HW18

①

Problem 1

107台聯大訊號與系統

八、(15%)

$x[n]$ ‖

If the input to an LTI system is $u[n]$, the output is $y[n] = (1/2)^{n-1}\, u[n]$.

(一) (6%) If $H(z)$ is the $z$-transform of the system impulse response, find its pole, zero, and the region of convergence.

(二) (5%) Find the impulse response $h[n]$ of this LTI system.

(三) (4%) Is the system stable? Is the system causal?

(2)

$$y[n] = \left(\frac{1}{2}\right)^{n-1} u[n] = 2\left(\frac{1}{2}\right)^{n} u[n]$$

$$Y(z) = 2\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$x[n] = u(n)$$

$$X(z) = \frac{1}{1-z^{-1}}$$

---

$$y(n) = h(n) * x(n)$$

$$Y(z) = H(z)\,X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 2\frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}}$$

zero $z = 1$

pole $z = \frac{1}{2}$

$$H(z) = \frac{2\frac{1}{1-\frac{1}{2}z^{-1}}}{\frac{1}{1-z^{-1}}} = 2\frac{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\boxed{H(z) = 2 - \frac{z^{-1}}{1-\frac{1}{2}z^{-1}}}$$

③

$x_1[n] = u[n] \qquad y_1[n] = 2\left(\frac{1}{2}\right)^n u[n]$

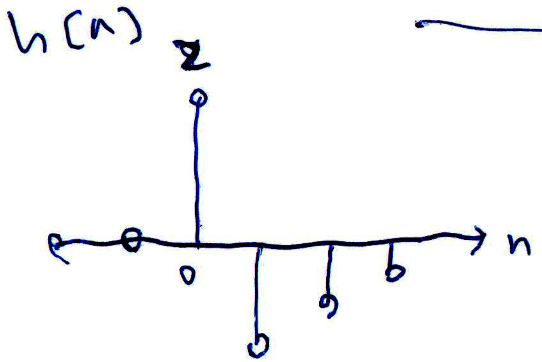
$x_2[n] = u[n-1] \qquad y_2[n] = 2\left(\frac{1}{2}\right)^{n-1} u[n-1]$

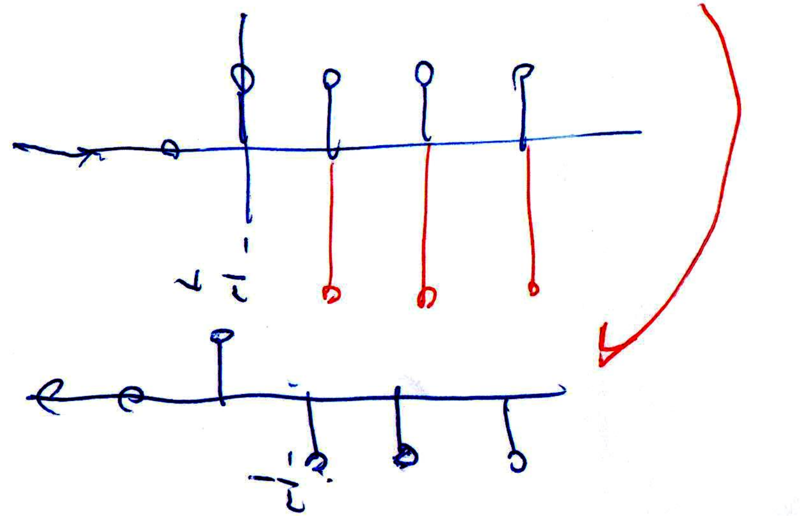
$x[n] = \delta[n] \qquad y[n] = 2\left[\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{2}\right)^{n-1} u[n-1]\right]$

$y[n] = h[n]$

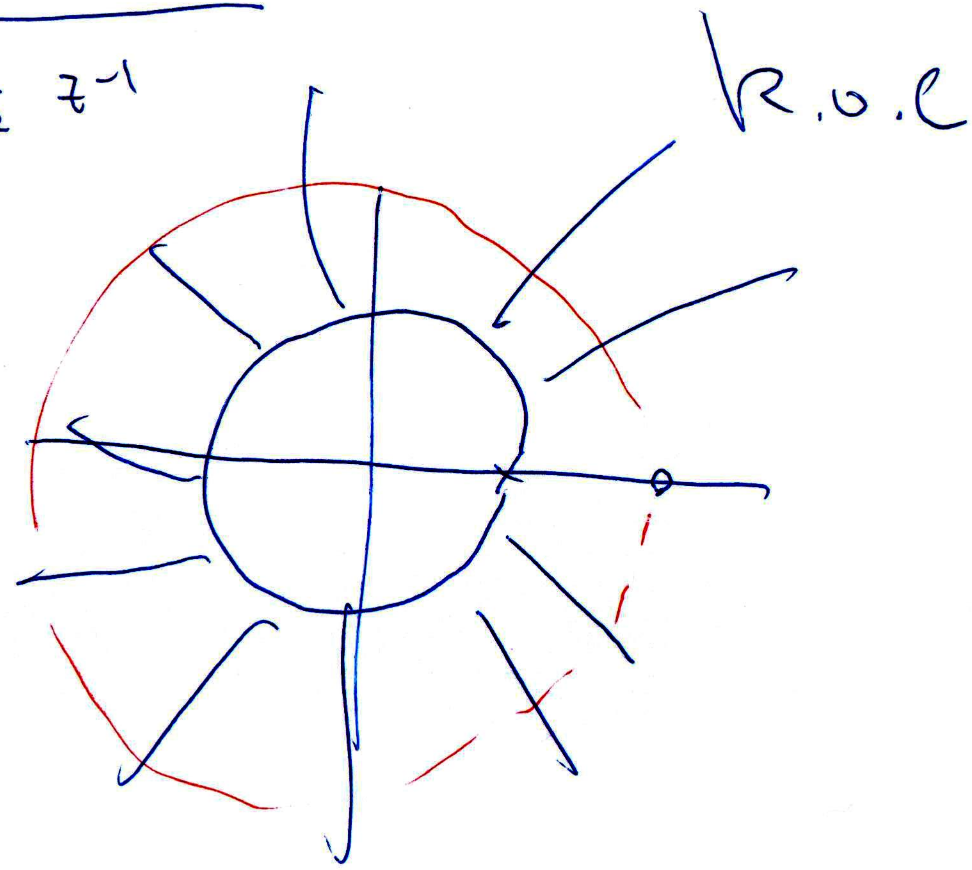
$h[n] = 2\left(\frac{1}{2}\right)^{n-1}\left[\frac{1}{2}u[n] - u[n-1]\right]$

$-\frac{1}{2} \times 2 \times \frac{1}{2}$



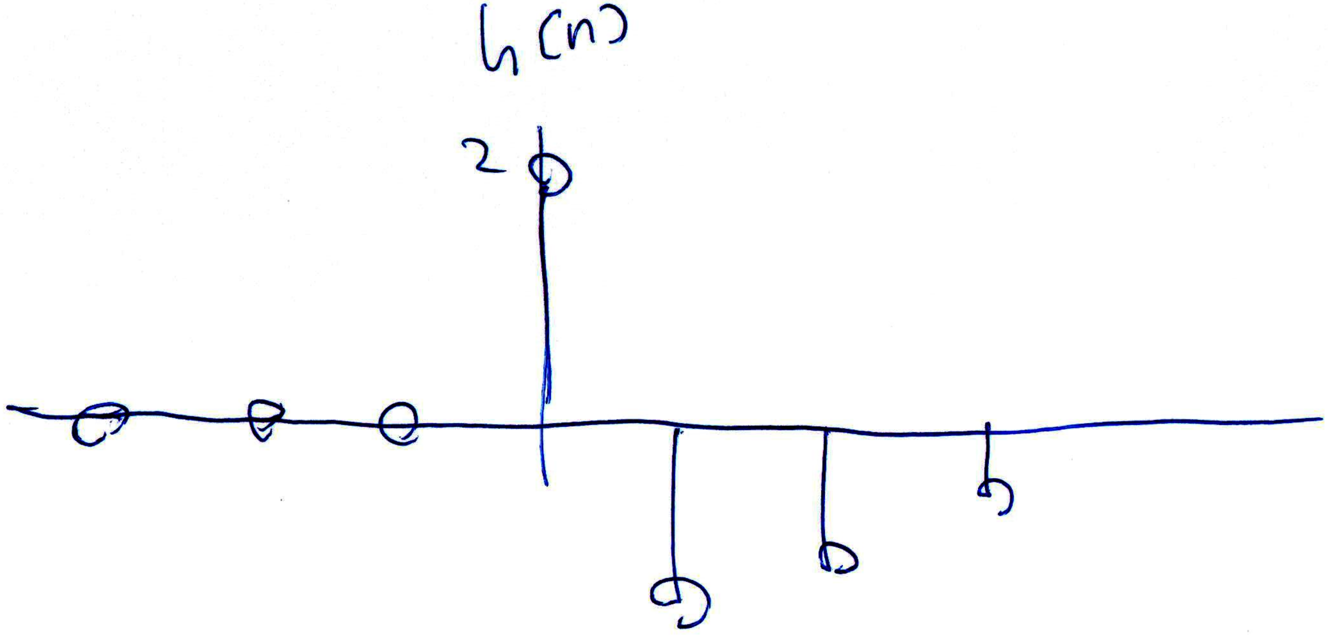
$$H(z) = 2\,\frac{1 - z^{-1}}{1 - \frac{1}{2} z^{-1}}$$




z plane

$$H(z) = 2 - \frac{z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$\boxed{h(n) = 2 \cdot \delta(n) - \left(\frac{1}{2}\right)^{n-1} u(n-1)}$$



Problem 2

106台聯大訊號與系統

二、(10%)

~~(a) (5%) The impulse response of an LTI system is~~ $\cancel{h(t) = \begin{cases} \cos(\pi t), & |t| < 0.5 \\ 0, & \text{otherwise} \end{cases}}$ ~~. Use linearity and time invariance to determine and plot the output $y(t)$ for $x(t) = \delta(t+1) - \delta(t-1)$.~~

(b) (5%) Evaluate the convolution sum: $y[n] = \left(u[n+3] - u[n-1]\right) * u[n-4]$.

$$y[n] = u[n+3] * u[n-4] - u[n-1] * u[n-4]$$

$$Y(z) = \frac{z^{+3}}{1-z^{-1}} \times \frac{z^{-4}}{1-z^{-1}} - \frac{z^{-1}}{1-z^{-1}} \times \frac{z^{-4}}{1-z^{-1}}$$

$$= \frac{z^{-1}}{(1-z^{-1})^2} - \frac{z^{-5}}{(1-z^{-1})^2}$$

$$y[n] = (\overset{n-1}{n}+1)\,u[\overset{n-1}{n}] - (\overset{n-5}{n}+1)\,u[\overset{n-5}{n}]$$

$$y[n] = n\,u[n-1] - (n-4)\,u[n-5]$$

$$u[n] \longleftrightarrow \frac{1}{1-z^{-1}} \qquad |z| > 1$$

$$(n+1)\,u(n) \longleftrightarrow \frac{1}{(1-z^{-1})^2} \qquad |z| > 1$$

$$(n+1)\,a^n u(n) \longleftrightarrow \frac{1}{(1-az^{-1})^2} \qquad |z| > |a|$$

Problem 3
106台聯大訊號與系統

causal

三、(15%)

(a) (10%) Find the frequency response (5%) and impulse response (5%) of the discrete-time system described by $8y[n]-2y[n-1]-y[n-2]=x[n]+x[n-1]$.

(b) (5%) Draw $\boxed{\text{direct form II implementation}}$ of the system in (a).

$$y(n)+\left(\frac{-2}{8}\right)y[n-1]+\left(\frac{-1}{8}\right)y(n-2)=$$

$$\frac{1}{8}x[n]+\frac{1}{8}x(n-1)+0\,x(n-2)$$

$$8y(n) - 2y(n-1) - y(n-2) = x(n) + x(n-1)$$ 9

$$8Y(z) - 2z^{-1}Y(z) - z^{-2}Y(z) = X(z) + z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{8 - 2z^{-1} - z^{-2}}$$

$$H(z) = \frac{1}{8}\,\frac{1+z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$H(z) = \frac{1}{8}\,\frac{1+z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

$$H(e^{j2\pi f}) = \frac{1}{8}\,\frac{1+z^{-1}}{1-\frac{1}{4}e^{-j2\pi f}-\frac{1}{8}e^{-j4\pi f}}$$

---

$$H(z) = \frac{1}{8}\,\frac{1+z^{-1}}{(1+\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$\frac{1+x}{(1+\frac{1}{4}x)(1-\frac{1}{2}x)} = \frac{a}{1+\frac{1}{4}x}\ (1) + \frac{b}{1-\frac{1}{2}x}\ (2)$$

$$\frac{1+x}{1-\frac{1}{2}x} = a + \frac{b}{1-\frac{1}{2}x}(1+\tfrac{1}{4}x)$$

$$a = \frac{-3}{3} = -1$$

$$\frac{1+x}{1+\frac{1}{4}x} = \frac{a}{1+\frac{1}{4}x}\left(1-\frac{1}{2}x\right) + b$$

11

$$b = \frac{3}{3/2} = \cancel{0}2$$

---

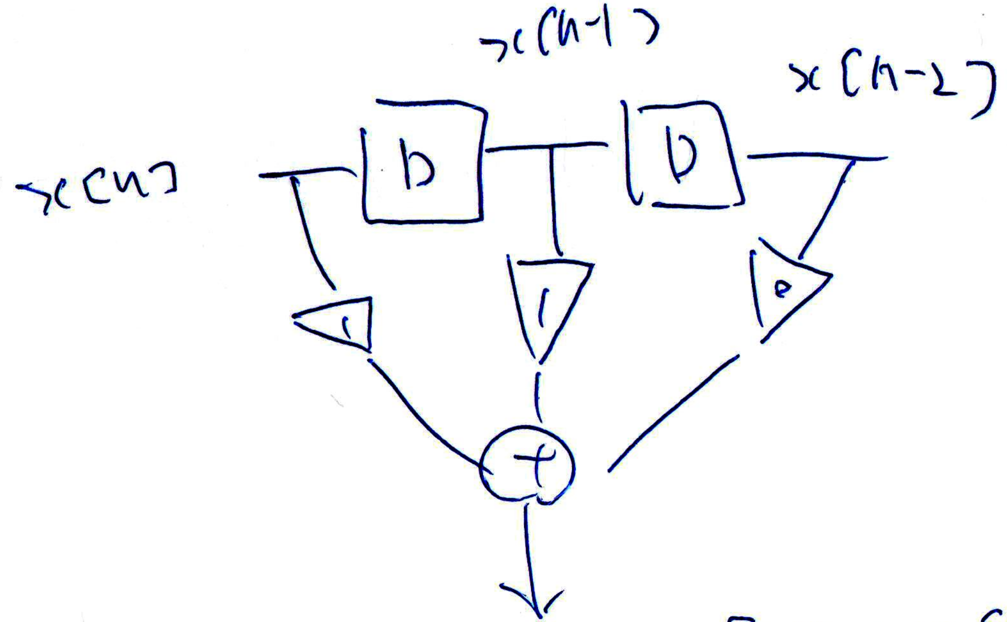
$$H(z) = \frac{1}{8}\,\frac{1}{1+\frac{1}{4}z^{-1}} + \frac{1}{8}\,\frac{2}{1-\frac{1}{2}z^{-1}}$$

---

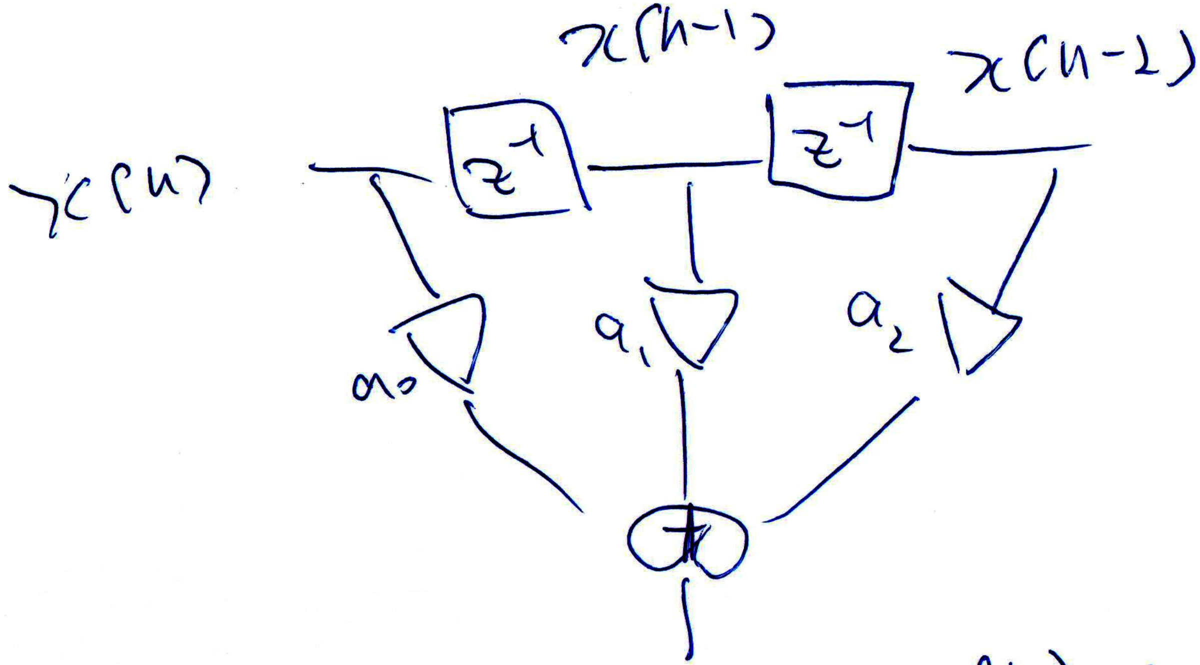
$$\boxed{h(n) = \frac{1}{8}\left(-\frac{1}{4}\right)^{n}u(n) + \frac{2}{8}\left(+\frac{1}{2}\right)^{n}u(n)}$$

$$8y[n] - 2y[n-1] - y[n-2] = x[n] + x[n-1]$$

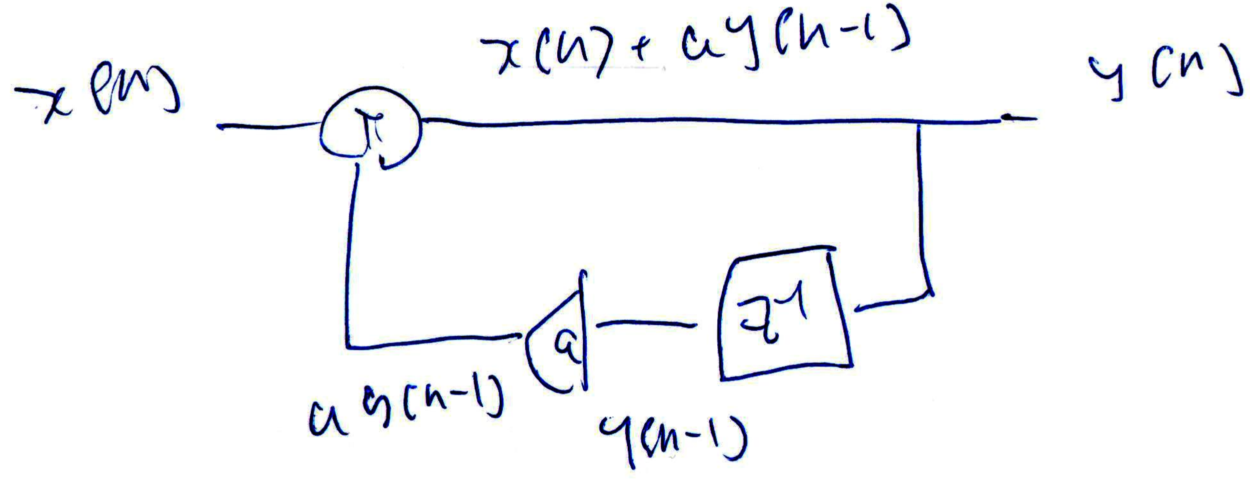
$$\boxed{\begin{cases} w[n] = x[n] + x[n-1] \\ 8y[n] - 2y[n-1] - y[n-2] = w[n] \end{cases}}$$




$$w[n] = x[n] + x[n-1]$$




$$w(n) = a_0 x(n) + a_1 x[n-1] + a_2 x[m-2]$$

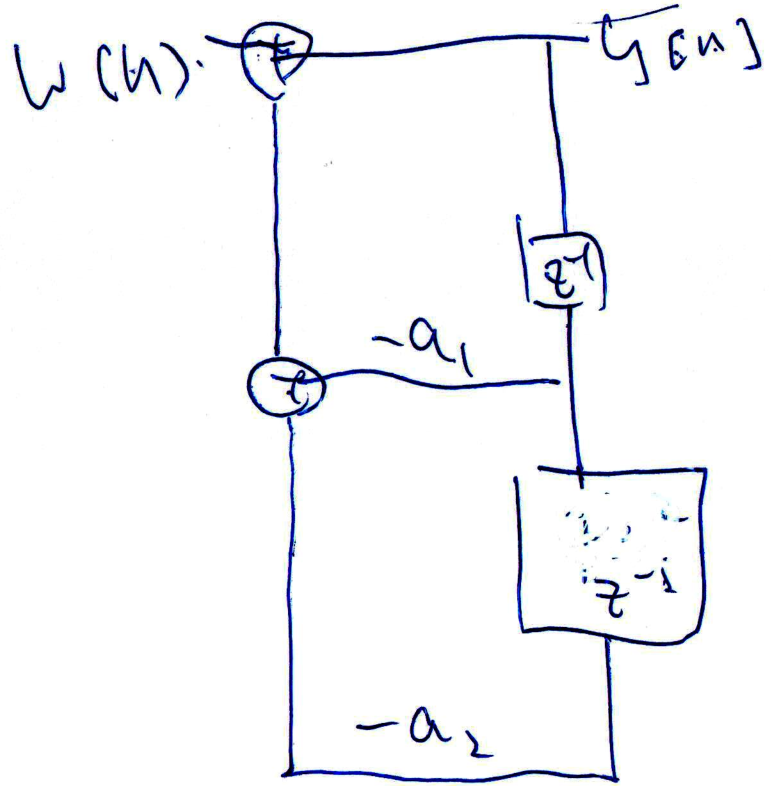



$$y(n) = a\,y(n-1) + x(n)$$

$$y(n) - a\,y(n-1) = x(n)$$

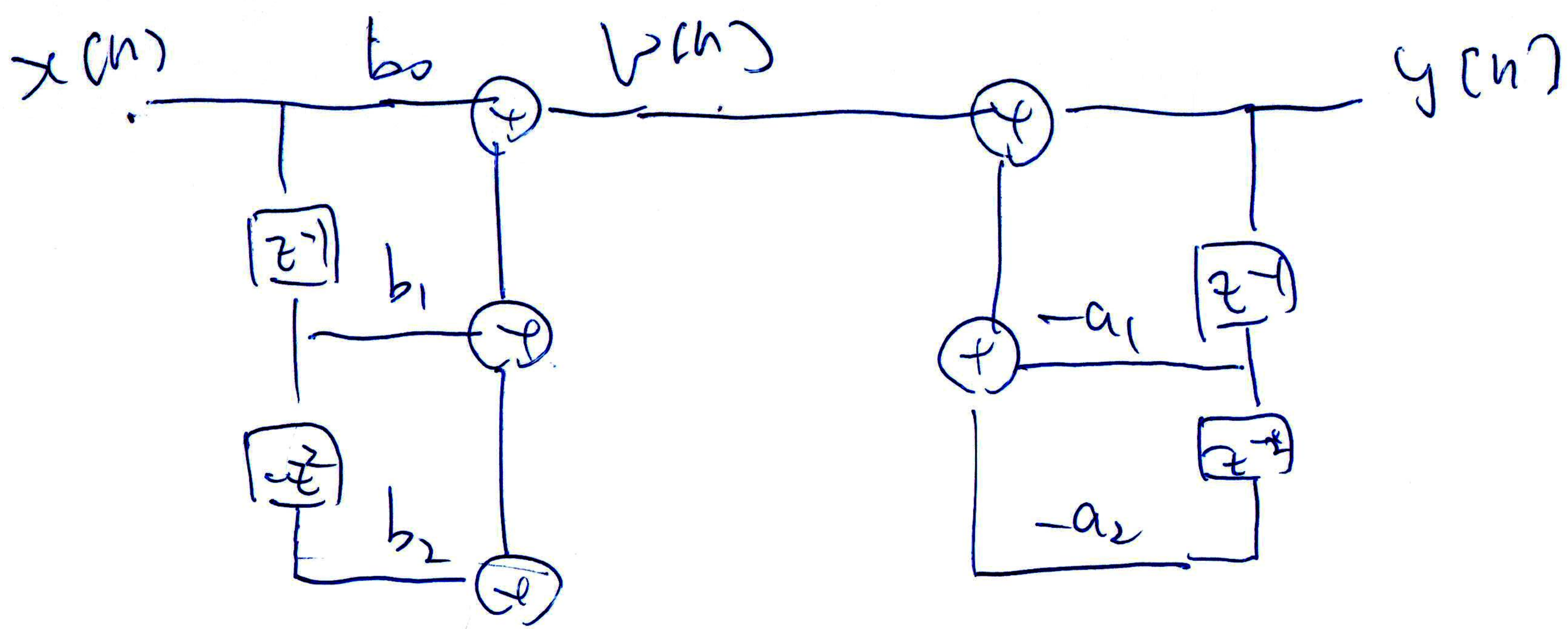
$$y[n] + a_1\, y(n-1) + a_2\, y(n-2) = w[n]$$

$$y[n] = w[n] + (-a_1)\, y(n-1) + (-a_2)\, y[n-2]$$



$$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

$$\begin{cases} w(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) \\ y(n) + a_1 y(n-1) + a_2 y(n-2) = w(n) \end{cases}$$

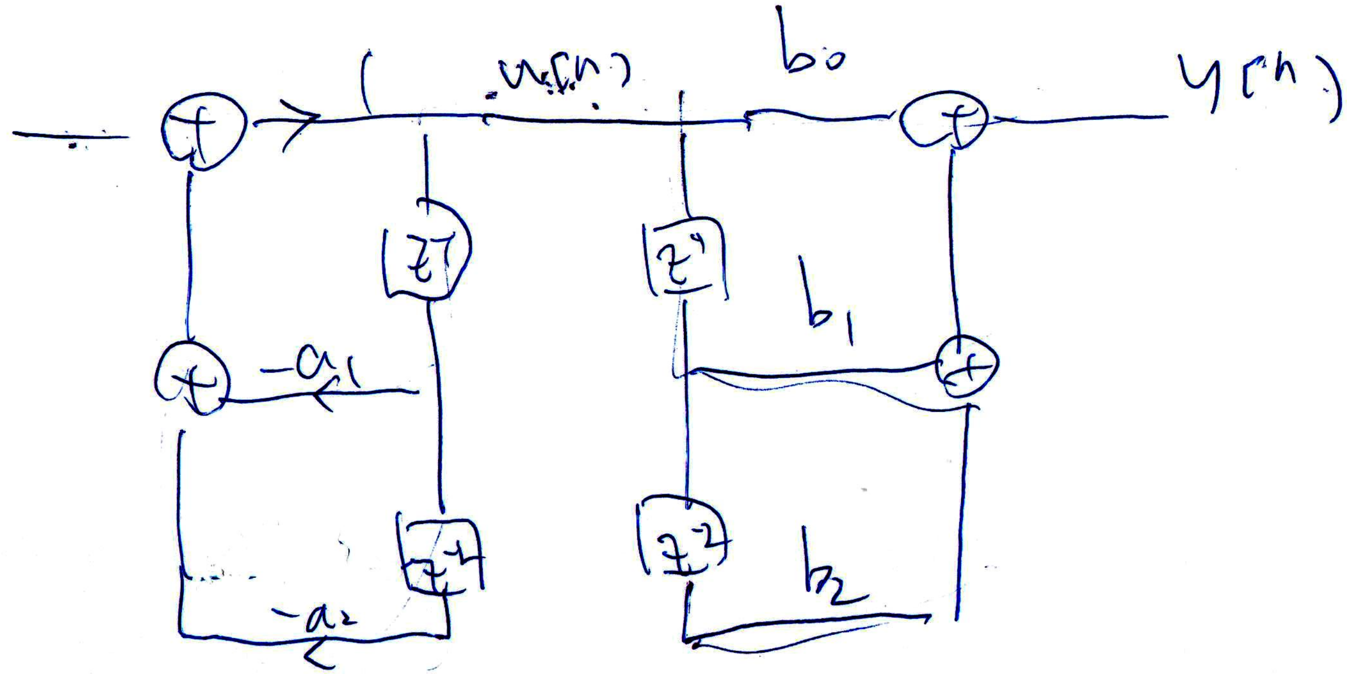


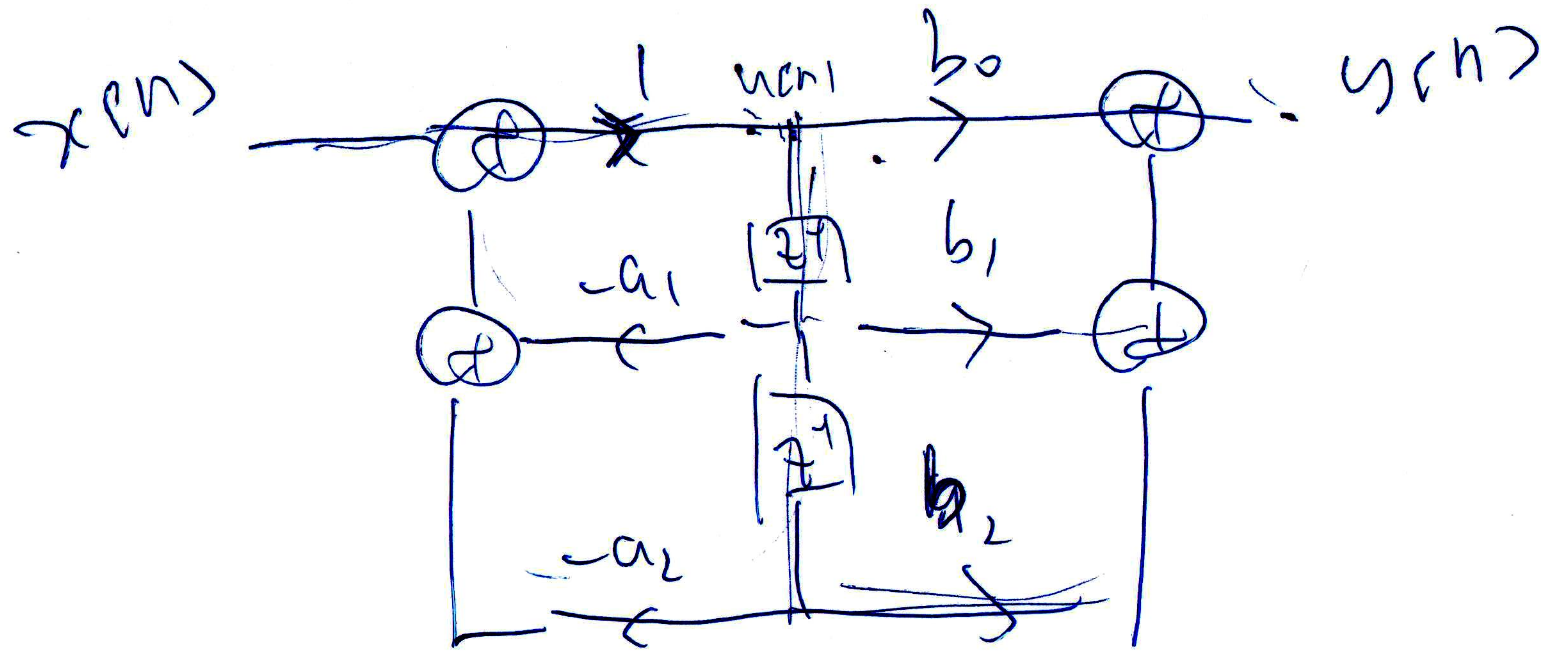
$$y[n] = h_2(n) * h_1(n) * x(n)$$

$$x(n) \longrightarrow \boxed{h_1(n)} \longrightarrow \boxed{h_2(n)} \longrightarrow y[n]$$

$$H(z) = H_1(z) \times H_2(z)$$

$$x(n) \longrightarrow \boxed{h_2(n)} \longrightarrow \boxed{h_1(n)} \longrightarrow y[n]$$

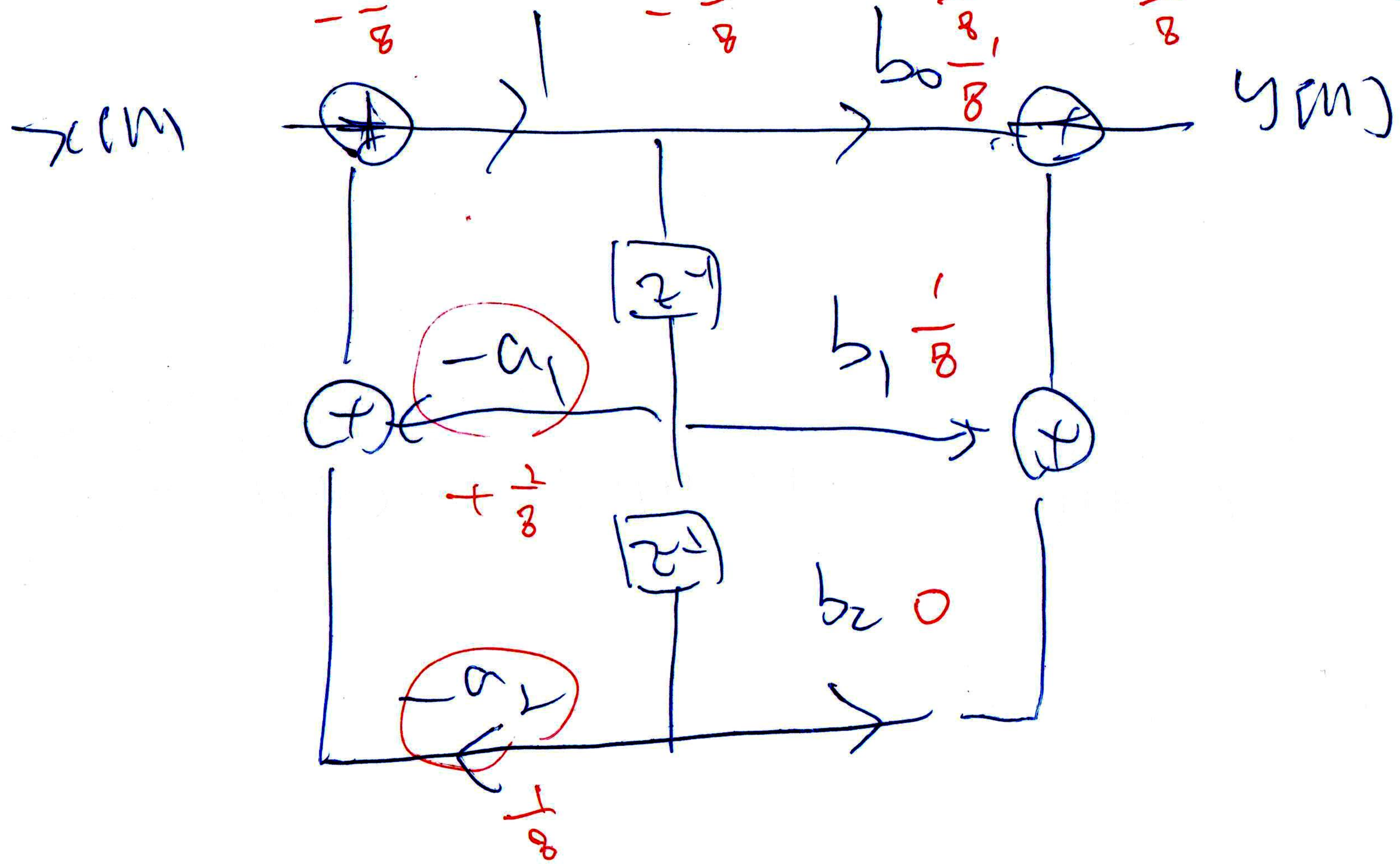

x(n)
u(n)
$b_0$
y(n)
$z^{-1}$
$z^{-1}$
$b_1$
$-a_1$
$z^{-1}$
$z^{-1}$
$b_2$
$-a_2$


x(n)
1
u(n)
$b_0$
y(n)
$z^{-1}$
$-a_1$
$b_1$
$z^{-1}$
$b_2$
$-a_2$

(9)

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x(n) + b_1 x(n-1) + b_2 x[n-2]$$

$-\frac{2}{8}$ $\quad -\frac{1}{8}$ $\quad \frac{1}{8}$ $\quad \frac{1}{8}$ $\quad 0$



Problem 4

105台聯大訊號與系統

20

八、(15%) When the input to a $\boxed{\text{causal LTI system}}$ is $x[n]=-\frac{1}{3}\left(\frac{1}{3}\right)^n u[n]-\frac{4}{3}2^n u[-n-1]$, the $z$-transform of the output is $Y(z)=\dfrac{1+z^{-1}}{(1-z^{-1})(1+0.5z^{-1})(1-2z^{-1})}$.

(a) (5%) Find the $z$-transform of $x[n]$.

(b) (4%) What is the region of convergence of ~~Y(z)~~ H(z)?

(c) (4%) Find the impulse response of the system.

(d) (2%) Is the system stable? not stable

參考

(a) $$x[n] = -\frac{1}{3}\left(\frac{1}{3}\right)^n u[n] + \frac{4}{3}\left(-(2)^n u[-n-1]\right)$$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \quad |z| > |a|$$

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \quad |z| < |a|$$

$$X(z) = -\frac{1}{3}\frac{1}{1-\frac{1}{3}z^{-1}} + \frac{4}{3}\frac{1}{1-2z^{-1}}$$

$|z| > \frac{1}{3}$ $\qquad$ $|z| < 2$

$$X(z) = \frac{-\frac{1}{3}(1-2z^{-1}) + \frac{4}{3}(1-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$$

22

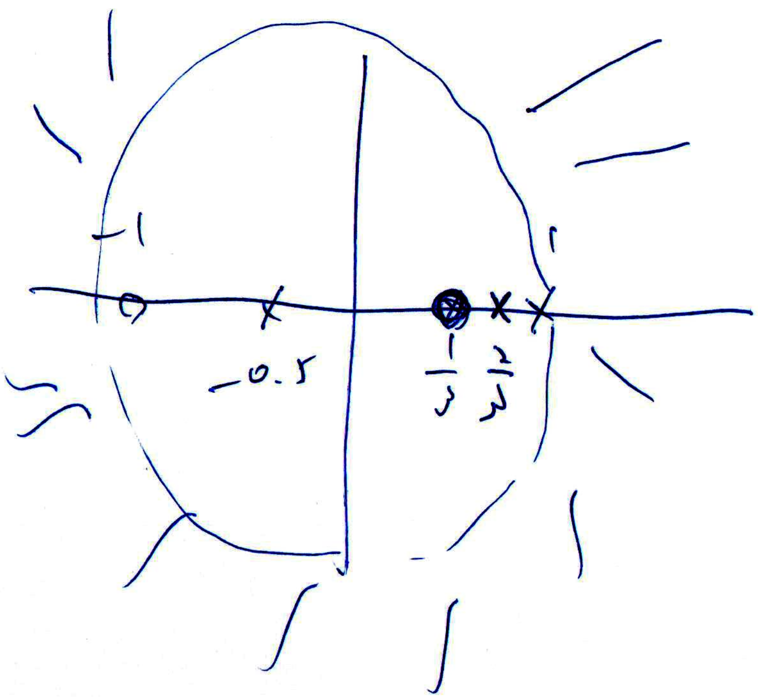
$+\frac{2}{3} - \frac{4}{3}$

$$\boxed{X(z) = \frac{1 - \frac{2}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}}$$

b) $$Y(z) = \frac{1+z^{-1}}{(1-z^{-1})(1+0.5z^{-1})(1-2z^{-1})}$$

c) $$H(z) = \frac{Y(z)}{X(z)} = \frac{\dfrac{1+z^{-1}}{(1-z^{-1})(1+0.5z^{-1})\cancel{(1-2z^{-1})}}}{\dfrac{1-\frac{2}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})\cancel{(1-2z^{-1})}}}$$

$$H(z) = \frac{(1+z^{-1})\cancel{(1-\frac{1}{3}z^{-1})}}{(1-z^{-1})(1+0.5z^{-1})\cancel{(1-\frac{2}{3}z^{-1})}}$$




ROC $|z| > 1$

C4

$$H(z) = \frac{(1+z^{-1})(1-\frac{1}{3}z^{-1})}{(1-z^{-1})(1+0.5z^{-1})(1-\frac{2}{3}z^{-1})}$$

$$= a\frac{1}{1-z^{-1}} + b\frac{1}{1+0.5z^{-1}} + c\frac{1}{1-\frac{2}{3}z^{-1}}$$

$$h(n) = a\,u[n] + b(-0.5)^n u(n) + (\tfrac{2}{3})^n u(n)$$

25

Problem 5

104台聯大訊號與系統

四、(15 pts) Let $H_1(z) = \dfrac{z^{-1}}{1-2.5z^{-1}+z^{-2}}$ and $H_2(z) = \dfrac{0.5z^{-1}}{1-2z^{-1}+0.75z^{-2}}$. Answer the following questions:

(一) (3 pts) Find the **region of convergence** so that $H_1(z)$ is stable. Also, find the corresponding time domain sequence $h_1[n]$.

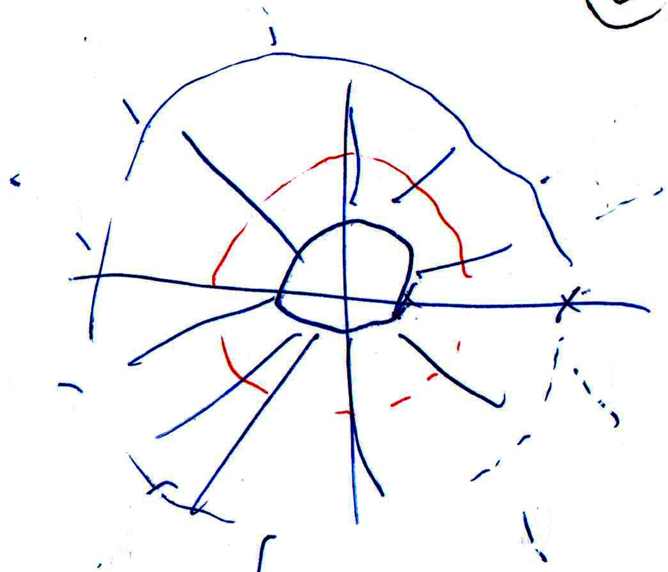
(二) (3 pts) Find the region of convergence so that $H_2(z)$ is causal. Also, find the corresponding time domain sequence $h_2[n]$.

~~(三) (9 pts) Let $X(z)$ be the input and $Y(z)$ be the output. $X(z)$ has the same condition in (a) and $Y(z)$ has the same condition in (b), find the system transfer function $H(z)$. Is $H(z)$ stable? Is $H(z)$ causal? Explain your answer.~~

$$H_1(z) = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

$$= \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$= \frac{a}{1-\frac{1}{2}z^{-1}} + \frac{b}{1-2z^{-1}}$$




$\frac{1}{2} < |z| < 2$

$$\frac{z^{-1}}{1-2z^{-1}} = a + \frac{b}{1-2z^{-1}}(1-\tfrac{1}{2}z^{-1})$$

$$a = -\frac{2}{3}$$

27

$$\frac{z^{-1}}{1-\frac{1}{2}z^{-1}} = \frac{a}{1-\frac{1}{2}z^{-1}}(1-2z^{-1}) + b$$

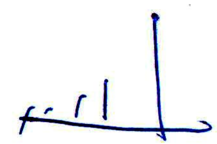
$$b = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$$H(z) = -\frac{2}{3}\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{2}{3}\frac{1}{1-2z^{-1}}$$

$\cancel{|z|>2}$

$\frac{1}{2}<|z|<2$

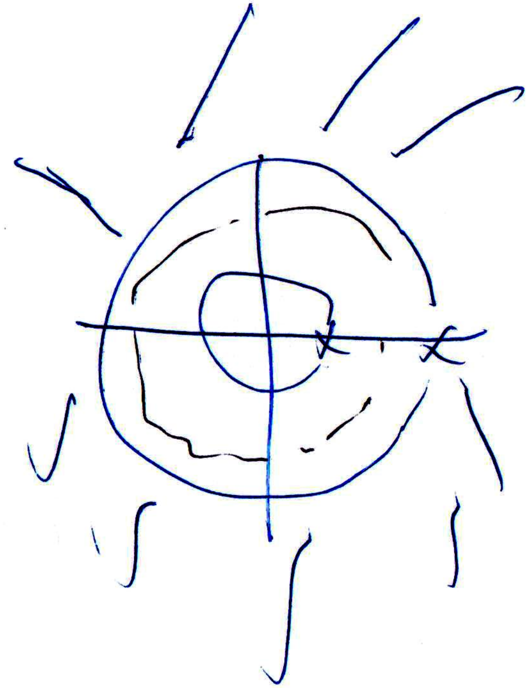
$$h(n) = -\frac{2}{3}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{3}\left(-2^n u(-n-1)\right)$$



$$H_2(z) = \frac{0{,}5\,z^{-1}}{1 - 2z^{-1} + \frac{3}{4}z^{-2}}$$

$$H_2(z) = \frac{0{,}5\,z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{3}{2}z^{-1}\right)}$$

$|z| > \frac{3}{2}$



29

$$\frac{0{,}5x}{\left(1-\frac{1}{2}x\right)\cdot\left(1-\frac{3}{2}x\right)} = \frac{a}{1-\frac{1}{2}x} + \frac{b}{1-\frac{3}{2}x}$$

$$\frac{0{,}5x}{1-\frac{1}{2}x} = \frac{a}{1-\frac{1}{2}x}\left(1-\frac{3}{2}x\right) + b$$

$$b = \frac{\frac{1}{2}\times\frac{2}{3}}{1-\frac{1}{2}\times\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$\frac{0{,}5x}{1-\frac{3}{2}x} = a + b\,\frac{1-\frac{1}{2}x}{1-\frac{3}{2}x}$$

$$a = \frac{1}{1-3} = -\frac{1}{2}$$

30

$$H(z) = \left(-\frac{1}{2}\right)\frac{1}{1-\frac{1}{2}z^{-1}} + \left(\frac{1}{2}\right)\frac{1}{1-\frac{3}{2}z^{-1}} \qquad |z| > \frac{3}{2}$$

$$h(n) = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)^n u(n) + \frac{1}{2}\left(\frac{3}{2}\right)^n u(n)$$