HW 19

Problem 1

103台聯大訊號與系統

$-\frac{1}{8}y[n-2]+\frac{1}{4}y[n-1]+y[n]$

$=-2x[n-2]+x[n-1]$

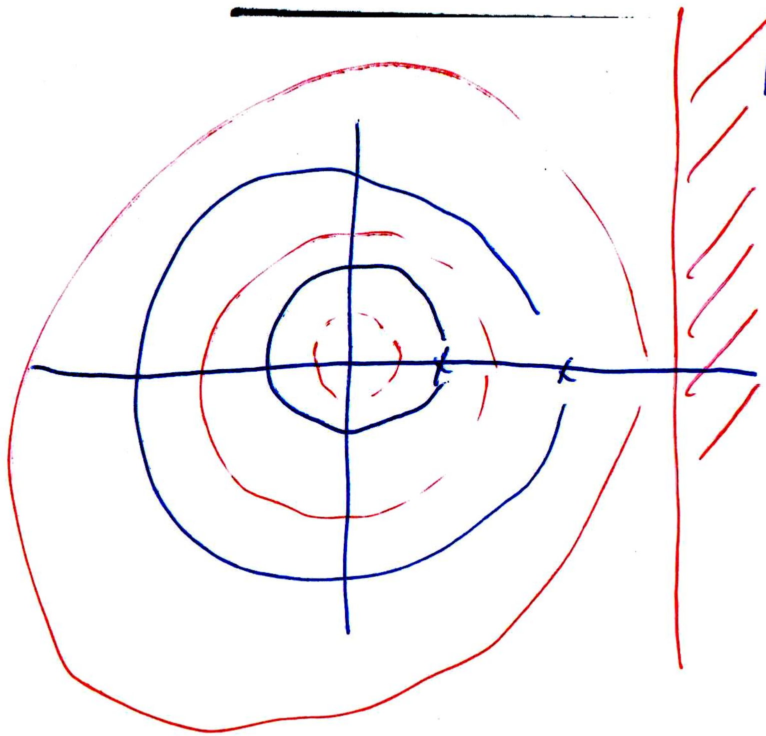
①

九、(20%) Consider a discrete-time system with input $x[n]$ and output $y[n]$ for which

$$|z|>\frac{1}{2} \qquad \boxed{-\frac{1}{8}y[n-1]+\frac{1}{4}y[n]+y[n+1]=-2x[n-1]+x[n].}$$

(一) (10%) Suppose all z with ~~Re{z}>5~~ are in the region of convergence of the system function H(z). Determine H(z) and indicate the region of convergence. What is the impulse response?

(二) (10%) Draw three block diagrams for the system in the direct form, cascade form, and parallel form, respectively. Note that each block diagram should have the minimum number of delay elements.



$$h(n)=\frac{1}{j2\pi}\oint_c \sum_{n=-\infty}^{+\infty} h[n]z^{-n}z^{n-1}\,dz$$

$$h(n)=\frac{1}{j2\pi}\oint_c H(z)z^{n-1}\,dz$$

(2)

$$-\frac{1}{8} z^{-1} Y(z) + \frac{1}{4} Y(z) + z^{+1} Y(z) = -2 z^{-1} X(z) + X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1}}{-\frac{1}{8} z^{-1} + \frac{1}{4} + z^{+1}} = \frac{0 + z^{-1} - 2z^{-2}}{1 + \frac{1}{4} z^{-1} - \frac{1}{8} z^{-2}}$$

$$H(z) = \frac{z^{-1}(1 - 2z^{-1})}{(1 + \frac{1}{2} z^{-1})(1 - \frac{1}{4} z^{-1})}$$

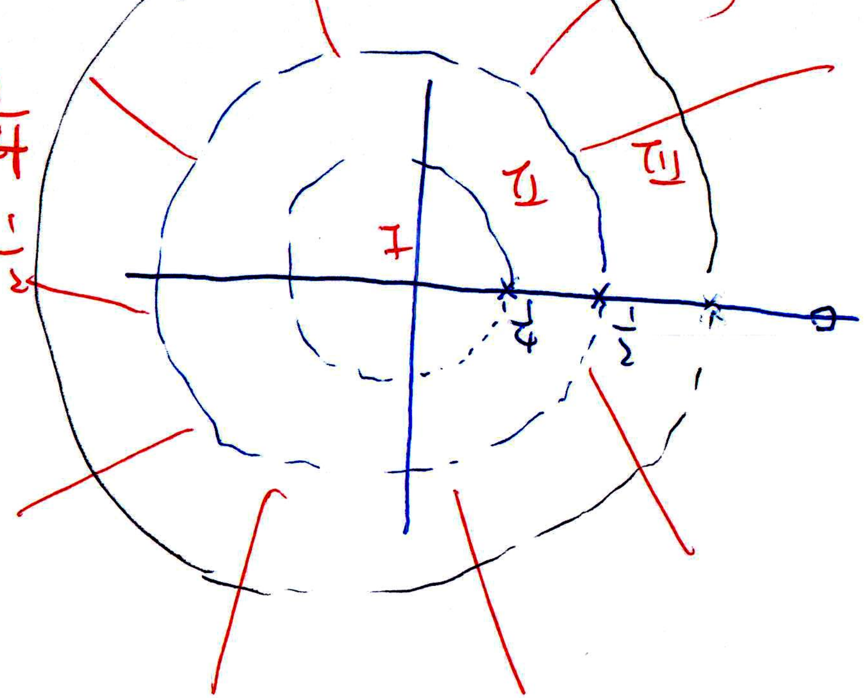
zero ∞ 2

pole $\frac{1}{2}$ $\frac{1}{4}$

I : $|z| < \frac{1}{4}$

II : $\frac{1}{4} < |z| < \frac{1}{2}$

III : $|z| > \frac{1}{2}$



Causal

BIBO stable

(3)

$$\frac{x(1-2x)}{(1+\frac{1}{2}x)(1-\frac{1}{4}x)} = a + \frac{b}{1+\frac{1}{2}x} + \frac{c}{1-\frac{1}{4}x}$$

$$b = \frac{-2(1+4)}{1+\frac{1}{2}} = -\frac{20}{3}$$

$$0 = a - \frac{20}{3} - \frac{28}{3}$$

$$c = \frac{4(1-8)}{1+\frac{1}{2}\times 4} = -\frac{28}{3}$$

$$a = \frac{48}{3} = 16$$

$$a = 16$$

$$\frac{1}{1-(-\frac{1}{2})z^{-1}}$$

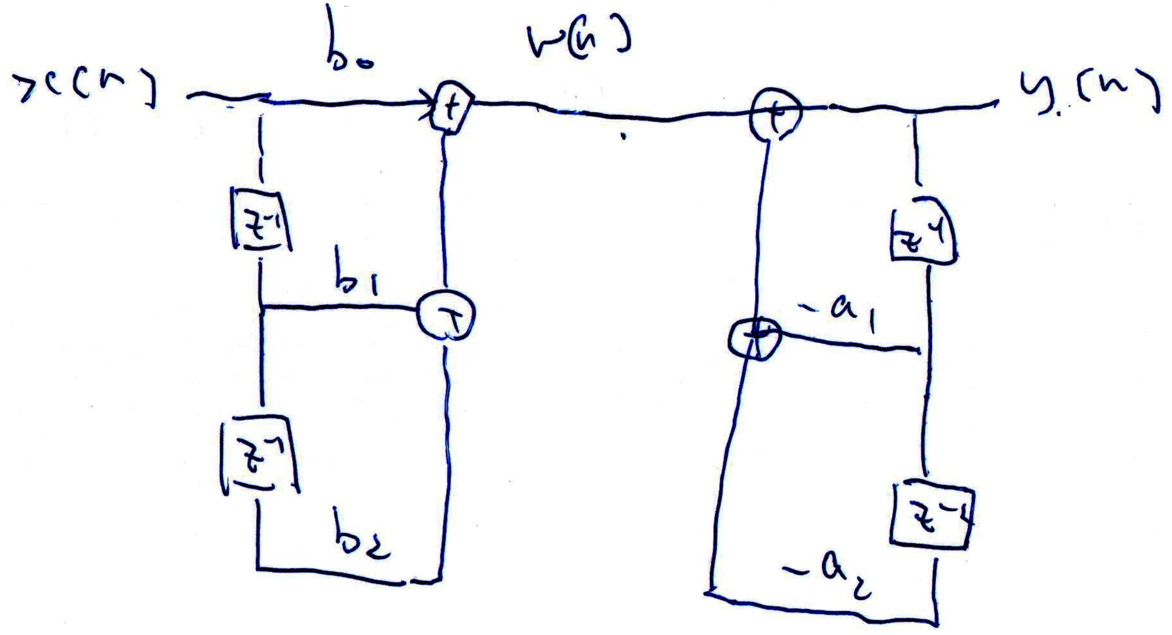
$$H(z) = 16 + (-\frac{20}{3})\frac{1}{1+\frac{1}{2}z^{-1}} + (-\frac{28}{3})\frac{1}{1-\frac{1}{4}z^{-1}}$$

$$h[n] = 16\delta[n] + (-\frac{20}{3})(-\frac{1}{2})^n u[n] + (-\frac{28}{3})(\frac{1}{4})^n u[n]$$

$y[n] = w[n] + (-a_1)\, y[n-1] + (-a_2)\, y[n-2]$ (4)

$a_2\, y[n-2] + a_1\, y(n-1) + y(n) = \boxed{b_2\, x[n-2] + b_1\, x[n-1] + b_0\, x[n]}$

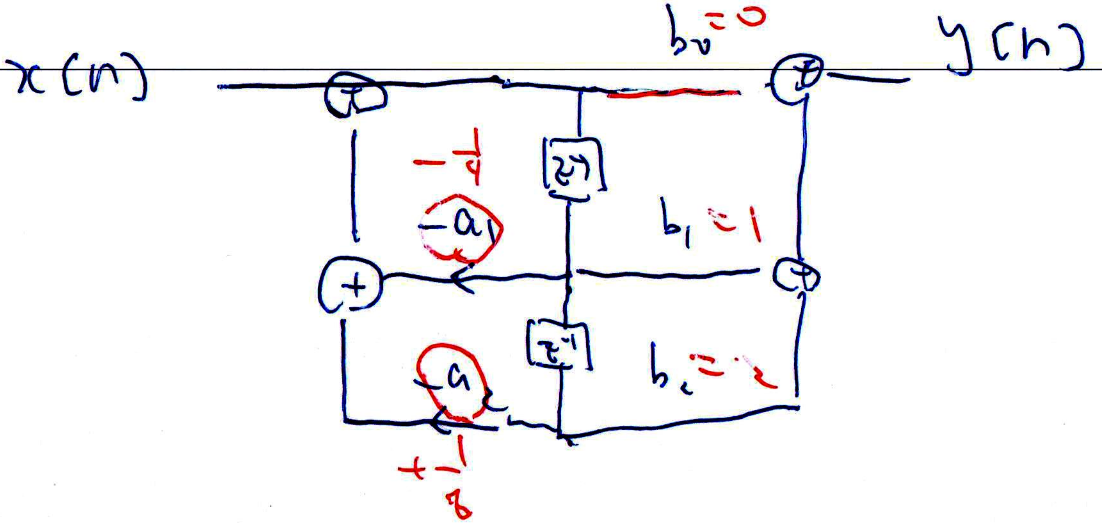
$w[n]$



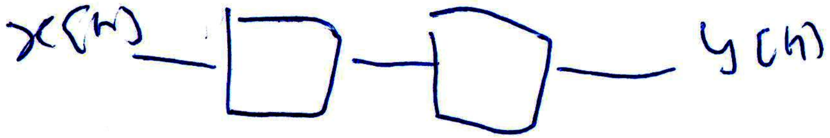
5

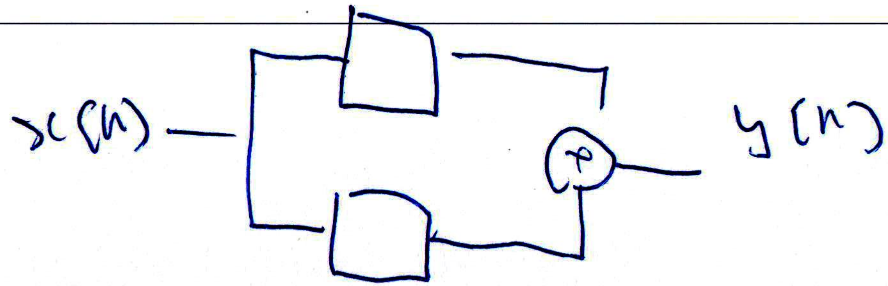
$$a_2\, y(n-2) + a_1\, y(n-1) + 1\, y(n) = b_2\, x(n-2) + b_1\, x(n-1) + b_0\, x(n)$$

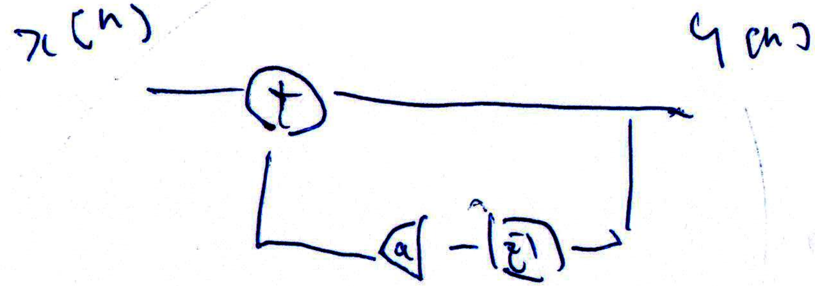
$$\underset{-\frac{1}{8}}{a_2} \quad \underset{\frac{1}{4}}{a_1} \quad \underset{-2}{b_2} \quad \underset{1}{b_1} \quad \underset{0}{b_0}$$



$$-\frac{1}{8}\, y(n-2) + \frac{1}{4}\, y(n-1) + 1\, y(n) = -2\, x(n-2) + x(n-1) + 0\, x(n)$$







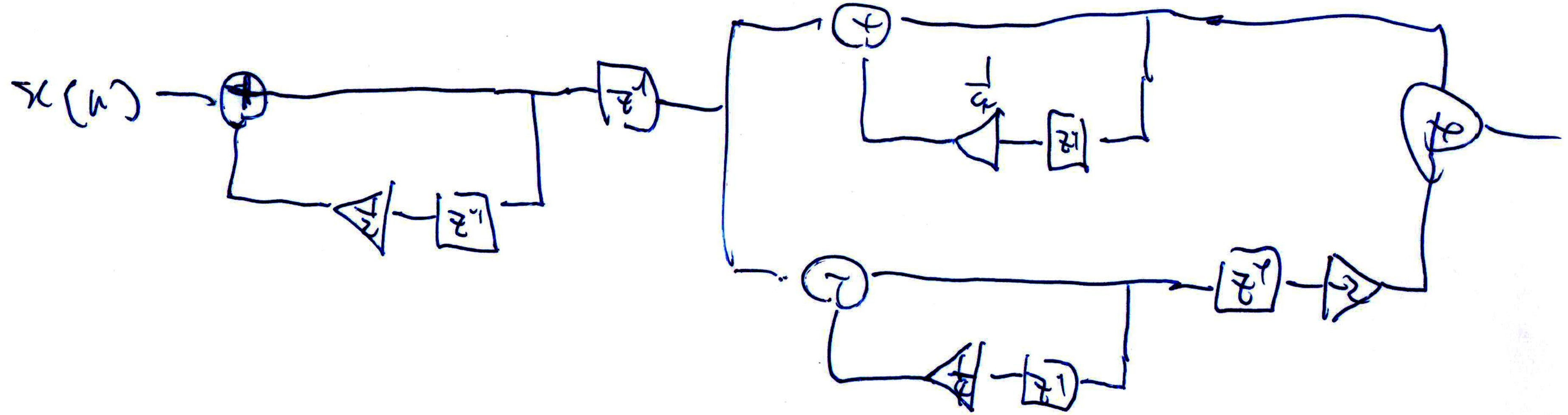
$$H(z) = \frac{1}{1 - az^{-1}}$$

Cascade

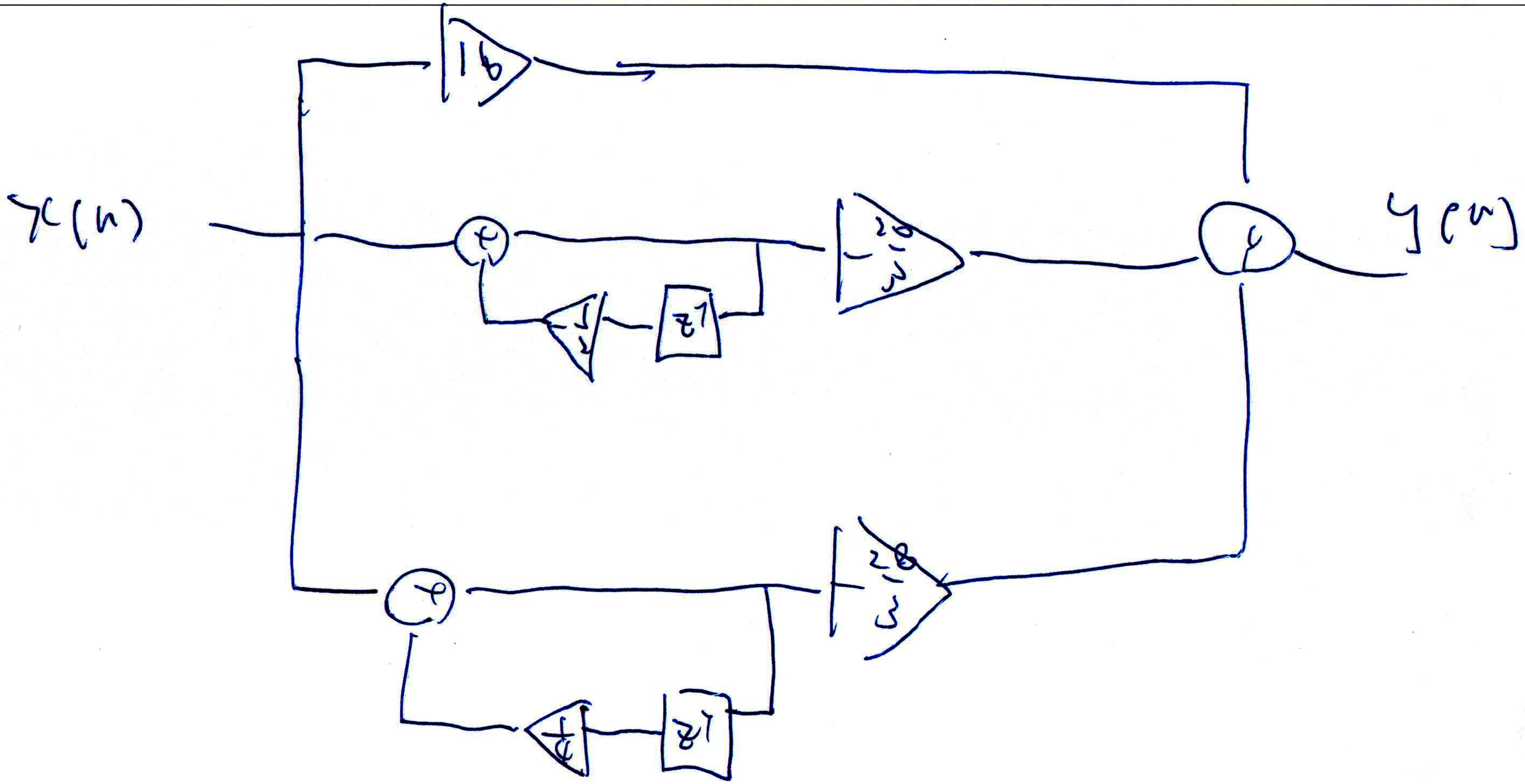
$$H(z) = \frac{z^{-1}(1-2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$= \frac{z^{-1}}{1-\frac{1}{2}z^{-1}} \; \frac{1-2z^{-1}}{1-\frac{1}{4}z^{-1}}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} z^{-1} \times \left(\frac{1}{1-\frac{1}{4}z^{-1}} - 2\frac{z^{-1}}{1-\frac{1}{4}z^{-1}} \right)$$



$$H(z) = 16 + \left(\frac{-20}{3}\right)\frac{1}{1-\left(-\frac{1}{2}\right)z^{-1}} + \left(-\frac{28}{3}\right)\frac{1}{1-\frac{1}{4}z^{-1}} \tag{8}$$



Problem 2

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stable

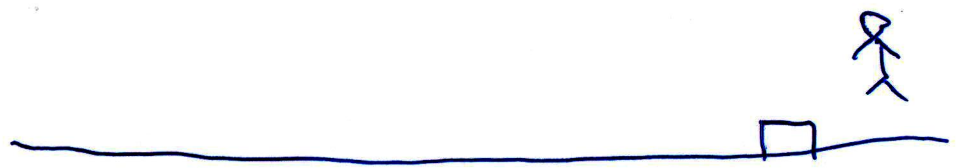
3. Consider the LTI system $\boxed{\text{initially at rest}}$ and described by the difference equation

time

$$y[n]+2y[n-1]=x[n]+2x[n-2]$$

Find the response of this system to the input $x[n]=\delta\cancel{[n+2]+2\delta[n+1]+3\delta[n]+2\delta[n-1]+2\delta[n-2]}$ (5%)

$\delta[n-2]$

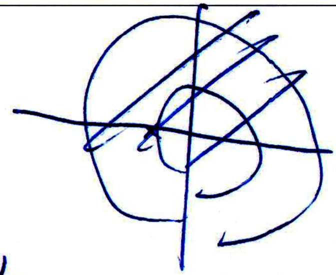


(10)

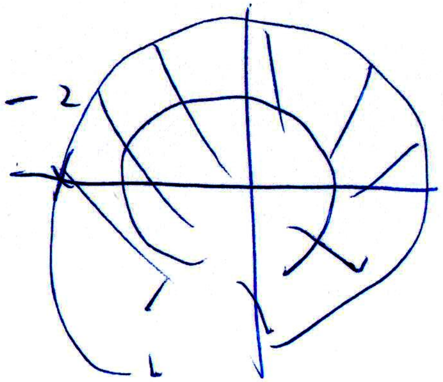
$$y(n) + 2y(n-1) = x(n) + 2x(n-2)$$

$$Y(z) + 2z^{-1}Y(z) = X(z) + 2z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-2}}{1+2z^{-1}}$$



$$= \frac{1+2z^{-1} - 2z^{-1} + 2z^{-2}}{1+2z^{-1}}$$



$$= 1 - 2z^{-1}\frac{1 - z^{-1}}{1+2z^{-1}}$$

$$= 1 - 2z^{-1}\frac{1+2z^{-1} - 3z^{-1}}{1+2z^{-1}}$$

$$= 1 - 2z^{-1}\left(1 - 3\frac{z^{-1}}{1+2z^{-1}}\right)$$

$$H(z) = 1 - 2z^{-1} + 6z^{-2} \frac{1}{1 - (-2)\, z^{-1}}$$

$$\cancel{H(z)}\; h(n) = \delta(n) - 2\,\delta(n-1) + 6\,(-2)^{n-2}\, u[-n-3]$$

$$y(n) = h(n) * x(n) \leftarrow \delta(n-2)$$

$$\boxed{y(n) = \delta(n-2) - 2\,\delta(n-3) + 6\,(-2)^{n-4} [-n-5]}$$

Problem 3

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6. Consider the following system.

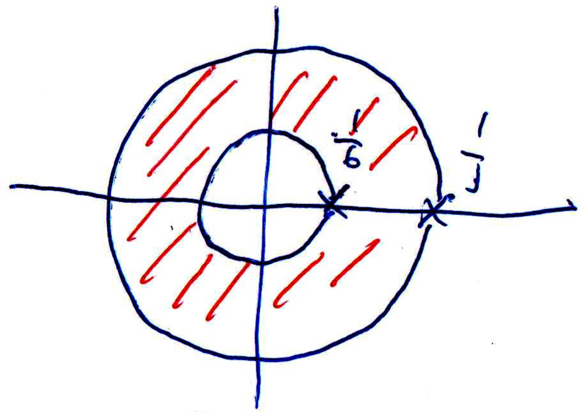
$$x[n] \longrightarrow \boxed{\begin{matrix} H(z) \\ h[n] \end{matrix}} \longrightarrow y[n]$$

(a) Let $H(z) = \frac{1-\frac{2}{9}z^{-1}}{1-\frac{1}{3}z^{-1}}$ and $x[n] = (\frac{1}{6})^n u[n]$, where $u[n]$ is the unit step function with unity gain for $n \geq 0$. If Region of Convergence (ROC) of $y[n]$ is a ring, determine the output $y[n]$. (5%)

$$H(z) = \frac{1-\frac{2}{9}z^{-1}}{1-\frac{1}{3}z^{-1}} \qquad X(z) = \frac{1}{1-\frac{1}{6}z^{-1}} \quad |z| > \frac{1}{6}$$

$$Y(z) = H(z) \times X(z)$$

$$= \frac{1-\frac{1}{9}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})}$$



z plane

$$\frac{1-\frac{1}{9}x}{\left(1-\frac{1}{3}x\right)\left(1-\frac{1}{6}x\right)} = \frac{a}{1-\frac{1}{3}x} + \frac{b}{1-\frac{1}{6}x}$$

$$a = \frac{1-\frac{1}{9}\times 3}{1-\frac{1}{6}\times 3} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{4}{3}$$

$$b = \frac{1-\frac{1}{9}\times 6}{1-\frac{1}{3}\times 6} = \frac{1-\frac{2}{3}}{-1} = -\frac{1}{3}$$

$$Y(z) = \frac{4}{3}\,\frac{1}{1-\frac{1}{3}z^{-1}} + \left(-\frac{1}{3}\right)\frac{1}{1-\frac{1}{6}z^{-1}}$$

$$y[n] = -\frac{4}{3}\left(\frac{1}{3}\right)^n u[-n-1] + \left(-\frac{1}{3}\right)\left(\frac{1}{6}\right)^n u[n]$$

Problem 4
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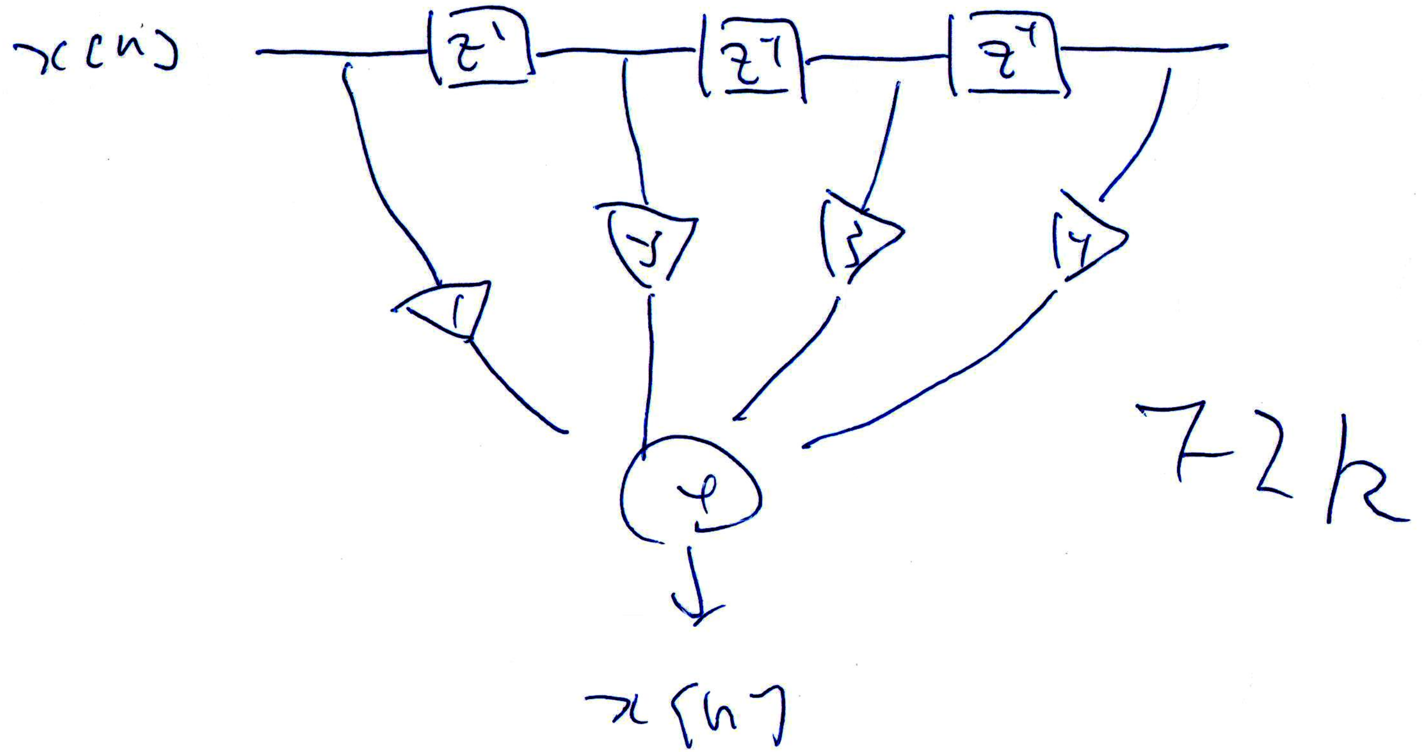
六、(15 %) An LTI system is described by the difference equation:

$$y[n] = x[n] - 3x[n-1] + 3x[n-2] - x[n-3].$$

What is the output if the input is $x[n] = 10 + 4\cos(0.5\pi n + \pi/4) + 5\delta[n-3]$?



$$y[n] = 10 + 4\cos(0.5\pi n + \tfrac{\pi}{4}) + 5\delta[n-3]$$

$$-3\left(10 + 4\cos(0.5\pi(n-1) + \tfrac{\pi}{4}) + 5\delta[n-4]\right)$$

$$+3\left(10 + 4\cos(0.5\pi(n-2) + \tfrac{\pi}{4}) + 5\delta[n-5]\right)$$

$$-\left(10 + 4\cos(0.5\pi(n-3) + \tfrac{\pi}{4}) + \delta[n-6]\right)$$

Problem 5

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causal

8. (10%) For a discrete-time LTI system, we know that if the input is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \frac{1}{4}\left(\frac{1}{3}\right)^{n-1} u[n-1]$$

then the output is

$$y[n] = \frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$$

(1) Find the corresponding impulse response $h[n]$ of the LTI system.

(2) Find a difference equation relating $x[n]$ and $y[n]$ that describes the LTI system.

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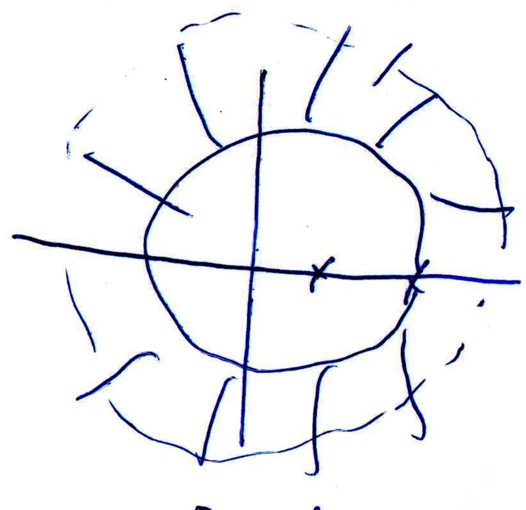
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{2}\,\frac{1}{1-\frac{1}{2}z^{-1}}}{\frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{4}\,\frac{z^{-1}}{1-\frac{1}{3}z^{-1}}}$$

$$= \frac{\frac{3}{2}\,\frac{1}{1-\frac{1}{2}z^{-1}}}{\frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{3}z^{-1}}}$$

$$= \frac{3}{2}\,\frac{1-\frac{1}{3}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$



z plane

$$|z| > \frac{1}{2}$$

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$$\frac{1-\frac{1}{3}x}{(1-\frac{1}{2}x)(1-\frac{1}{4}x)} = \frac{a}{1-\frac{1}{2}x} + \frac{b}{1-\frac{1}{4}x}$$

$$a = \frac{1-\frac{2}{3}}{1-\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$b = \frac{1-\frac{4}{3}}{1-2} = \frac{1}{3}$$

$$H(z) = \frac{3}{2} \times \frac{2}{3} \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{3}{2} \times \frac{1}{3} \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$h(n) = \left(\frac{1}{2}\right)^n u(n) + \frac{1}{2}\left(\frac{1}{4}\right)^n u(n)$$

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$$\frac{Y(z)}{X(z)} = H(z) = \frac{3}{2} \frac{1 - \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)} = \frac{\frac{3}{2} - \frac{1}{2} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$\frac{1}{8} z^{-2} Y(z) - \frac{3}{4} z^{-1} Y(z) + Y(z) = -\frac{1}{2} z^{-1} X(z) + \frac{3}{2} X(z)$$

$$\frac{1}{8} y(n-2) - \frac{3}{4} y(n-1) + y(n) = -\frac{1}{2} x(n-1) + \frac{3}{2} x(n)$$