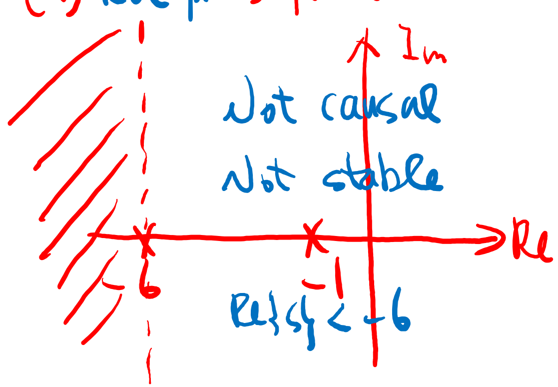
pole : $s_p = -6, -1$

zero : $s_z = \infty$ (order = 2 $\because$ order of $D(s)$ > order of $N(s)$)

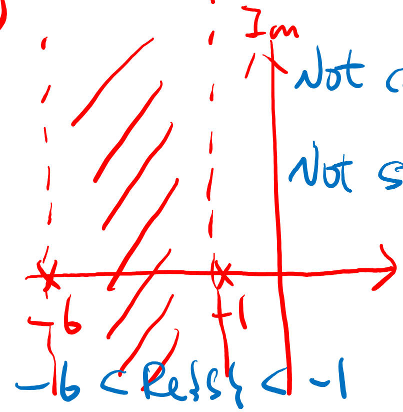


(2). $H(s)$: rational system fx

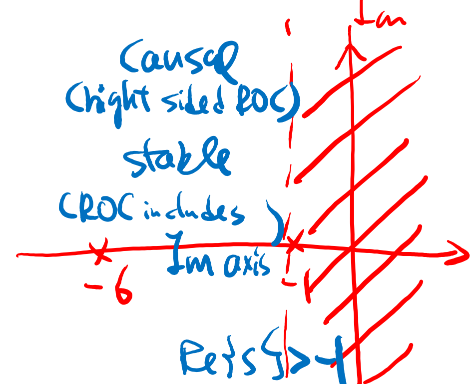
(a) ROC plot s plane



(b) ROC plot s plane



(c) ROC plot s plane



(3) $H(s) = \dfrac{1}{(s+6)(s+1)} = \dfrac{A}{s+1} + \dfrac{B}{s+6}$

$A = (s+1)\,H(s)\Big|_{s=-1} = \dfrac{1}{s+6}\Big|_{s=-1} = \dfrac{1}{5}$

$B = (s+6)\,H(s)\Big|_{s=-6} = \dfrac{1}{s+1}\Big|_{s=-6} = -\dfrac{1}{5}$

$\Rightarrow H(s) = \underbrace{\dfrac{\frac{1}{5}}{s+1}}_{(1)} - \underbrace{\dfrac{\frac{1}{5}}{s+6}}_{(2)}$

ROC(a)

$h(n)$: left sided signal $\Rightarrow$ (1) & (2) left sided

from the table

$h(n) = -\frac{1}{5}e^{-t}u(-t) + \frac{1}{5}e^{-6t}u(-t)$ #

ROC(b)

$h(n)$: two sided signal $\Rightarrow$ (1) left sided

(2) right sided.

from the table

$h(n) = -\frac{1}{5}e^{-t}u(-t) - \frac{1}{5}e^{-6t}u(t)$ #

ROC(c)

$h(n)$: right sided signal

$\Rightarrow$ (1) & (2) right sided signals

from the table

$h(n) = \frac{1}{5}e^{-t}u(t) - \frac{1}{5}e^{-6t}u(t)$ #

Problem 6 (10%)

A DT LTI system is described by the following transfection.

$$H(z) = \frac{2 - 0.75z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

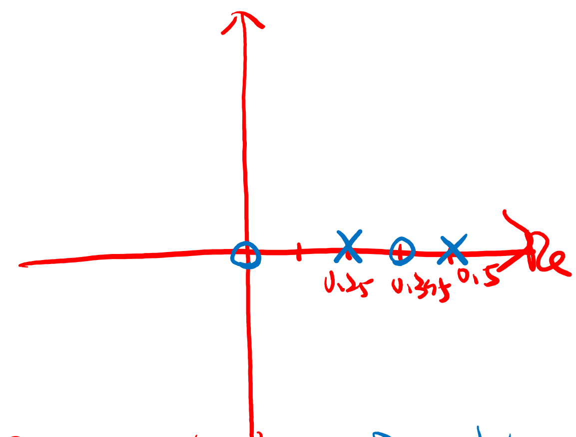
(1) Plot the pole-zero plot. (1%)

(2) Discuss the causality and stability of different ROCs. (3%)

(3) Find the impulse response of different ROCs. (6%)

pole-zero plot

z plane



(1) $H(z) = \frac{N(z)}{D(z)} = \frac{2 - 0.75z^{-1}}{(1-0.5z^{-1})(1-0.25z^{-1})} = \frac{z(2z-0.75)}{(z-0.5)(z-0.25)}$

pole-zero form

pole : $z_p = 0.5,\ 0.25$

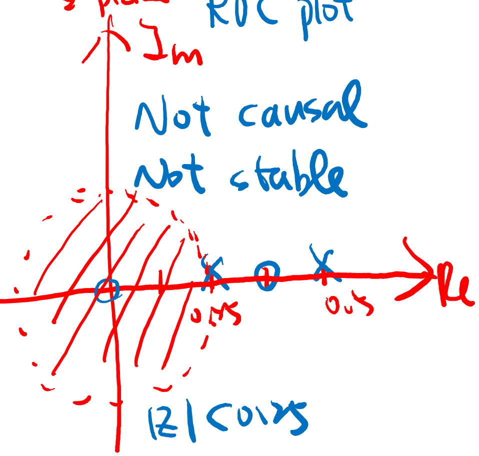
zero : $z_z = 0,\ 0.375$

(order of $N(z)$ = order of $D(z)$
No pole & No zero at ∞)

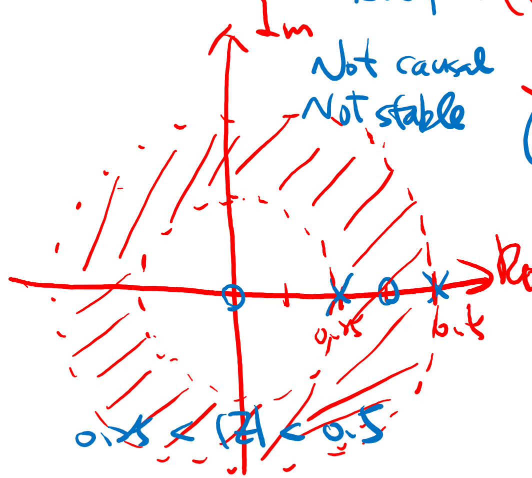
(2) (a) z plane ROC plot

H(z) rational system fx

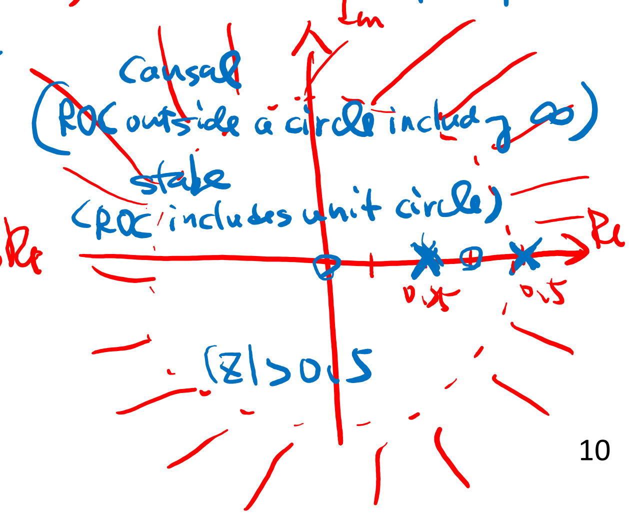
No pole at infinite.



(b) z-plane ROC plot



(c) z-plane ROC plot



(3) $H(z) = \dfrac{2-0.175z^{-1}}{(1-0.5z^{-1})(1-0.25z^{-1})} = \dfrac{A}{1-0.5z^{-1}} + \dfrac{B}{1-0.25z^{-1}}$

$A = (1-0.5z^{-1})H(z)\Big|_{z=0.5} = \dfrac{2-0.175z^{-1}}{1-0.25z^{-1}}\Big|_{z=0.5} = 1$

$B = (1-0.25z^{-1})H(z)\Big|_{z=0.25} = 1$

$H(z) = \underbrace{\dfrac{1}{1-0.5z^{-1}}}_{(1)} + \underbrace{\dfrac{1}{1-0.25z^{-1}}}_{(2)}$

ROC (a) $\Rightarrow h[n]$: left sided $\Rightarrow$ (1) & (2) left sided.

from the table

$h[n] = -(0.5)^n u[-n-1] - (0.25)^n u[-n-1]$ #

ROC (b) $\Rightarrow h[n]$: two sided $\Rightarrow$ (1) left sided

(2) right sided.

from the table

$h[n] = -(0.5)^n u[-n-1] + (0.25)^n u[n]$ #

ROC (c) $\Rightarrow h[n]$: right sided

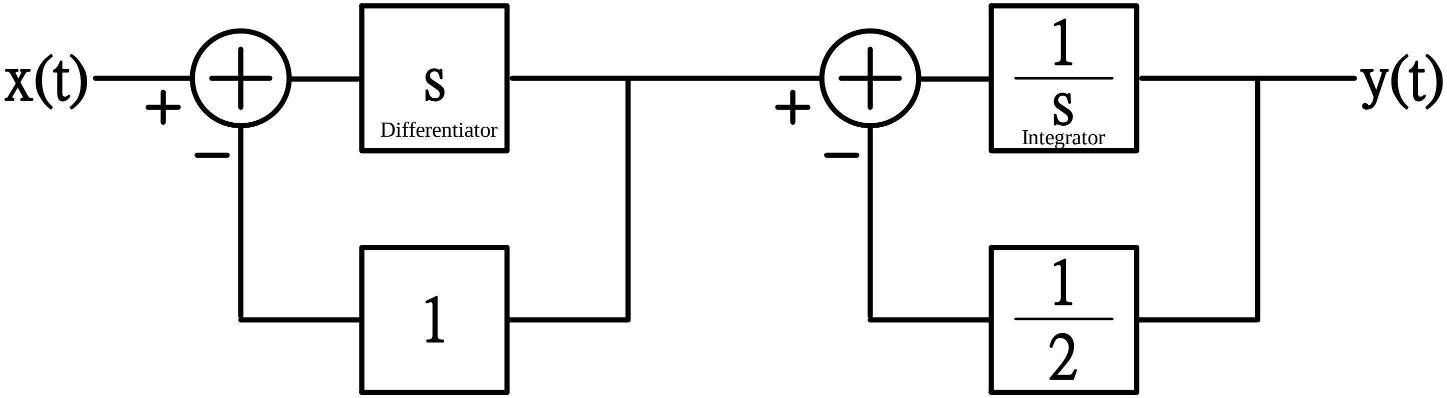
$\Rightarrow$ (1) & (2) right sided.

from the table

$h[n] = (0.5)^n u[n] + (0.25)^n u[n]$ #

Problem 7 (15%)

A CT "**causal**" LTI system is represented by the following block diagram.



algebraic form(2分), ROC(2分)

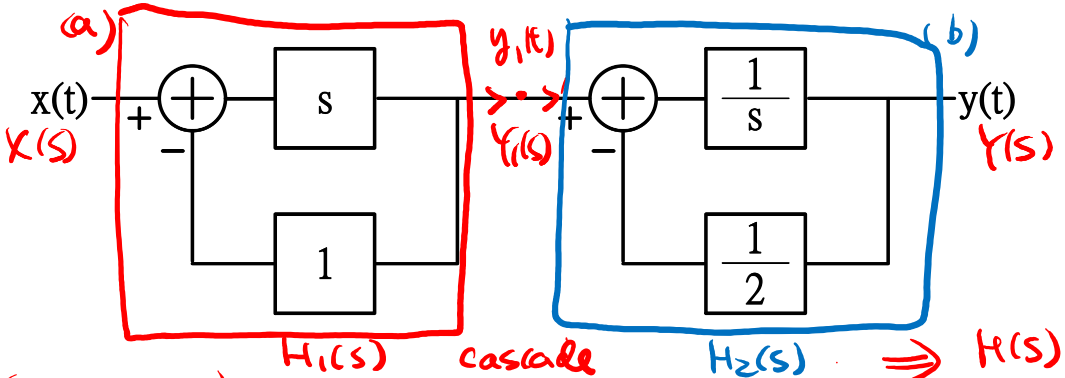
(1) Find the transfer function of the system H(s). Note that the transfer function should include the region of convergence (ROC). (4%)

(2) Give a linear constant coefficient differential equation describing this system. (3%)

(3) Is the system stable? (3%)

(4) Assume that a computer-aided design (CAD) package will be used to design a circuit implementing this LTI system, and this CAD package only has built-in integrator modules and has no differentiator modules built in, re-draw a new block diagram which allows you to use this CAD package to design the circuit. (5%)

(1)



$\Rightarrow H(s) = H_1(s) \cdot H_2(s).$

(a) $Y_1(s) = (X(s) - Y_1(s)) \cdot s$

$H_1(s) = \frac{Y_1(s)}{X(s)} = \frac{s}{s+1}$

(b) $Y(s) = \left(Y_1(s) - \frac{1}{2}Y(s)\right) \cdot \frac{1}{s}$

$sY(s) = Y_1(s) - \frac{1}{2}Y(s)$

$H_2(s) = \frac{Y(s)}{Y_1(s)} = \frac{1}{s+\frac{1}{2}}$

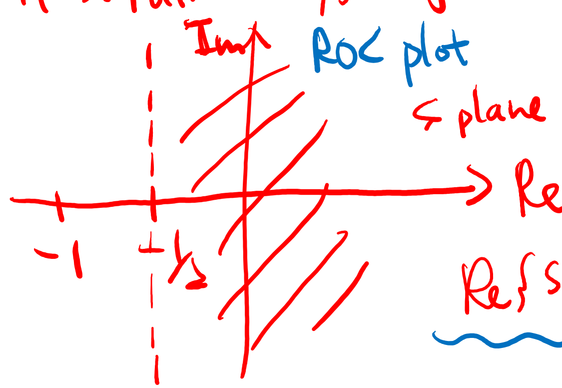
$H(s) = H_1(s) \cdot H_2(s) = \underbrace{\frac{s}{(s+1)} \cdot \frac{1}{(s+\frac{1}{2})}}_{\text{pole zero form}}$ # $= \frac{s}{\underbrace{s^2 + \frac{3}{2}s + \frac{1}{2}}_{\text{rational form.}}}$ # $\underbrace{Re\{s\} > \frac{-1}{2}}$ #

pole: $s_p = -1, -\frac{1}{2}.$

"causal" LTI system & H(s): rational system fx

$\Rightarrow$ ROC: right sided ROC

$\Rightarrow$



$\underbrace{Re\{s\} > -\frac{1}{2}}$ #

(2) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{s}{s^2 + \frac{3}{2}s + \frac{1}{2}}$

system eq: $s^2 Y(s) + \frac{3}{2} s Y(s) + \frac{1}{2} Y(s) = s X(s)$

ICT $\Rightarrow$ $\dfrac{d^2 y(t)}{dt^2} + \dfrac{3}{2}\dfrac{dy(t)}{dt} + \dfrac{1}{2} y(t) = \dfrac{dx(t)}{dt}$ #

(3) ROC includes Im axis $(j\omega)$ $\Rightarrow$ stable #

$\frac{1}{s}$ in s domain

(4) (a) only Part (a) has a differentiator, which needs to be replaced by an integrator.

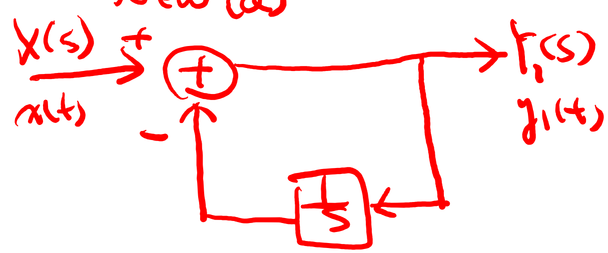
$H_1(s) = \dfrac{Y_1(s)}{X(s)} = \dfrac{s}{s+1} \Rightarrow s Y_1(s) + Y_1(s) = s X(s).$

$\xrightarrow[\text{/s in both sides}]{\text{get } \frac{1}{s} \text{ term in system eq}}$ $Y_1(s) + \frac{1}{s} Y_1(s) = X(s).$

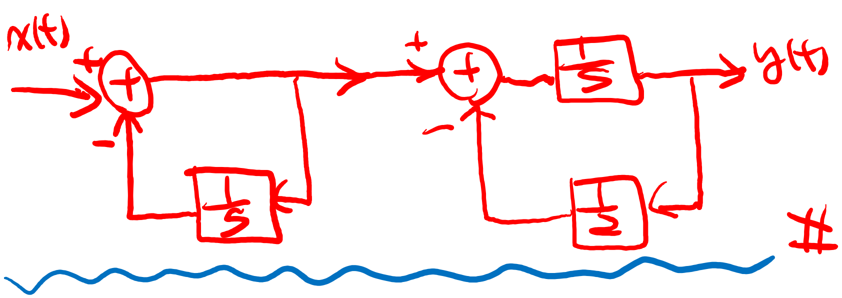
$\Rightarrow Y_1(s) = X(s) - \frac{1}{s} Y_1(s).$

$\Downarrow$

New (a)



$\Leftarrow$



#

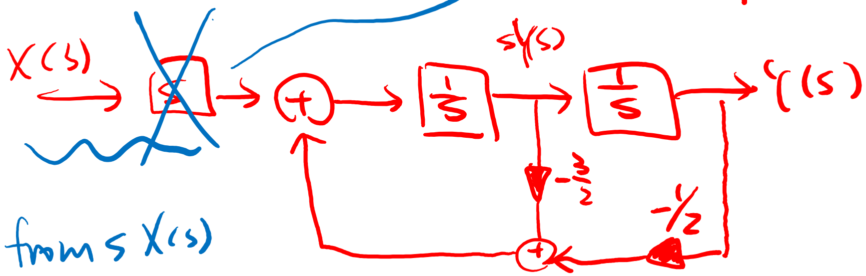
(b)

$$H(s) = \frac{s}{s^2 + \frac{3}{2}s + \frac{1}{2}} = \frac{Y(s)}{X(s)}$$

system eq $\Rightarrow$ $s^2 Y(s) + \frac{3}{2} s Y(s) + \frac{1}{2} Y(s) = s X(s)$

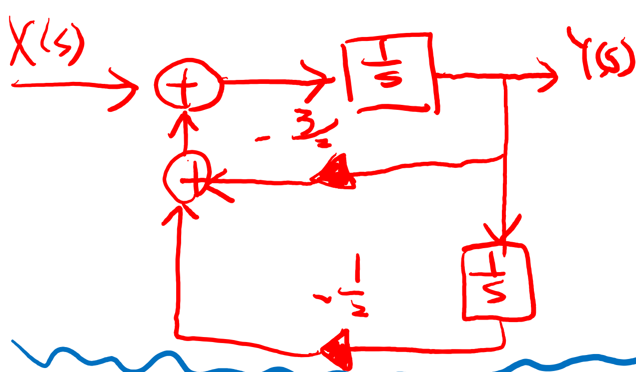
$$s^2 Y(s) = s X(s) - \frac{3}{2} s Y(s) - \frac{1}{2} Y(s)$$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s} \left(s X(s) - \frac{3}{2} s Y(s) - \frac{1}{2} Y(s) \right) \quad (*)$$



need to remove s from s X(s)

$$(*) \Rightarrow Y(s) = \frac{1}{s} \cdot \left(X(s) - \frac{3}{2} Y(s) - \frac{1}{2} \frac{1}{s} Y(s) \right)$$

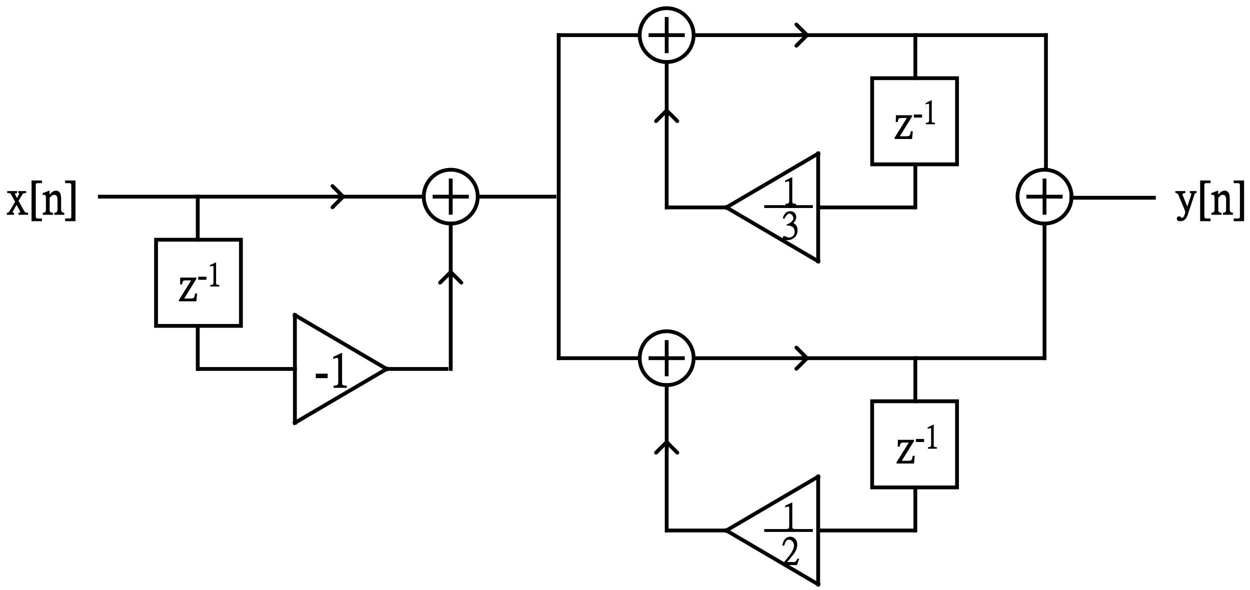


#

((a) or (b) block diagram any of them is good for (+))

Problem 8 (15%)

A DT "**causal**" LTI system is represented by the following block diagram.



algebraic form (2分), ROC (2分)

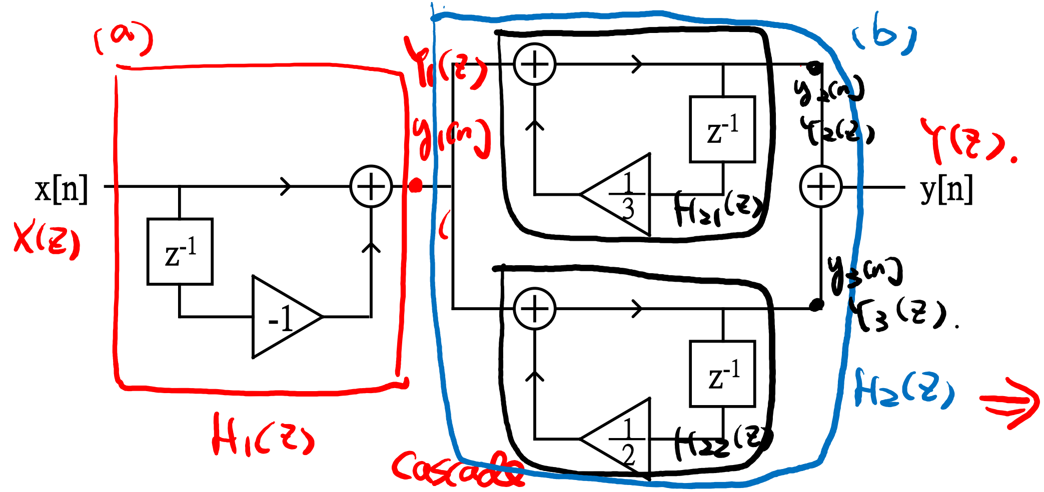
(1) Find the transfer function of the system H(z). Note that the transfer function should include the region of convergence (ROC). (4%)

(2) Give a linear constant coefficient difference equation describing this system. (3%)

(3) Is the system stable? (3%)

(4) Re-draw the block diagram so that at most two unit delayers are required to implement this DT system. (5%)

(1)



$\Rightarrow H(z) = H_1(z)\cdot H_2(z)$.

(a) $Y_1(z) = X(z) - z^{-1}X(z)$.

$H_1(z) = \frac{Y_1(z)}{X(z)} = 1 - z^{-1}$

(b) $H_{21}(z)$, $H_{22}(z)$ parallel connection $\Rightarrow H_2(z) = H_{21}(z) + H_{22}(z)$

$H_{21}(z)$: $Y_2(z) = Y_1(z) + \frac{1}{3}z^{-1}Y_2(z)$

$H_{21}(z) = \frac{Y_2(z)}{Y_1(z)} = \frac{1}{1-\frac{1}{3}z^{-1}}$

$H_{22}(z)$: $Y_3(z) = Y_1(z) + \frac{1}{2}z^{-1}Y_3(z)$

$H_{22}(z) = \frac{Y_3(z)}{Y_1(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$

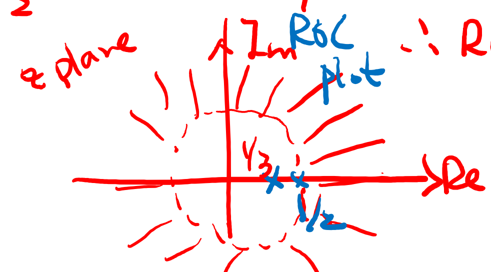
$H_2(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$

$H(z) = (1-z^{-1})\cdot\left(\frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}\right)$ # $= (1-z^{-1})\cdot\frac{2-\frac{5}{6}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})}$ # $= \frac{N(z)}{D(z)}$

$= \frac{(z-1)(2z-\frac{5}{6})}{(z-\frac{1}{3})(z-\frac{1}{2})}$ #, $|z| > \frac{1}{2}$ #

pole $z_p = \frac{1}{3}, \frac{1}{2}$ ∵ causal system ∴ ROC: outside a circle

∴ ROC: $|z| > \frac{1}{2}$ #



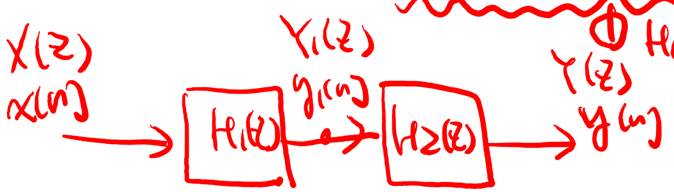
(2) $$H(z)=\frac{(1-z^{-1})(2-\frac{5}{6}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})}=\frac{2-\frac{17}{6}z^{-1}+\frac{5}{6}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}=\frac{Y(z)}{X(z)}$$

system eq. $\Rightarrow Y(z)-\frac{5}{6}z^{-1}Y(z)+\frac{1}{6}z^{-2}Y(z)=2X(z)-\frac{17}{6}z^{-1}X(z)+\frac{5}{6}z^{-2}X(z)$

$\xRightarrow{IZT}$ $y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=2x[n]-\frac{17}{6}x[n-1]+\frac{5}{6}x[n-2]$ #

(3) ROC includes unit circle $\Rightarrow$ stable #

(4) (a) $$H(z)=\underbrace{\frac{1}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}}_{①\ H_1(z)}\cdot\underbrace{\left(2-\frac{17}{6}z^{-1}+\frac{5}{6}z^{-2}\right)}_{②\ H_2(z)}$$

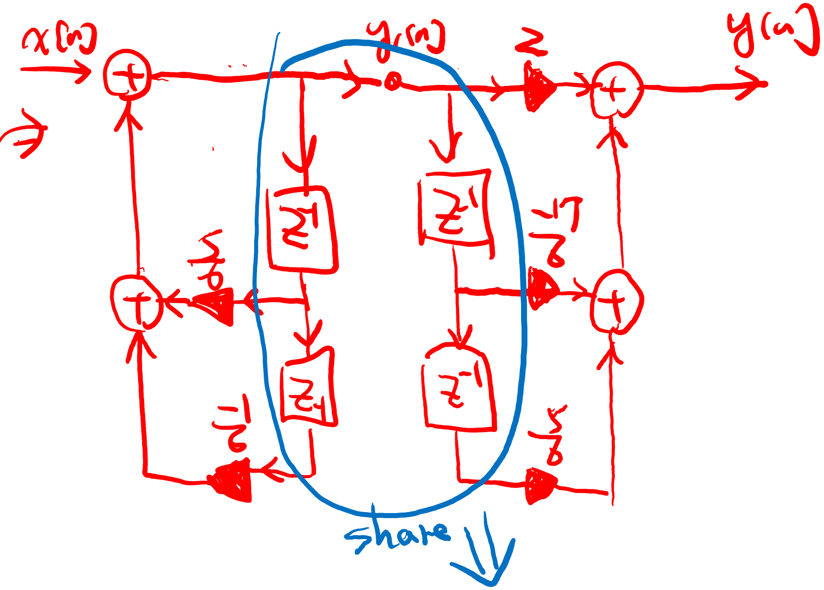


from $H_1(z)=\frac{Y_1(z)}{X(z)}=\frac{1}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$

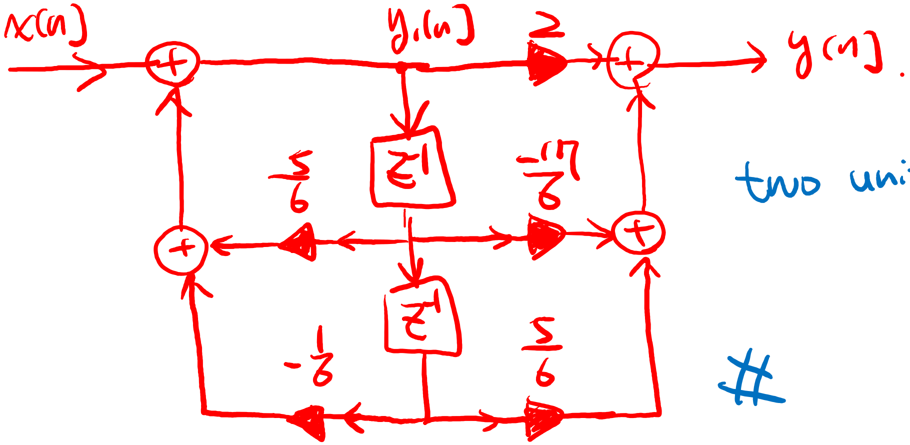
$\Rightarrow Y_1(z)=X(z)+\frac{5}{6}z^{-1}Y_1(z)-\frac{1}{6}z^{-2}Y_1(z)$ ①

from $H_2(z)$

$\Rightarrow Y(z)=2Y_1(z)-\frac{17}{6}z^{-1}Y_1(z)+\frac{5}{6}z^{-2}Y_1(z)$ ②



(a)



two unit delayers.

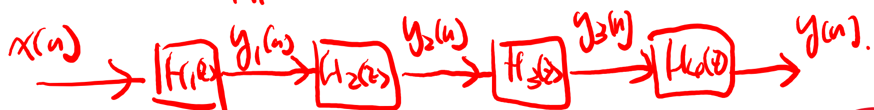
#

(b) $$H(z) = \underbrace{\frac{1}{1-\frac{1}{3}z^{-1}}}_{①\,H_1(z)} \cdot \underbrace{(1-z^{-1})}_{②\,H_2(z)} \cdot \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}_{③\,H_3(z)} \underbrace{\left(2-\frac{5}{6}z^{-1}\right)}_{④\,H_4(z)}$$

all these cascading sequences with two unit delayers

1	2	3	4
1	4	3	2
3	4	1	2
3	2	1	4

#

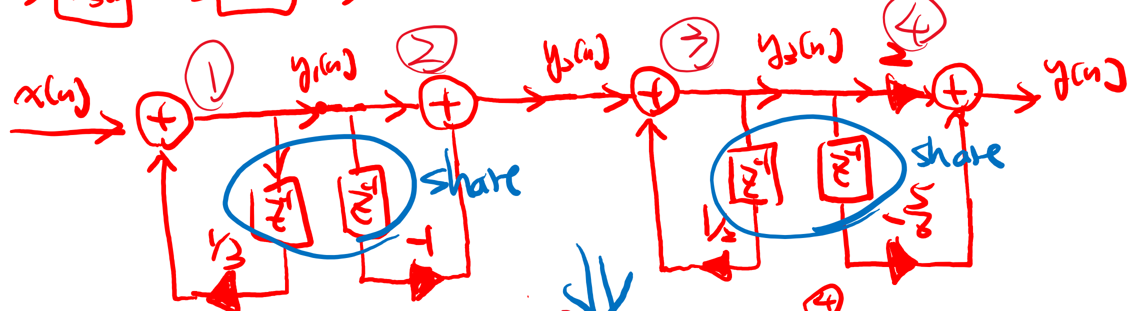


from ① ⇒ $Y_1(z) = X(z) + \frac{1}{3}z^{-1}Y_1(z)$

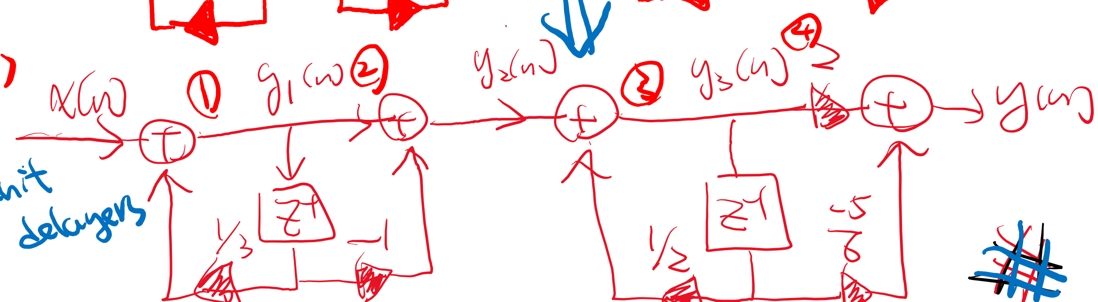
② ⇒ $Y_2(z) = Y_1(z) - z^{-1}Y_1(z)$

③ ⇒ $Y_3(z) = Y_2(z) + \frac{1}{2}z^{-1}Y_3(z)$

④ ⇒ $Y(z) = 2Y_3(z) - \frac{5}{6}z^{-1}Y_3(z)$



⇓



((a) or (b) block diagram any of them is good. for (4)) two unit delayers

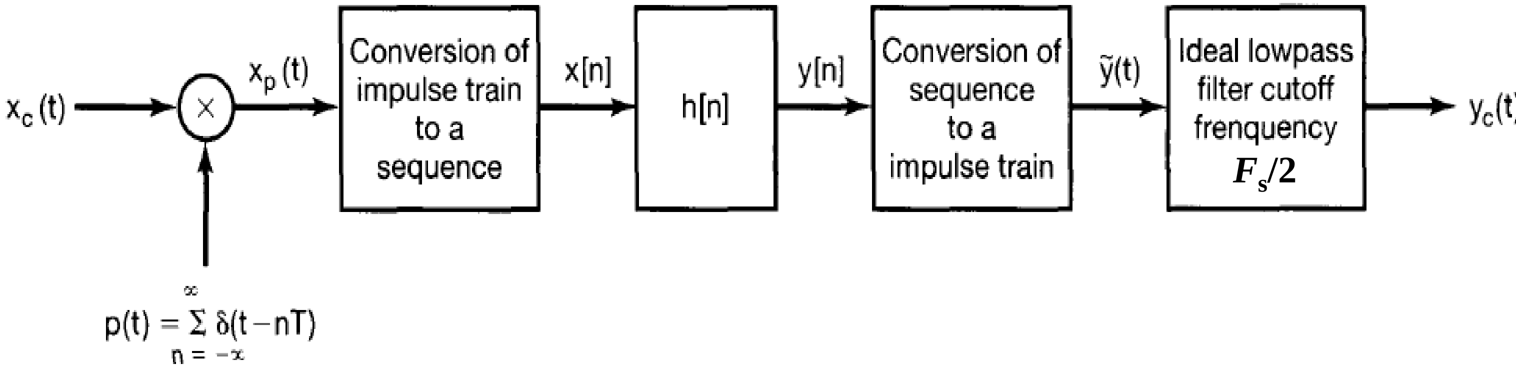
#

Problem 9 (10%)

The figure shown below is a system that processes continuous-time signals using a discrete time (DT) linear time-invariant (LTI) filter with impulse response $h[n]$. For input signals which are band limited within Nyquist rate (i.e., after sampling, there will be no aliasing in frequency domain), the overall system in the figure is equivalent to a continuous-time (CT) LTI system. Note that T is the sampling interval, $F_s = 1/T = 1000$ Hz is the sampling rate, and $h[n]$ is real. If this effective CT LTI system we would like to design is a stable, causal and non-ideal bandpass filter and has its maximum magnitude response at 125 Hz,

(1) At what (normalized) frequency should the DT LTI system $h[n]$ have the maximum magnitude response? Justify your answer. (5%)

(2) Provide the possible pole-zero placement (i.e., pole-zero plot of the transfer function of the system $h[n]$) which you will use to design this DT LTI system $h[n]$ with the assumption that the transfer function of the system $h[n]$ only has one zero at 0. Justify your answer. (5%)



(1) impulse invariant transformation

$F_{peak} = 125\,Hz$, $F_s = 1000\,Hz$ for CT LTI system.

$\Rightarrow$ for DT LTI system, $f_{peak} = \dfrac{\pm F_{peak}}{F_s} = \pm\dfrac{1}{8}$ in normalized freq.

(with or without $\pm$, both are ok.)

(or by freq mapping $\omega = \Omega \cdot T$) between CTFT & DTFT.

(2) ① $f_{peak} = \pm\dfrac{1}{8} \Rightarrow \omega_{peak} = \pm\dfrac{\pi}{4}$ (max magnitude response at $\pm\frac{\pi}{4}$)

because $h[n]$ is real. (freq response : conjugate symmetry)

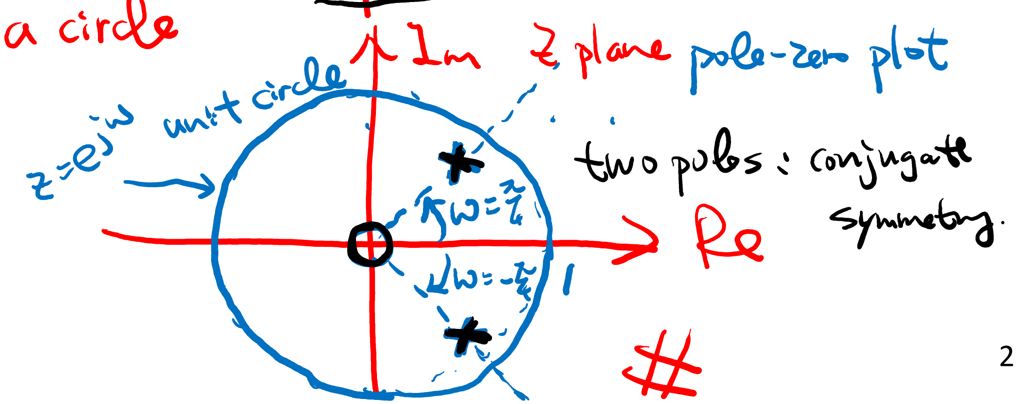
$\Rightarrow$ The poles are the closest to $z = e^{j\omega}$ ($r=1$, DTFT) at $\omega = \pm\frac{\pi}{4}$

($\Rightarrow$ get the shortest pole vectors $\Rightarrow$ max magnitude response.)

② stable $\Rightarrow$ ROC includes unit circle $\Rightarrow$ poles within unit circle

causal $\Rightarrow$ ROC : outside a circle

③ one zero $z_z = 0$.



#

Problem 10 (5%)

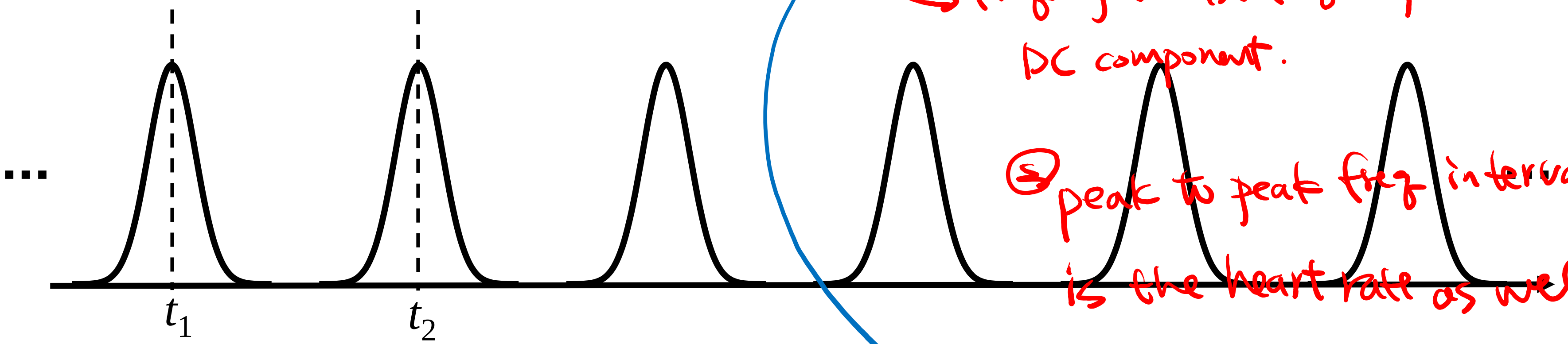
There is an abnormal person whose heart rate (i.e., the heart beat frequency) is constant. His every heart beat is exactly the same. That is, if we listen to the beating of his heart, we will hear the sound $x(t)$ in time domain as shown in the following figure. The heart rate is defined as the inverse of the peak-to-peak time duration in the heartbeat sound signal. For example, $1/(t_2-t_1)$ is defined as the heart rate at t_2. In this case, can Fourier transform of $x(t)$ be used to estimate the heart rate? Yes. ① the fundament freq is the heart rate in this case.

No matter your answer is yes or no, tell the reason why you give such an answer.

Remember to justify your answer.

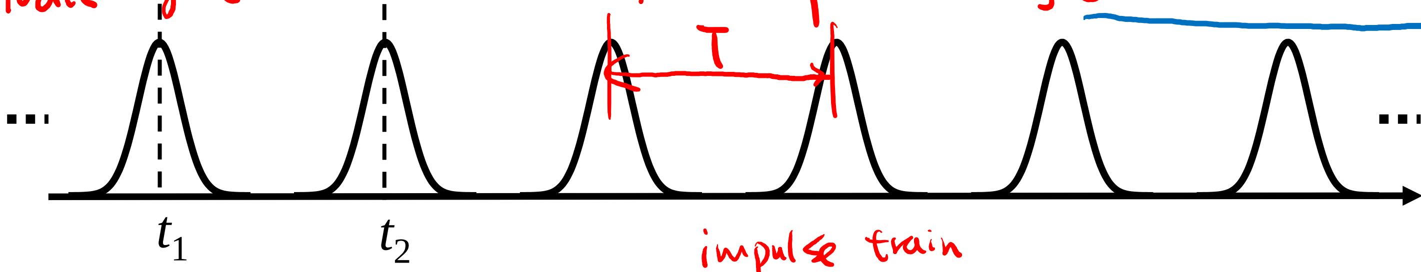
freq of the 1st freq component except DC component.

from CTFS pt of view.

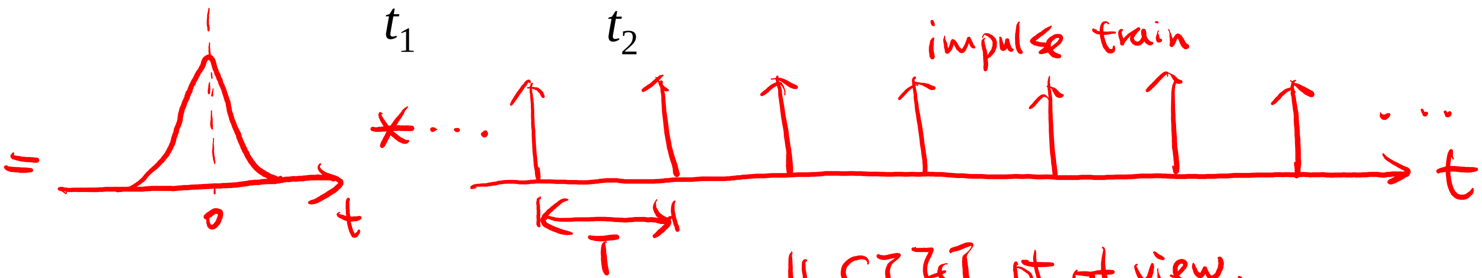


② peak to peak freq interval is the heart rate as well.

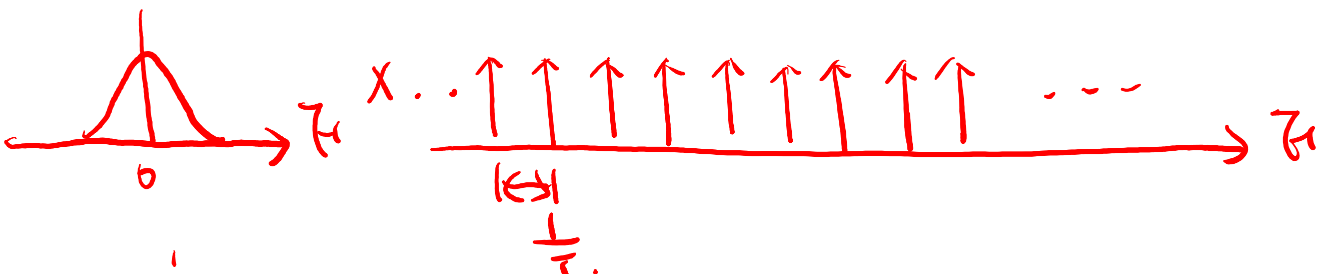
periodic signal with fundamental period: $T = t_2 - t_1$, fundamental freq $= \frac{1}{T} = \frac{1}{t_2 - t_1}$ = heart rate.

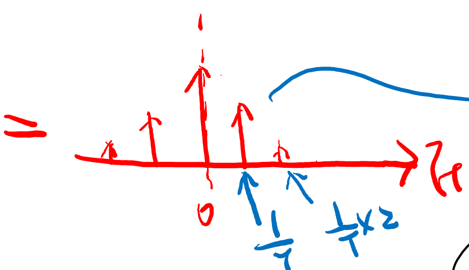


impulse train



⇓ CTFT pt of view.





① freq of the 1st freq component (except the DC component), i.e., the fundamental freq = heart rate

② peak to peak freq interval = heart rate.

③ from CTFS pt of view, fundamental freq = heart rate

(CTFT of periodic signals is from their CTFS)