Signal and System Midterm Exam

Hint:

CTFT

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi ft} df$$

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

DTFT

$$x[n] = \int_{-\frac{1}{2}}^{+\frac{1}{2}} X(f) e^{+j2\pi fn} df$$

$$X(f) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j2\pi fn}$$

CTFS

$$x(t) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X[k] e^{+j\frac{k2\pi t}{T}}$$

$$X[k] = \int_{0}^{T} x(t) e^{-j\frac{k2\pi t}{T}} dt$$

DTFS

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{k2\pi n}{N}}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{N}}$$

Problem 1 CTFT

(1) Please prove $sinc(t) \overset{CTFT}{\leftrightarrow} rect(f)$. (3%)

P.S. $sinc(t) = \dfrac{\sin(\pi t)}{\pi t}$ $rect(f) = \begin{cases} 1, & -\dfrac{1}{2} \le f \le -\dfrac{1}{2} \\ 0, & \text{else} \end{cases}$

(2) Please prove $x(t) * y(t) \overset{CTFT}{\leftrightarrow} X(f) \times Y(f)$. (3%)

P.S. $x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau) y(t-\tau) d\tau$

(3) $x(t)$ is a real function . $x^*(t) = x(t)$

Please prove $X^*(f) = X(-f)$. (3%)

Please prove $\int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$. (3%)

(4) Please find the following convolution. (hint: use the statement of Problem 1 (3).)

$z(t) = \dfrac{\sin 4\pi t}{\pi t} * \dfrac{\sin 8\pi t}{\pi t}$ (3%)

Problem 2 CTFT

Please prove $e^{-at}u(t) \overset{CTFT}{\leftrightarrow} \dfrac{1}{a + j2\pi f}$, where the real part of a is positive. (3%)

Problem 3 CTFT

Please prove find the following DTFTs.

(1) $x(t) = e^{-t}\sin(\pi t)u(t)$ (hint: use the statement of Problem 2.) (3%)

(2) $x(t) = \sin(3\pi t) + \cos(5\pi t)$ (3%)

(3) $x(t) = e^{-jt}u(t-3)$ (3%)

(4) $x(t) = \delta(t) * \delta(t)$ (3%)

Problem 4 DTFT

Please prove $a^n u[n] \overset{DTFT}{\leftrightarrow} \frac{1}{1-ae^{-j2\pi f}}$, where the real part of a is smaller than 1. (3%)

Problem 5 DTFT

Please find the following DTFTs.

(1) $x[n]=\delta[n]$ (3%)

(2) $x[n]=(n-1)(u[n-2]-u[n-4])$ (3%)

(3) $x[n]=\frac{1}{2^n}u[n-4]$ (hint: use the statement of Problem 4.) (3%)

Problem 6 DTFT

Please find the following DT convolutions

(1) $z[n]=u[n]*u[n]$ (3%)

hint: $x[n]*y[n]=\sum_{n=-\infty}^{+\infty} x[m]*y[n-m]$

(2) $z[n]=e^{-n}*\delta[n-2]$ (3%)

Problem 7 CTFS

$x(t)$ is a periodic function with a period of 8.

$x(t+8) = x(t)$

$x(t) = \delta(t), 0 \le t < 8$

(1) Plot $x(t)$. (3%)

(2) Find $X[k]$, which is the DTFS of $x(t)$. (3%)

Problem 8 CTFS

$x(t)$ is a periodic function with the period of T.

$x(t+2) = x(t)$

$x(t) = t$

(1) Plot $x(t)$. (3%)

(2) Find $X[0]$, where $X[k]$ is the DTFS of $x(t)$. (3%)

Problem 9 DTFS

Please prove the following DTFFs.

(1) $x[n]=\sum_{l=-\infty}^{+\infty}\delta[n-lN] \overset{DTFS}{\leftrightarrow} X[k]=1$ (3%)

(2) $e^{+j2\pi\frac{m}{N}n} \overset{DTFS}{\leftrightarrow} X[k]=\begin{cases} N, k=m \\ 0, else \end{cases}$ (3%)

Problem 10 DTFS

(1) $x[n-n_0] \overset{DTFS}{\leftrightarrow} e^{-j\frac{k2\pi n_0}{N}}X[k]$ (3%)

(2) $e^{+j2\pi f_0 n}x[n] \overset{DTFS}{\leftrightarrow} X[k-Nf_0]$ (3%)

(3) $x[\frac{n}{p}] \overset{DTFS}{\leftrightarrow} X[k]$ $(p=2)$ (3%)

Problem 11 DTFS

$x[n]=\delta[n]+2\delta[n-6]$

$x[n+N]=x[n]$

$N=8$

(1) Plot $x[n]$ (3%)

(2) Find DFT of $x[n]$ (3%)

Problem 12 DTFS

Find the 8-point DFT,

where $x[n]=1+2\sin(\frac{\pi}{4}n)$ (3%)

Problem 13 DTFS

Find the 8-point inverse DFT,

where $X[k]=\begin{cases} +j, & k=2 \\ -j, & k=6 \\ 0, & otherwise \end{cases}$ (3%)

Problem 14 DTFS

Find the matrix A of 4-point DFT.

$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j\frac{k2\pi n}{N}}$

$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix}=\begin{bmatrix} & \text{A} & \end{bmatrix}\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$ (3%)

Problem 15 Down Sampling

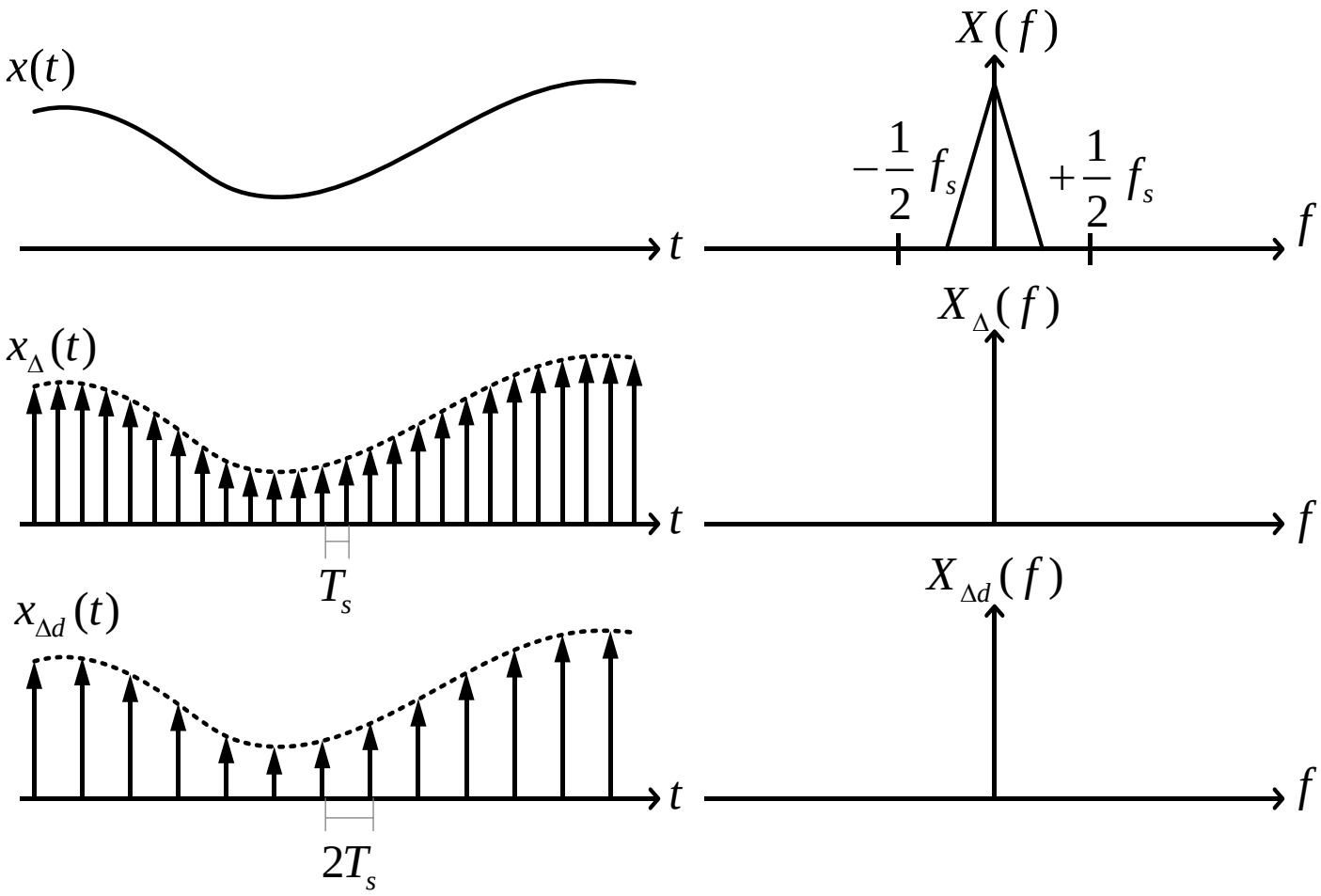
Assume the sampling frequency is fs, and signal bandwidth is B. Under the condition of

(1) No aliasing in frequency domain.

(2) Perfect reconstruction in time domain.

What is the relationship between fs and B? (3%)

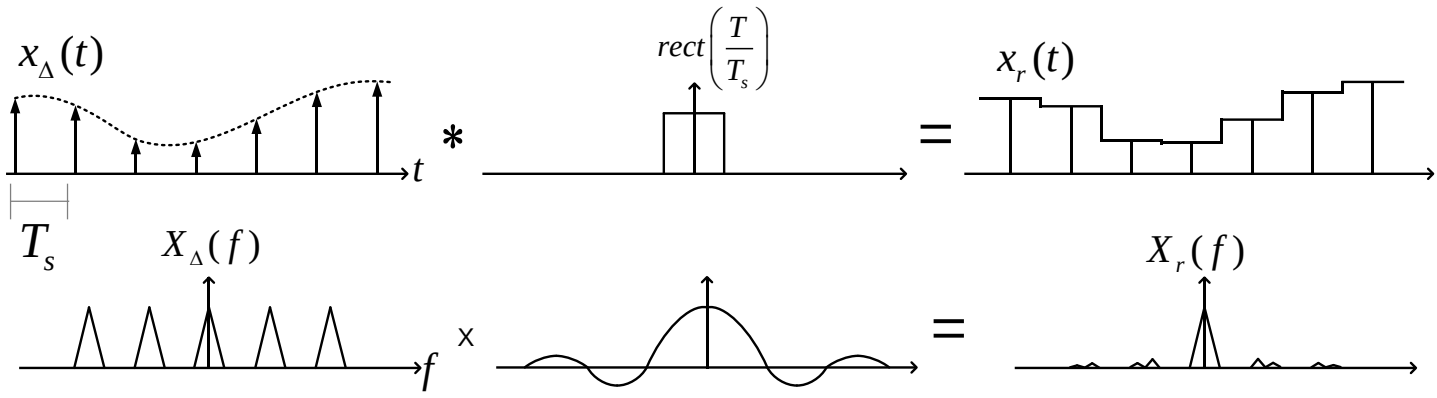
Problem 16 Down Sampling



Please use $X(f)$ and f_s to represent $X_\Delta(f)$ and $X_{\Delta d}(f)$. (3%)

Please plot $X(f)$, $X_\Delta(f)$ and $X_{\Delta d}(f)$. (3%)

Problem 17 Reconstruction



The time-domain and frequency domain processes of zero-order hold are shown in above.

The time-domain reconstruction signal is $x_r(t) = x_\Delta(t) * rect\left(\frac{T}{T_s}\right)$,

please find the frequency representation of the reconstruction signal $X_r(f)$.

(3%)