Problem 1 HW20

①

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一. Consider a discrete-time, causal LTI system with input $x[n]$ and output $y[n]$. The system is described by the following pairs of difference equations, involving an intermediate signal $w[n]$:

$$\begin{cases} y[n]-\frac{5}{4}y[n-1]+w[n]+\frac{1}{4}w[n-1]=\frac{1}{10}x[n] \\ y[n]-\frac{3}{2}y[n-1]+2w[n]=-\frac{2}{5}x[n] \end{cases}$$

(一) (3%)Derive the frequency response of this system. $H(z)$ $H(e^{j\omega t})$

(二) (3%)Derive the impulse response of this system. $h(n)$

(三) (4%)Find a single difference equation relating $x[n]$ and $y[n]$.

(2)

$$w[n] = -\frac{1}{5}x[n] + \frac{3}{4}y[n-1] - \frac{1}{2}y[n]$$

$$w[n-1] = -\frac{1}{5}x[n-1] + \frac{3}{4}y[n-2] - \frac{1}{2}y[n-1]$$

$$y[n] - \frac{5}{4}y[n-1] - \frac{1}{5}x[n] + \frac{3}{4}y[n-1] - \frac{1}{2}y[n] - \frac{1}{20}x[n-1] + \frac{3}{16}y[n-2] - \frac{1}{8}y[n-1] = \frac{1}{10}x[n]$$

$$\frac{3}{16}y[n-2] + \left(-\frac{1}{8} + \frac{3}{4} - \frac{5}{4}\right)y[n-1] + \left(1 - \frac{1}{2}\right)y[n] = +\frac{1}{20}x[n-1] + \frac{3}{10}x[n]$$

$$\frac{3}{16}y[n-2] + \left(-\frac{5}{8}\right)y[n-1] + \frac{1}{2}y[n] = \frac{1}{20}x[n-1] + \frac{3}{10}x[n]$$

3

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{10} + \frac{1}{20} z^{-1}}{\frac{1}{2} - \frac{5}{8} z^{-1} + \frac{3}{16} z^{-2}}$$

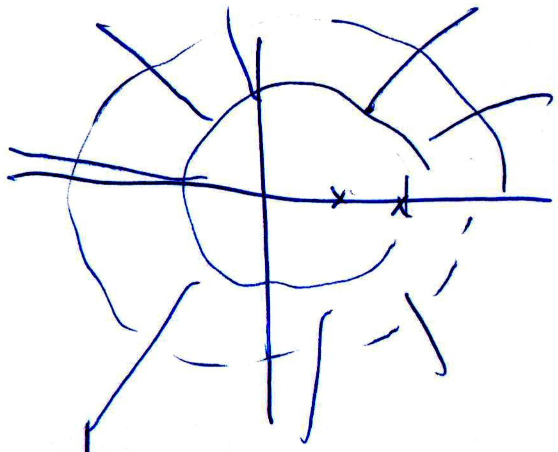
$$= \frac{1}{10} \frac{6 + z^{-1}}{1 - \frac{5}{4} z^{-1} + \frac{3}{8} z^{-2}}$$

$\frac{3}{4}$, $\frac{1}{2}$

$$= \frac{1}{10} \frac{6 + z^{-1}}{(1 - \frac{3}{4} z^{-1})(1 - \frac{1}{2} z^{-1})}$$

$$H(e^{j2\pi f}) = \frac{1}{10} \frac{6 + e^{-j2\pi f}}{1 - \frac{5}{4} e^{-j2\pi f} + \frac{3}{8} e^{-j4\pi f}}$$

④



$$H(z) = \frac{1}{10} \frac{6 + z^{-1}}{(1 - \frac{3}{4} z^{-1})(1 - \frac{1}{2} z^{-1})}$$

$$\frac{6 + x}{(1 - \frac{3}{4} x)(1 - \frac{1}{2} x)} = \frac{a}{1 - \frac{3}{4} x} + \frac{b}{1 - \frac{1}{2} x}$$

$$a = \frac{6 + \frac{4}{3}}{1 - \frac{1}{2} \times \frac{4}{3}} = \frac{\frac{22}{3}}{1 - \frac{2}{3}} = 22$$

$$b = \frac{6 + 2}{1 - \frac{3}{4} \times 2} = \frac{8}{-\frac{1}{2}} = 16$$

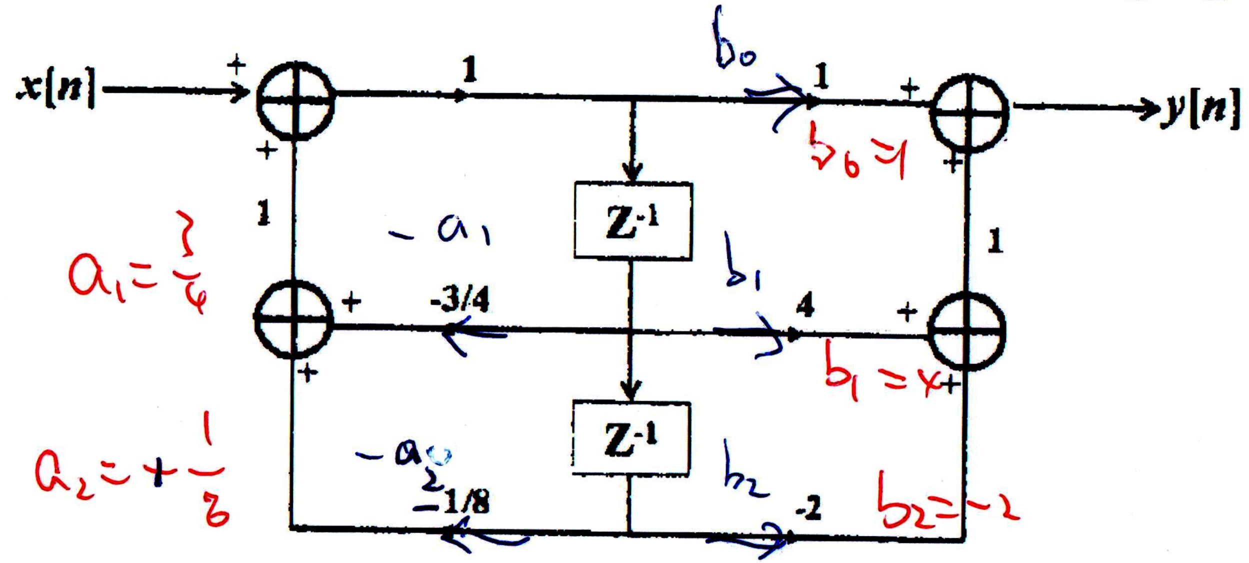
$$H(z) = \frac{22}{10} \frac{1}{1 - \frac{3}{4} z^{-1}} + \frac{16}{10} \frac{1}{1 - \frac{1}{2} z^{-1}}$$

$$h(n) = \frac{22}{10}\left(\frac{3}{4}\right)^n u(n) + \frac{16}{10}\left(\frac{1}{2}\right)^n u(n)$$

5

Problem 2
101台聯大訊號與系統

八、(15%) Consider a signal flow graph shown in the following figure.



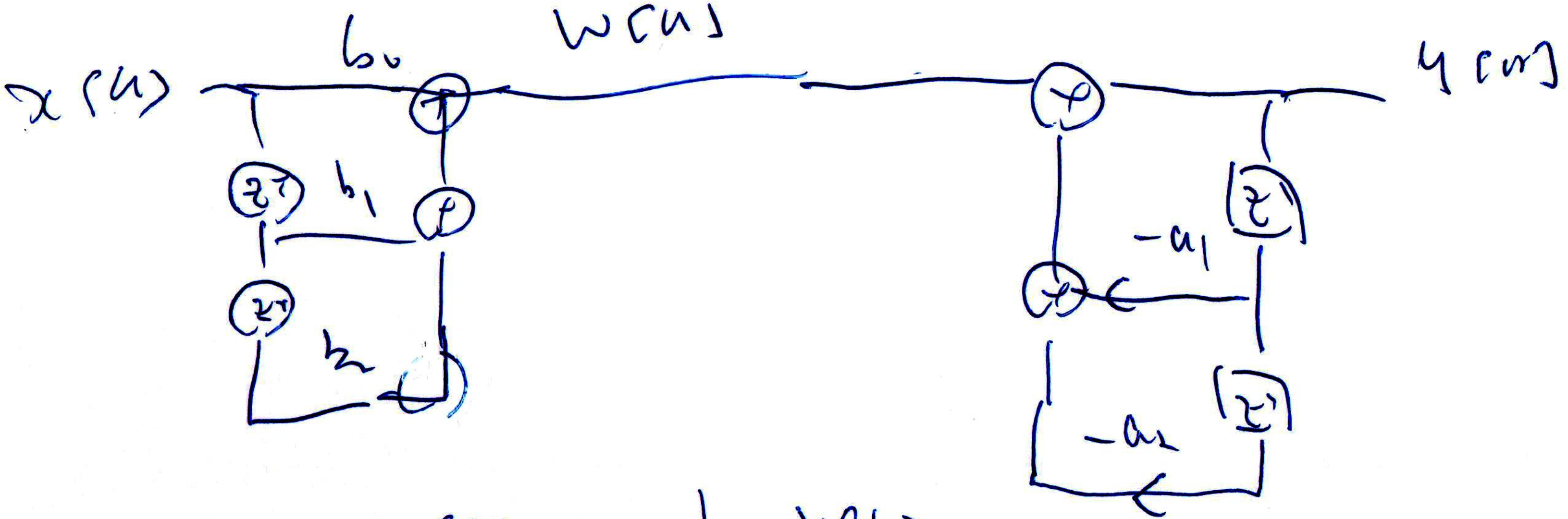
(一) (8%) Please find the transfer function H(z).

(二) (5%) Please find the impulse response, h[n], of the system.

(三) (2%) $\boxed{\text{Is the system stable?}}$ Please explain it.

$$y[n] + \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n] + 4x[n-1] - 2x[n-1]$$

1



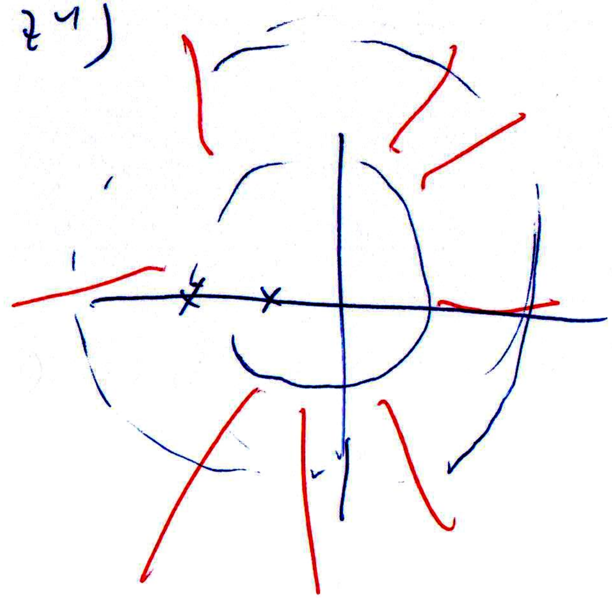
$$w(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

$$y[n] = w(n) + (-a_1) y[n-1] + (-a_2) y(n-2)$$

$$\boxed{y(n) + a_1 y[n-1] + a_2 y(n-2) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 4z^{-1} - 2z^{-2}}{1 + \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$= \frac{1 + 4z^{-1} - 2z^{-2}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{4} z^{-1}\right)}$$



9

$$\frac{1+4x-2x^2}{(1+\frac{1}{2}x)(1+\frac{1}{4}x)} = a + \frac{b}{1+\frac{1}{2}x} + \frac{c}{1+\frac{1}{4}x}$$

$$b = \frac{1-8-8}{1-\frac{1}{2}} = \frac{-15}{+\frac{1}{2}} = -30$$

$$c = \frac{1-16-32}{1-2} = +47$$

$$1 = a - 30 + 47 = \qquad a = -16$$

$$H(z) = -16 - 30\frac{1}{1+\frac{1}{2}z^{-1}} + 47\frac{1}{1+\frac{1}{4}z^{-1}}$$

$$h(n) = -16\,\delta(n) - 30\left(-\frac{1}{2}\right)^n u(n) + 47\left(\frac{1}{4}\right)^n u(n)$$

Problem 3

100台聯大訊號與系統

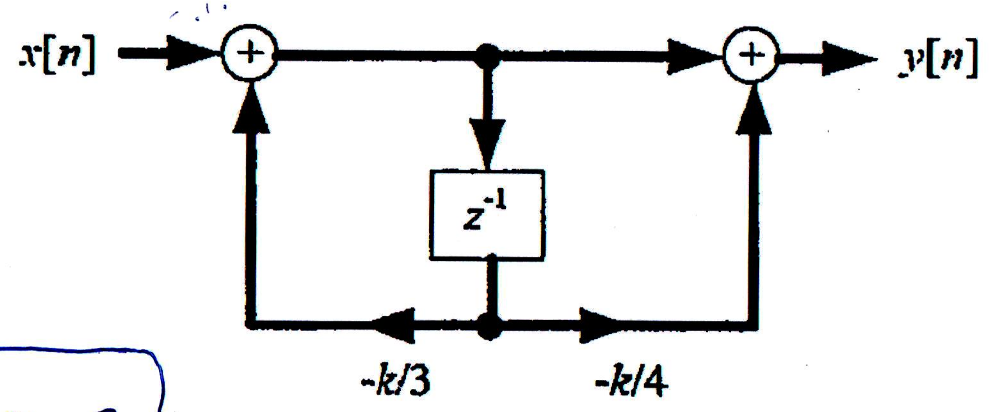
(11)

7. (10%) Consider the causal digital filter structure shown below.

(a) (4%) Find $H(z)$ for this $\boxed{\text{causal}}$ filter ~~and its region of convergence~~. ROC $|z| > \left|-\frac{1}{3}k\right|$

(b) (2%) For what values of k is the system stable? $|k| < 3$

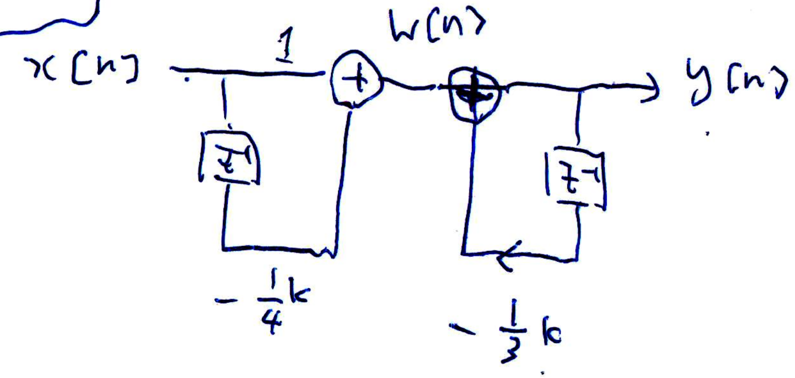
(c) (4%) Determine $y[n]$ if $k = 1$ and $x[n] = (2/3)^n$ for all n.



$w[n] = x[n] - \frac{k}{4} x[n-1]$

$y[n] = -\frac{k}{3} y[n-1] + w[n]$

$$\boxed{y[n] + \frac{k}{3} y[n-1] = x[n] - \frac{k}{4} x[n-1]}$$

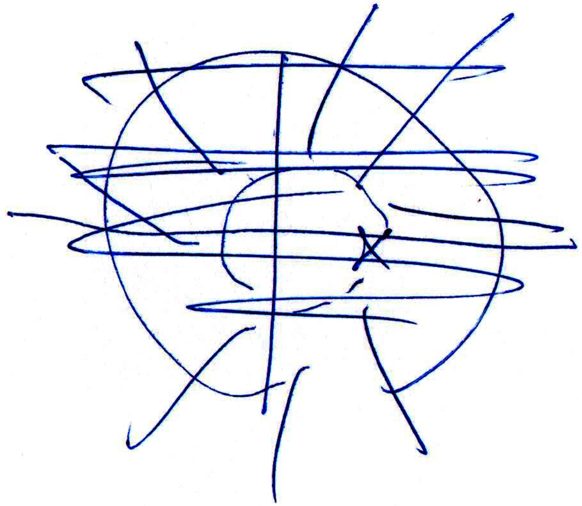


$$Y(z) + \frac{k}{3} z^{-1} Y(z) = X(z) - \frac{k}{4} z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{4} k z^{-1}}{1 + \frac{1}{3} k z^{-1}}$$

~~pole = $\frac{3}{k}$~~

$\cancel{\left|\frac{3}{k}\right| < \ }$



$$1 + \frac{1}{3} k z^{-1} = 0$$

$$-\frac{1}{3} k z^{-1} = 1$$

$$-\frac{1}{3} k = z$$

pole : $-\frac{1}{3} k$

$$\left| -\frac{1}{3} k \right| < 1$$

$$|k| < 3$$

$k=1$

$$y(n) + \frac{1}{3}\, y(n-1) = \left(\frac{2}{3}\right)^n - \frac{1}{4}\left(\frac{2}{3}\right)^{n-1}$$

$$= \left(\frac{2}{3}\right)^n - \frac{1}{4}\,\frac{3}{2}\left(\frac{2}{3}\right)^n$$

$$= \left(\frac{2}{3}\right)^n \left[1 - \frac{3}{8}\right]$$

$$= \frac{5}{8}\left(\frac{2}{3}\right)^n$$

$$y(n) + \frac{1}{3}\, y(n-1) = \frac{5}{8}\left(\frac{2}{3}\right)^n$$

$$y(n) = c \cdot \left(\frac{2}{3}\right)^n$$

$$c\left(\frac{2}{3}\right)^n + \frac{1}{3}\, c\left(\frac{2}{3}\right)^{n-1} = \frac{5}{8}\left(\frac{2}{3}\right)^n$$

b

14

$$c\left(\frac{2}{3}\right)^n + \frac{1}{2}c\left(\frac{2}{3}\right)^n = \frac{5}{8}\left(\frac{2}{3}\right)^n$$

$$\frac{3}{2}c = \frac{5}{8}$$

$$c = \frac{10}{24}$$

$$y(n) = \frac{10}{24}\left(\frac{2}{3}\right)^n$$

Problem 4
106台聯大訊號與系統

15

九、(15%)

A causal LTI system with system function $H(z)$ is represented by the following block diagram.

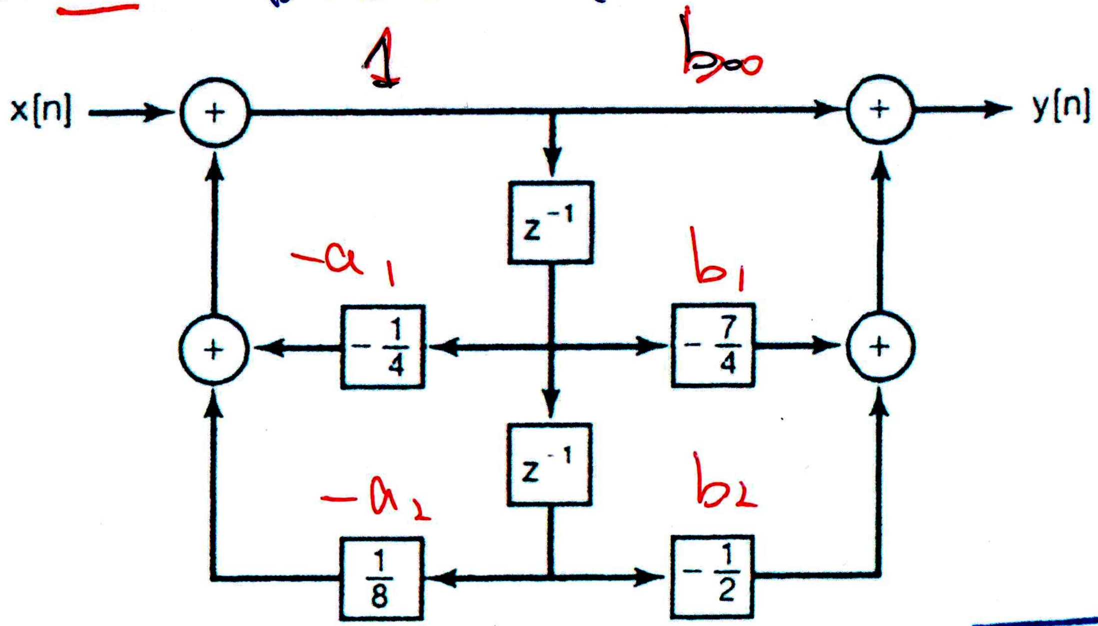
(a) (3%) Determine the system function $H(z)$.

(b) (3%) Give a linear constant coefficient difference equation describing the system.

(c) (3%) What is the region of convergence of $H(z)$? $|z| > \frac{1}{2}$

(d) (4%) Find the impulse response of the system.

(e) (2%) Is the system stable? BIBO stable

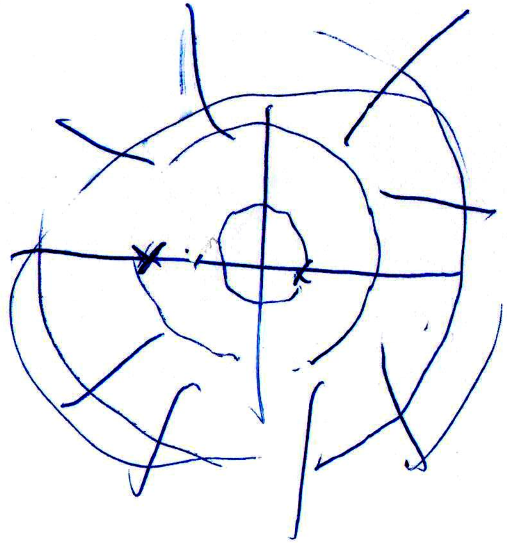


$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$

$$y(n) + \frac{1}{4}y(n-1) + \left(-\frac{1}{8}\right)y(n-2) = 1\,x(n) + \left(-\frac{7}{4}\right)x(n-1) + \left(-\frac{1}{2}\right)x(n-2) \quad (16)$$

$$Y(z) + \frac{1}{4}z^{-1}Y(z) + \left(-\frac{1}{8}\right)z^{-2}Y(z) = X(z) + \left(-\frac{7}{4}\right)z^{-1}X(z) + \left(-\frac{1}{2}\right)z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{\boxed{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}}$$



$$= \frac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$|z| > \frac{1}{2}$$

$$\frac{1-\frac{7}{4}x-\frac{1}{2}x^2}{(1+\frac{1}{2}x)(1-\frac{1}{4}x)} = a + \frac{b}{1+\frac{1}{2}x} + \frac{c}{1-\frac{1}{4}x}$$

$$b = \frac{1+\frac{7}{2}-2}{1+\frac{1}{4}2} = \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3}$$

$$c = \frac{1-7-2}{3} = -\frac{8}{3}$$

$$1 = a + \frac{5}{3} - \frac{8}{3}$$

$$a = 2$$

18

$$H(z) = 2 + \left(\frac{5}{3}\right) \frac{1}{1 - \left(-\frac{1}{2}\right) z^{-1}} + \left(-\frac{8}{3}\right) \frac{1}{1 - \frac{1}{4} z^{-1}}$$

$$h(n) = 2\delta(n) + \frac{5}{3} \times \left(-\frac{1}{2}\right)^n u(n)$$

$$+ \left(-\frac{8}{3}\right) \left(\frac{1}{4}\right)^n u(n)$$

Problem 5

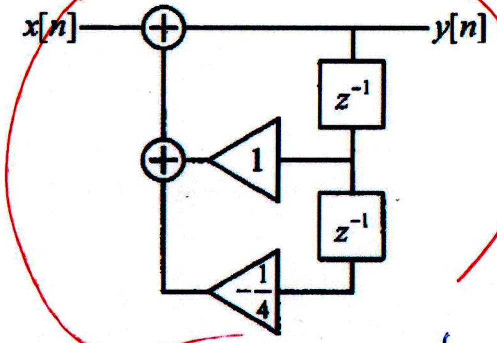
108台聯大訊號與系統

$H(z)$ $h[n]$

19

11. **(8%)**

Find the impulse response (i.e., h[n]) and the transfer function (i.e., H(z)) of the following DT LTI system. The input signal is x[n] and the output signal is y[n].



$h[n] = (n+1)\left(\frac{1}{2}\right)^n u[n]$

$y[n] = x[n] + y[n-1] - \frac{1}{4} y[n-2]$

$y[n] - y[n-1] + \frac{1}{4} y[n-2] = x[n]$

$Y(z) - z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1} + \frac{1}{4} z^{-2}}$

$H(z) = \frac{1}{(1 - \frac{1}{2} z^{-1})^2}$