Problem 1 HW 12 ①

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三、(10 points) Find the **Laplace transform** of $x(t) = \frac{d^2}{dt^2}(e^{-3(t-2)}u(t-2))$.

$$e^{-3t}u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s+3}, \quad Re\{s\} > -3$$

$$e^{-3(t-2)}u(t-2) \overset{LT}{\longleftrightarrow} e^{-2s}\frac{1}{s+3}, \quad Re\{s\} > -3$$

$$\frac{d^2}{dt^2}\left(e^{-3(t-2)}u(t-2)\right) \longleftrightarrow e^{-2s}\frac{s^2}{s+3}, \quad Re\{s\} > -3$$

Problem 2
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六、(10%) Consider a continuous-time LTI system for which whose the input $x(t)$ and output $y(t)$ are related by the differential equation

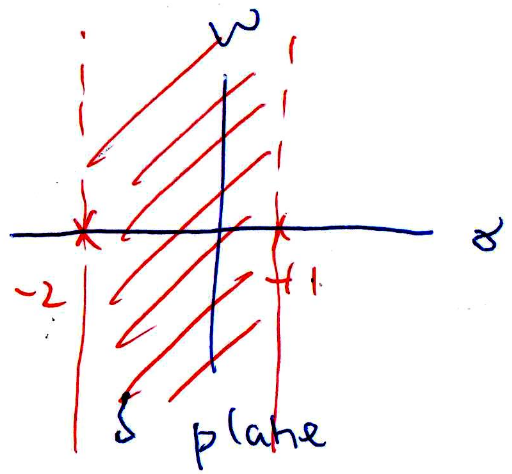
$$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}-2y(t)=x(t).$$

Suppose the system is stable. Determine $y(t)$ as $x(t)=\sum_{n=1}^{\infty}u(t-n)$, where $u(t)$ denotes the unit step function.

$$y''(t)+y'(t)-2y(t)=x(t)$$

$$s^2Y(s)+sY(s)-2Y(s)=X(s)$$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{1}{s^2+s-2}=\frac{1}{(s+2)(s-1)}$$



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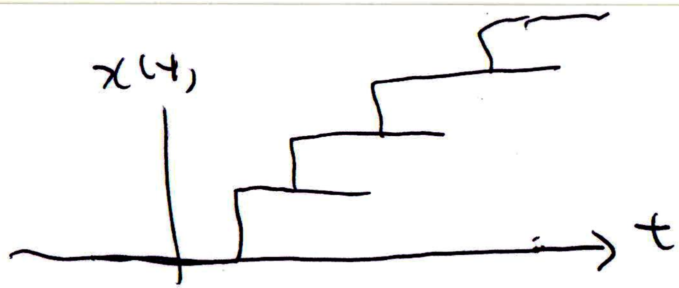

$$x(t) = \sum_{n=1}^{\infty} x_n(t)$$

$$x_n(t) = u(t-n)$$

$$X_n(s) = \frac{1}{s} e^{-ns}$$

ROC: $Re\{s\} > 0$

$$Y_n(s) = H(s) X_n(s)$$

$$= \frac{1}{(s+2)(s-1)} \cdot \frac{1}{s} e^{-ns}$$

(4)

$$Y_n(s) = \frac{1}{s(s+2)(s-1)} e^{-ns}$$

$$= \left(\frac{a}{s} + \frac{b}{s+2} + \frac{c}{s-1} \right) e^{-ns}$$

$$a = -\frac{1}{2} \quad b = +\frac{1}{6} \quad c = \frac{1}{3}$$

$$Y_n(s) = \left(-\frac{1}{2}\frac{1}{s} + \frac{1}{6}\frac{1}{s+2} + \frac{1}{3}\frac{1}{s-1} \right) e^{-ns}$$

$$y_n(t) = -\frac{1}{2} u(t_{-n}) + \frac{1}{6} e^{-2(t-n)} u(t_{-n}) + \frac{1}{3} e^{(t-n)} u(-(t-n))$$

$$y(t) = \sum_{n=1}^{\infty} y_n(t)$$

Problem 3

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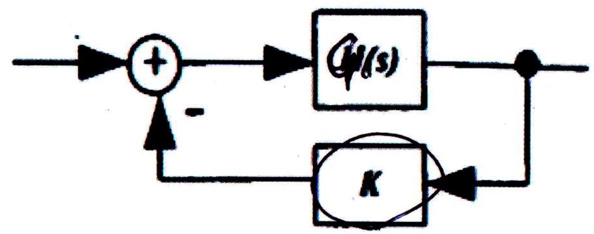
5. Consider a feedback system shown below, where $\hat{G}(s) = \dfrac{s+2}{s^2+2s+4}$

(a) Find the smallest positive value of K for which the closed-loop impulse response doesn't exhibit any oscillatory behavior (5%)

unstable



(5)

$$H(s) = \frac{\dfrac{s+2}{s^2+2s+4}}{1 + K\dfrac{s+2}{s^2+2s+4}}$$

$$= \frac{s+2}{s^2+2s+4+Ks+2K}$$

$$= \frac{s+2}{s^2+(K+2)s+(4+2K)}$$

(6)

$$s^2 + (k+2)s + (2k+4) = 0$$

$$s = \frac{-(k+2) \pm \sqrt{(k+2)(k-6)}}{2}$$

$$(k+2)^2 - 4 \times 2\,(k+2)$$

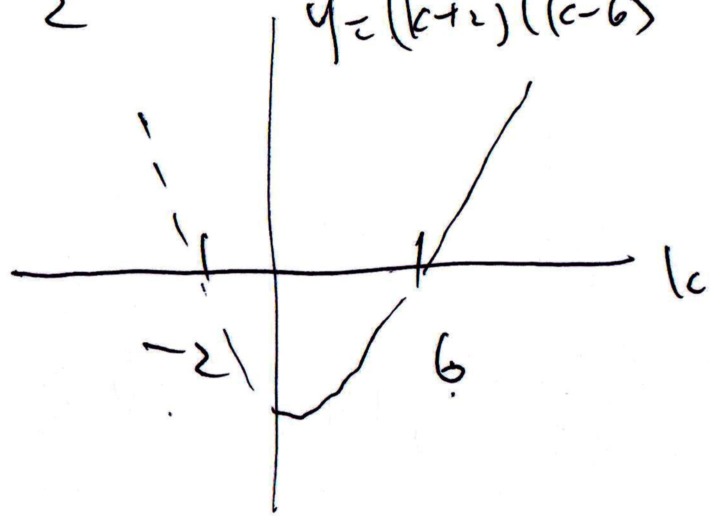
$$(k+2)[\,k-6\,]$$

$$k^2 + 4k + 4$$

$$-8k - 16$$

$$k^2 - 4k - 12$$

$$(k(k+2) - (k-6))$$



I. $k < -2$

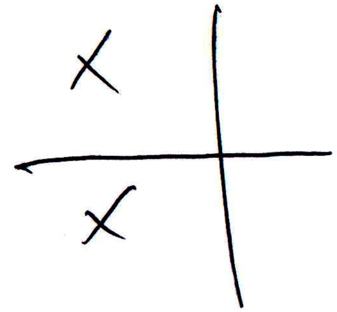
II. $\boxed{-2 < k < 6}$

III. $k > 6$

~~$s = \square + j\square$~~

II $-2 < k < 6$ Stable

$s = -\boxed{+} \pm j \boxed{\cdot}$

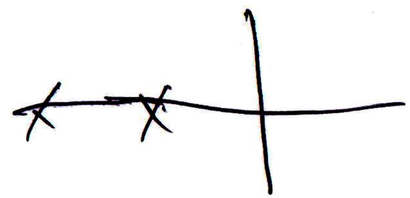


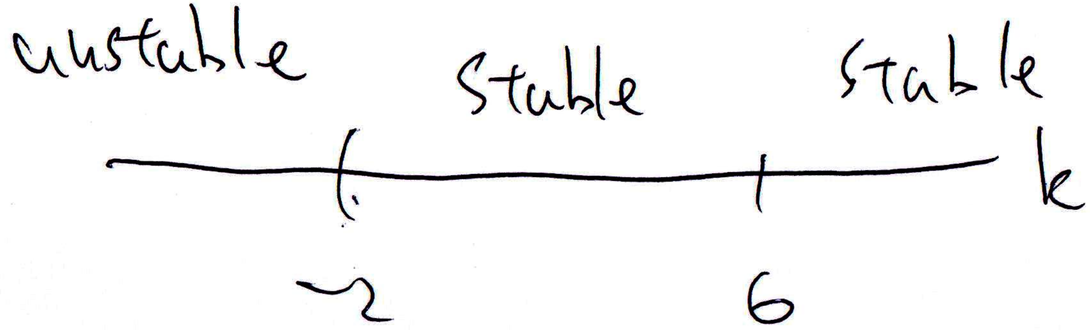
I $k < -2$ unstable

$k = -3$ $s = \dfrac{+1 \pm \sqrt{(-1)(-9)}}{2} = \dfrac{+1 \pm 3}{2}$

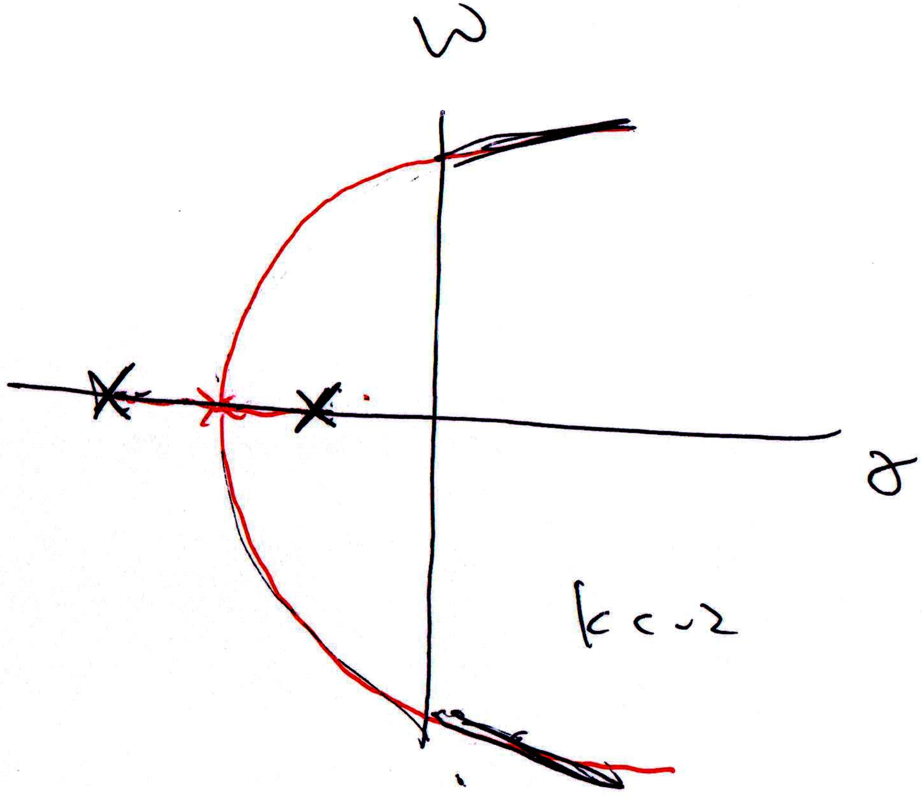
III $k > 6$

$s = 7$ $s = \dfrac{-9 \pm \sqrt{9 \times 1}}{2} = \dfrac{-9 \pm 3}{2}$




unstable
stable
stable
k
-2
6

Root Locus



S plane

Problem 4

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一、(5%) Please define a linear system in terms of mathematical expression.

二、(10%) Please define the properties of causality and stability for a Linear Time-Invariant (LTI) System in terms of mathematical expression.

三、(10%) Please define an eigenfunction for an LTI system with impulse response $h(t)$; and show its transfer function H(s) as the corresponding eigenvalue.

一、 $x \to \boxed{} \to y$ $\quad ax \to \boxed{} \to ay$

$x_1 \to \boxed{} \to y_1$ $\quad x_1+x_2 \to \boxed{} \to y_1+y_2$

$x_2 \to \boxed{} \to y_2$

二、 causal ROC 在最右邊 pole 的右邊

BIBO stability [illegible]

(11)

$$x \rightarrow \boxed{[A]} \rightarrow y$$

$$y = Ax = \lambda x$$

eigenvalue

$x =$ eigenvector

$$y = \lambda x$$

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) = h(t) * x(t) = H(s)\, x(t)$$

$$x(t) = e^{st}$$

$$y(t) = H(s)\, \underbrace{e^{st}}_{x(t)}$$

Problem 5

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b)

3. (10%) A second-order continuous-time linear dynamic system is characterized by the following equation:

$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = \alpha x,$$

where $x(t)$ denotes the input, $y(t)$ denotes the output, and let us assume that $\beta > 0$ and $\beta^2 < \omega_0^2$.

~~(a) (3%) Make a sketch of the impulse response of the system.~~

(b) (3%) Calculate the transfer function $H(j\omega) \triangleq \frac{Y(j\omega)}{X(j\omega)}$.

~~(c) (4%) Make a sketch of the magnitude response of $H(j\omega)$. If $\beta^2 \ll \omega_0^2$, what is the approximate frequency at which the magnitude response reaches its maximum?~~

$$(s^2 + 2\beta s + \omega_0^2)\,Y(s) = \alpha\,X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\alpha}{s^2 + 2\beta s + \omega_0^2}$$

$$H(j\omega) = \frac{\alpha}{(\omega_0^2 - \omega^2) + j2\beta\omega}$$

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Problem 6

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CT LTI

四.(一)(10%) Consider a continuous-time linear time-invariant system with impulse response $h(t)=e^{-t}u(t)$. Determine the output $y(t)$ of the system when the input is $x(t)=u(t+1)-u(t-1)$.

$$y(t)=\left(1-e^{-(t+1)}\right)u(t+1)-\left(1-e^{-(t-1)}\right)u(t-1)$$

$$y(t)=h(t)*x(t)$$

$$Y(s)=H(s)\times X(s)$$

$$=\frac{1}{s+1}\left[\frac{1}{s}e^{+s}-\frac{1}{s}e^{-s}\right]$$

$$=\frac{1}{s(s+1)}e^{+s}-\frac{1}{s(s+1)}e^{-s}$$

$$=\left(\frac{1}{s}e^{+s}-\frac{1}{s+1}e^{+s}\right)-\left(\frac{1}{s}e^{-s}-\frac{1}{s+1}e^{-s}\right)$$

$$y(t)=\left(u(t+1)-e^{-(t+1)}u(t+1)\right)-\left(u(t-1)-e^{-(t-1)}u(t-1)\right)$$

Problem 7

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六. Given a linear time-invariant (LTI) system with system function

$$H(s) = \frac{s-1}{(s+1)(s-2)},$$ please determine the impulse response $h(t)$ and show its corresponding region of convergence (ROC) if

(一)(10%) the system is known to be causal; 最右邊 pole 的右邊

(二)(10%) the system is known to be stable. 包含虛軸

$$H(s) = \frac{s-1}{(s+1)(s-2)} = \frac{a}{s+1} + \frac{b}{s-2}$$

$$a = -\frac{2}{3} \qquad b = \frac{1}{3}$$

$$H(s) = -\frac{2}{3}\,\frac{1}{s+1} + \frac{1}{3}\,\frac{1}{s-2}$$

$\cancel{h(t) =}$

(16)

$$H(s) = -\frac{2}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}$$

anti-causal	non causal	causal
unstable	stable	unstable
-1		2
III	II	I
$h(t) = +\frac{2}{3}e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$	$h(t) = -\frac{2}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(-t)$	$h(t) = -\frac{2}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$

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Problem 8
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causal

Bilateral

14. Determine the continuous-time signal corresponding to the following ~~unilateral~~ Laplace transform,

$$X(s) = s\frac{d^2}{ds^2}\left(\frac{1}{s^2+25}\right).$$

(10%)

$\sin 5t\, u(t) \xleftrightarrow{CT} \frac{5}{s^2+25}$ $Re\{s\} > 0$

$\frac{1}{5}\sin 5t\, u(t) \xleftrightarrow{CT} \frac{1}{s^2+25}$

$\boxed{\frac{1}{5}t^2 \sin 5t\, u(t)} \xleftrightarrow{CT} \frac{d^2}{ds^2}\frac{1}{s^2+25}$

$\longleftrightarrow s\frac{d^2}{ds^2}\frac{1}{s^2+25}$

$(fgh)' = f'gh + fg'h + fgh'$

$\frac{1}{5}2t\sin 5t * u(t)$

$\frac{1}{5}t^2\, 5\cos 5t * u(t)$

$\frac{1}{5}t^2 \sin 5t\, \delta(t)$

Problem 9

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11. Consider a linear time-invariant system with impulse response $h(t) = e^{-t}u(t+1)$. Determine the output $y(t)$ of the system when the input is $x(t) = \sin^2 t$.
(10%)

$y(t) = h(t) * x(t)$

$e^{-t}u(t) \longleftrightarrow \frac{1}{s+1}$

$h(t) = e^{+1} e^{-(t+1)} u(t+1)$

$H(s) = e^{+1} \frac{1}{s+1} e^{+s}$

$H(s) = \frac{1}{s+1} e^{s+1}$

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$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$x(t) = \sin^2 t$$

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos 2t$$

$$x(t) = \frac{1}{2} - \frac{1}{2}\,\frac{e^{j2t} + e^{-j2t}}{2}$$

$$x(t) = \frac{1}{2} e^{0} - \frac{1}{4} e^{j2t} - \frac{1}{4} e^{-j2t}$$

$$\frac{1}{2} e^{0} \rightarrow \boxed{} \rightarrow \frac{1}{2} H(0)\, e^{0}$$

$$-\frac{1}{4} e^{j2t} \rightarrow \boxed{} \rightarrow -\frac{1}{4} H(j2)\, e^{j2t}$$

$$-\frac{1}{4} e^{-j2t} \rightarrow \boxed{} \rightarrow -\frac{1}{4} H(-j2)\, e^{-j2t}$$

$$y(t) = \frac{1}{2} H(0)\, e^{0} - \frac{1}{4} H(j2)\, e^{+j2t} - \frac{1}{4} H(-j2)\, e^{-j2t}$$

2

$$\boxed{y(t) = \frac{1}{2} e^{1} - \frac{1}{4} \frac{1}{2j+1} e^{2j+1} \cdot e^{+j2t} - \frac{1}{4} \frac{1}{-2j+1} e^{-2j+1}\, e^{-j2t}}$$

$$H(s) = \frac{1}{s+1} e^{s+1}$$

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Problem 10

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10. The bilateral Laplace transform of a continuous-time signal $x(t)$ is specified by,

$$H(s) = \frac{s+4}{(s+2)(s^2+6s+13)} \quad \text{with ROC} : -3 < \text{Re}(s) < -2$$

Which of following answers is (are) correct?

(A) $x(t) = \frac{2}{5}e^{-2t}u(t) - \frac{2}{5}e^{-3t}\cos(2t)u(t) + \frac{3}{10}e^{-3t}\sin(2t)u(t)$

(B) $x(t) = \boxed{-\frac{2}{5}e^{-2t}u(-t)} + \frac{2}{5}e^{-3t}\cos(2t)u(t) + \frac{3}{10}e^{-3t}\sin(2t)u(t)$

(C) $x(t) = \boxed{-\frac{2}{5}e^{-2t}u(-t)} + \frac{2}{5}e^{-3t}\cos(2t)u(t) - \frac{3}{10}e^{-3t}\sin(2t)u(t)$

(D) $x(t) = \frac{2}{5}e^{-2t}u(t) + \frac{2}{5}e^{-3t}\cos(2t)u(-t) - \frac{3}{10}e^{-3t}\sin(2t)u(-t)$

(6%)

沒有答案

A	B
C	D

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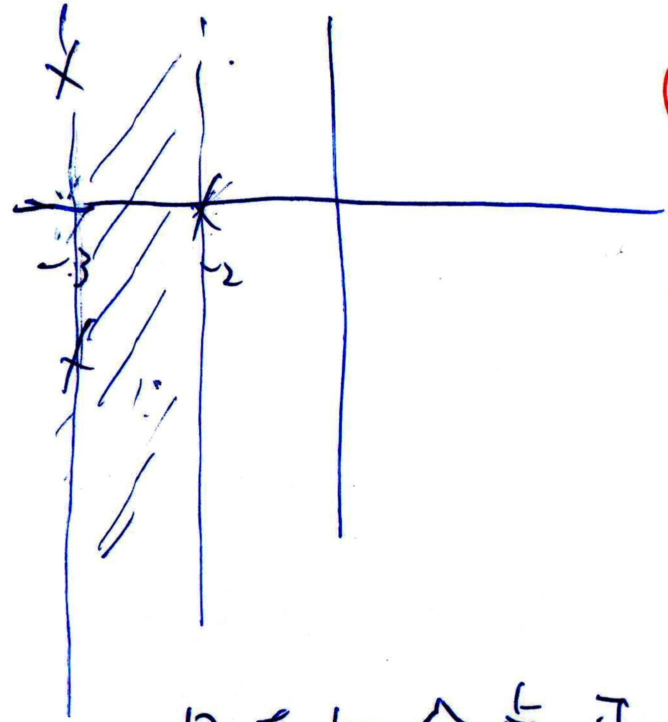
$$H(s) = \frac{s+4}{(s+2)(s^2+6s+13)}$$

$s=-2$

$$s = \frac{-6 \pm \sqrt{36-52}}{2}$$

$$= -3 \pm j2$$



$$H(s) = \frac{a}{s+2} + \frac{bs+c}{s^2+6s+13}$$

沒有包含虛軸 unstable

non causal

$$h(t) = \underline{-e^{-2t}u(-t)} + \alpha e^{-(3+2j)}u(t) + \beta e^{-(3-2j)}u(t)$$

$$\frac{s+4}{(s+2)(s^2+6s+13)} = \frac{a}{s+2} + \frac{bs+c}{s^2+6s+13}$$

$$a = \frac{2}{4-12+13} = \frac{2}{5}$$

$$\frac{s+4}{s^2+6s+13}\ {}^{2} = a + \frac{\quad}{\quad}\ (s+2)$$

$4-12+13$