HW 17

①

Problem 1

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3. (10%)

Given the impulse response of an LTI DT system, $h[n]=(0.5)^n u[n]+2(-0.25)^n u[n]$. Determine

(a) the frequency response (5%) and

(b) the linear, constant-coefficient difference equation (5%) of the system.

$$a^n u[n] \xleftrightarrow{ZT} \frac{1}{1-az^{-1}} \qquad |z|>|a|$$

$$0.5^n u[n] \longleftrightarrow \frac{1}{1-0.5z^{-1}} \qquad |z|>0.5$$

$$2\times(-0.25)^n u[n] \longleftrightarrow 2\,\frac{1}{1-0.25z^{-1}} \qquad |z|>0.25$$

$$H(z)=\frac{1}{1-0.5z^{-1}}+2\,\frac{1}{1-0.25z^{-1}} \qquad |z|>0.5$$

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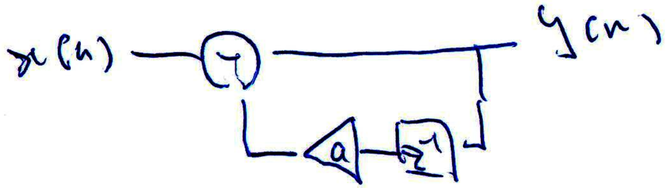


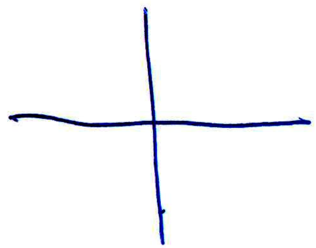
$y[n] + a\,y[n-1] = x[n]$

$H(z)$

$z = e^{j2\pi f}$

$H(e^{j2\pi f})$

$h[n]$



pole-zero plot

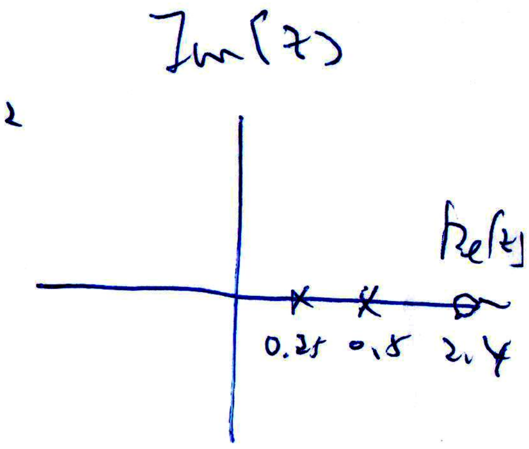
(3)

$$H(z) = \frac{1}{1-0.5z^{-1}} + 2\frac{1}{1-0.25z^{-1}}$$

$$= \frac{(1-0.25z^{-1}) + 2(1-0.5z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$$

$$= \frac{3 - 1.25z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$= \frac{3 - \frac{5}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$



z-plane

$$H(z) = \frac{3 - \frac{5}{4} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$\boxed{H(e^{j2\pi f}) = \frac{3 - \frac{5}{4} e^{-j2\pi f}}{1 - \frac{3}{4} e^{-j2\pi f} + \frac{1}{8} e^{-j4\pi f}}}$$

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$$y(n) = h(n) * x(n)$$

$$Y(z) = H(z) \times X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{3 - \frac{5}{4} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}} = \frac{Y(z)}{X(z)}$$

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = 3X(z) - \frac{5}{4} z^{-1} X(z)$$

$$\boxed{y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = 3x(n) - \frac{5}{4} x(n-1)}$$

Problem 2

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五. Please determine the z-transform or the inverse z-transform of the following signals.

(一)(10%) Please find the z-transform of the signal $x[n]$,

$$x[n]=\left(\frac{1}{2}\right)^{n}u[n]*\left(n\left(\frac{-1}{4}\right)^{n}u[n]\right),$$

where $u[n]$ is discrete-time unit-step function, and $*$ denotes the discrete-time convolution operator.

(二)(10%) Please find the inverse z-transform of $X(z)$,

$$X(z)=\left(\frac{1}{1-az^{-1}}\right)^{2}$$

$$x(n) = a(n) * b(n)$$

$$a(n) = \left(\tfrac{1}{2}\right)^n u(n) \qquad b(n) = n\left(\tfrac{-1}{4}\right)^n u(n)$$

$$X(z) = A(z) \times B(z)$$

$$a(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$$A(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} \qquad |z| > |\tfrac{1}{2}|$$

$$h(n) = n\left(-\frac{1}{4}\right)^n u(n)$$

$$H(z) = \sum_{n=-\infty}^{+\infty} h(n)\, z^{-n}$$

$$\frac{dH(z)}{dz} = \sum_{n=-\infty}^{+\infty} -n\, h(n)\, z^{-n-1}$$

$$-z\frac{dH(z)}{dz} = \sum_{n=-\infty}^{+\infty} n\, h(n)\, z^{-n}$$

$$B(z) = -z\frac{d}{dz}\frac{1}{1+\frac{1}{4}z^{-1}} \qquad |z| > \frac{1}{4}$$

$$= -z\,\frac{-\left(-\frac{1}{4}z^{-2}\right)}{\left(1+\frac{1}{4}z^{-1}\right)^2}$$

$$= -\frac{1}{4}\,\frac{z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)^2}$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \times \frac{-\frac{1}{4}z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)^2} \qquad |z| > \frac{1}{4}$$

$$a^n u[n] \xleftrightarrow{ZT} \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

$$-a^n u[-n-1] \xleftrightarrow{ZT} \frac{1}{1-az^{-1}} \qquad |z| < |a|$$

$$(n+1)\, a^n u[n] \xleftrightarrow{ZT} \frac{1}{(1-az^{-1})^2} \qquad |z| > |a|$$

$$-(n+1)\, a^n u[-n-1] \xleftrightarrow{ZT} \frac{1}{(1-az^{-1})^2} \qquad |z| < |a|$$

$$X(z) = \left(\frac{1}{1-az^{-1}}\right)^2 \qquad x[n] = \begin{cases} (n+1)\, a^n u[n] & |z| > |a| \\ -(n+1)\, a^n u[-n-1] & |z| < |a| \end{cases}$$

Problem 3
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7. (6%)

Find the z transforms of the following functions.

(1) $h[n] = 0.5^n u[n]$

$$H(z) = \frac{1}{1 - 0.5z^{-1}}, \quad |z| > 0.5$$

(2) $h[n] = (n+1)0.5^n u[n]$

$$H(z) = \frac{1}{(1 - 0.5z^{-1})^2}, \quad |z| > 0.5$$

Problem 4

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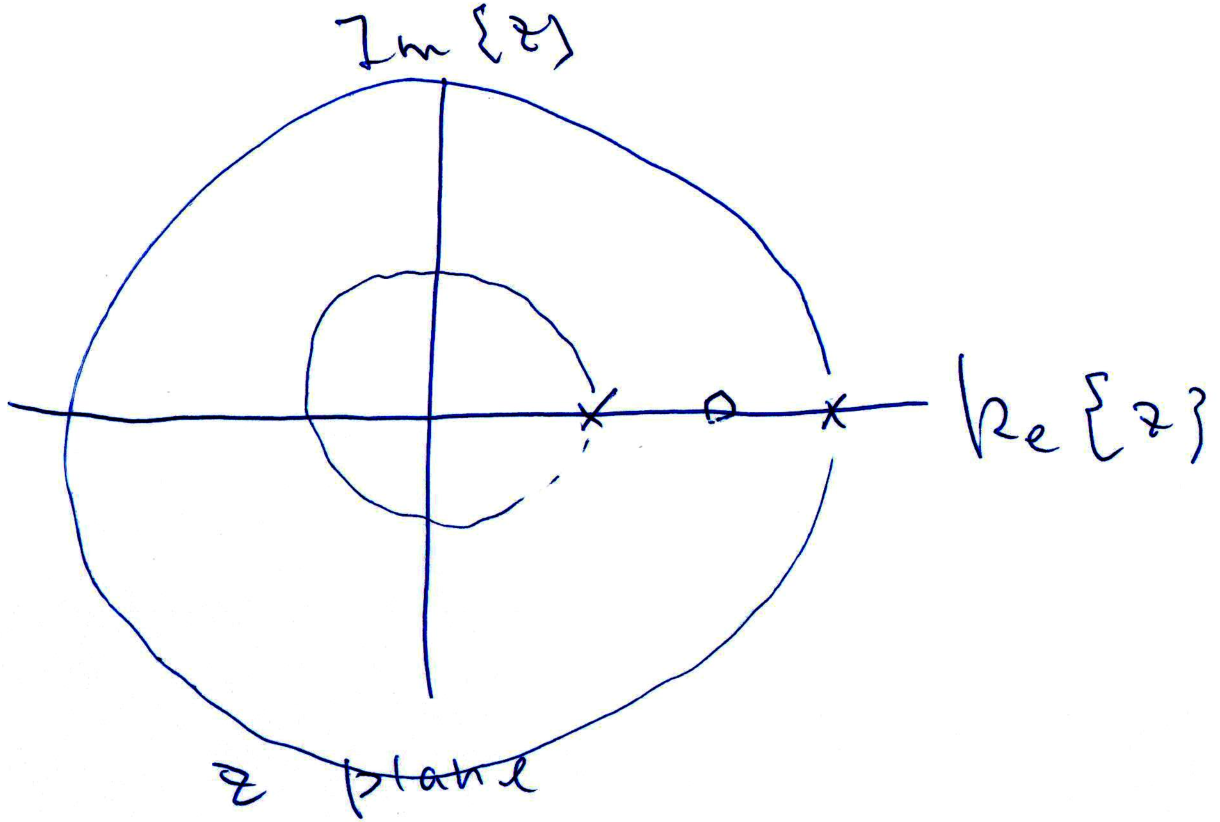
9. (6%)

Find the locations of poles and zeros and discuss the $\boxed{\text{causality}}$ and $\boxed{\text{stability}}$ of the following z-domain transfer function.

$$H(z) = \frac{2 - 2z^{-1}}{1 - 2z^{-1} + 0.75z^{-2}}$$

$$H(z) = \frac{2(1 - z^{-1})}{1 - 2z^{-1} + \frac{3}{4}z^{-2}}$$

$$H(z) = 2\,\frac{1 - z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{3}{2}z^{-1})}$$



$\lvert z\rvert < \frac{1}{2}$	anticausal	non-stable
$\frac{1}{2} < \lvert z\rvert < \frac{3}{2}$	non causal	BIBO stable
$\lvert z\rvert > \frac{3}{2}$	causal	non-stable

Problem 5
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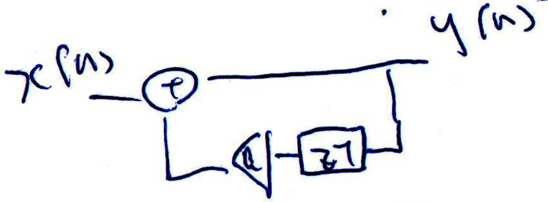
三、(15%)

Given an input $x[n]=\left(\frac{1}{3}\right)^n u[n]$, the output of a DT LTI system is $y[n]=\frac{1}{2}\left(\frac{1}{3}\right)^n u[n]+\frac{1}{4}\left(\frac{1}{6}\right)^n u[n]$

(一) (10%) Find the frequency response (5%) and impulse response (5%) of the system.

(二) (5%) Find the difference equation relating input and output.



$y(n)+a\,y(n-1)=x(n)$

$H(z)$

$h(n)$

$H(e^{j2\pi f})$

$y(n)=h(n)*x(n)$

$Y(z)=H(z)\times X(z)$

$H(z)=\frac{Y(z)}{X(z)}$

$$x(n) = \left(\tfrac{1}{3}\right)^n u(n)$$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} \qquad |z| > \tfrac{1}{3}$$

$$y(n) = \tfrac{1}{2}\left(\tfrac{1}{3}\right)^n u(n) + \tfrac{1}{4}\left(\tfrac{1}{9}\right)^n u(n)$$

$$Y(z) = \tfrac{1}{2}\,\frac{1}{1-\frac{1}{3}z^{-1}} + \tfrac{1}{4}\,\frac{1}{1-\frac{1}{9}z^{-1}} \qquad |z| > \tfrac{1}{3}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\tfrac{1}{2}\,\frac{1}{1-\frac{1}{3}z^{-1}} + \tfrac{1}{4}\,\frac{1}{1-\frac{1}{9}z^{-1}}}{\frac{1}{1-\frac{1}{3}z^{-1}}}$$

$$H(z) = \tfrac{1}{2} + \tfrac{1}{4}\,\frac{1-\frac{1}{3}z^{-1}}{1-\frac{1}{9}z^{-1}}$$

(15)

$$H(z) = \frac{1}{2} + \frac{1}{4}\,\frac{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{1}{6}\,\frac{z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{3}{4} - \frac{1}{6}\,\frac{z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$\boxed{h(n) = \frac{3}{4}\,\delta(n) - \frac{1}{6}\left(\frac{1}{6}\right)^{n-1} u(n-1)}$$

$$H(z) = \frac{1}{2} + \frac{1}{4}\,\frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{\frac{1}{2}\left(1 - \frac{1}{6}z^{-1}\right) + \frac{1}{4}\left(1 - \frac{1}{3}z^{-1}\right)}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{\frac{3}{4} - \frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$H(e^{j2\pi f}) = \frac{\frac{3}{4} - \frac{1}{6}e^{-j2\pi f}}{1 - \frac{1}{6}e^{-j2\pi f}}$$

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$$y[n] = h[n] * x[n]$$

$$Y(z) = H(z)\,X(z)$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\frac{3}{4} - \frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$Y(z) - \frac{1}{6}z^{-1}Y(z) = \frac{3}{4}X(z) - \frac{1}{6}z^{-1}X(z)$$

$$\boxed{y[n] - \frac{1}{6}y[n-1] = \frac{1}{4}x[n] - \frac{1}{6}x[n-1]}$$