HW 11

Problem 1

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1. (5%)

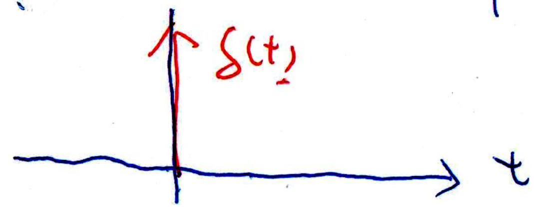
The output ($y(t)$) of a continuous-time system is related to its input ($x(t)$) as $\boxed{y(t) = e^{-t}x(t-2),\ t>0}$. Determine whether the system is (a) stable (2%), (b) causal (1%), (c) linear (1%), and (d) time invariant (1%).

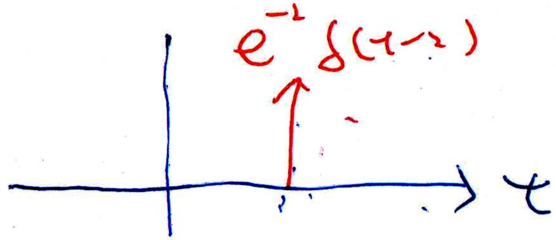
yes. yes yes yes

$$y(t) = e^{-t} x(t-2)$$

$$h(t) = e^{-t}\delta(t-2)$$

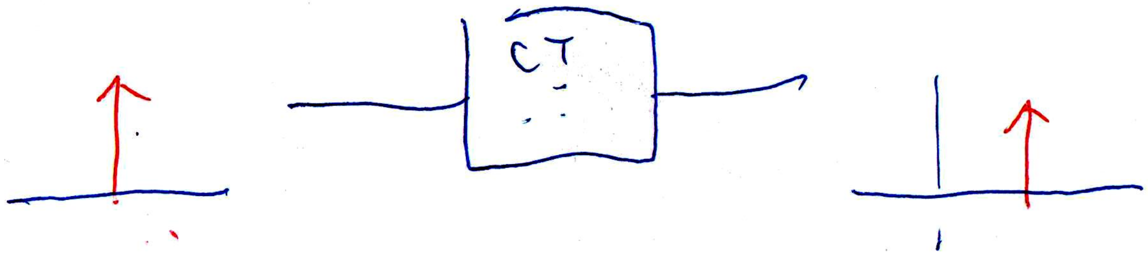
$$h(t) = e^{-2}\delta(t-2)$$





$$\int_{-\infty}^{+\infty} |e^{-2}\delta(t-2)|\,dt$$

$$= e^{-2}\cdot 1$$



Problem 2
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6. (6%)

Find the Laplace transforms of the following functions.

(1) $h(t) = e^{-t}u(t)$

(2) $h(t) = te^{-t}u(t)$

(1) $h(t) = e^{-t}u(t)$

$$H(s) = \frac{1}{s+1}, \quad Re\{s\} > -1$$

(2) $h(t) = t \cdot e^{-t}u(t)$

$$H(s) = \frac{1}{(s+1)^2} \quad Re\{s\} > -1$$

Problem 3

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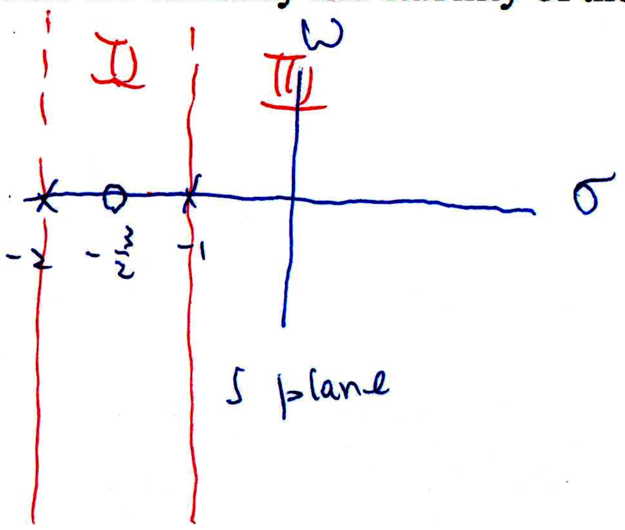
8. **(6%)**

Find the locations of poles and zeros and discuss the causality and stability of the following s-domain transfer function.

$$H(s) = \frac{2s+3}{s^2+3s+2}$$

$$H(s) = \frac{2(s+\frac{3}{2})}{(s+1)(s+2)}$$



	I	II	III
causality	anti-causal	non-causal	causal
BIBO stability	not BIBO stability	not BIBO stability	BIBO stability

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Problem 4

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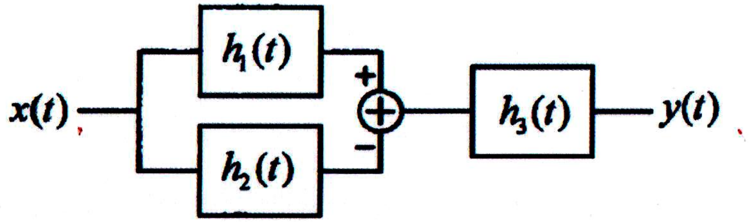
10. (8%)

Find the impulse response (i.e., h(t)) and the transfer function (i.e., H(s)) of the following CT LTI system. The input signal is x(t) and the output signal is y(t).



$h_1(t) = \delta(t),\ h_2(t) = e^{-t}u(t), and\ h_3(t) = e^{-t}u(t).$

$$h(t) = (h_1(t) + h_2(t)) * h_3(t)$$

$$H(s) = (H_1(s) + H_2(s)) \times H_3(s)$$

$h_1(t) = \delta(t)$

$H_1(s) = 1$

$h_2(t) = e^{-t}u(t)$

$H_2(s) = \frac{1}{s+1}$

$Re\{s\} > -1$

$h_3(t) = e^{-t}u(t)$

$H_3(s) = \frac{1}{s+1}$

$Re\{s\} > -1$

(5)

$$H(s) = \left(1 + \frac{1}{s+1}\right)\frac{1}{s+1}$$

$$\boxed{H(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2}}$$

$$H(s) = \frac{s+2}{(s+1)^2} \qquad \mathrm{Re}\{s\} > -1$$

$$h(t) = e^{-t}u(t) + t\,e^{-t}u(t)$$

Problem 5

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五、(15%)

(一) (5%) Determine the Fourier representation of the following signal

$$x(t) = 2e^{-t}u(t) - 3e^{-2t}u(t)$$

(二) (10%) Find the time-domain signals corresponding to the following Fourier transform representations:

$$\cancel{(5\%) \quad X(e^{j\Omega}) = \frac{1}{1-\alpha e^{-j(\Omega+\frac{\pi}{4})}}, \ |\alpha| < 1}$$

(5%) $$Y(jw) = \frac{1}{2+j(w-3)} + \frac{1}{2+j(w+3)}$$

(一) $h(t) = 2e^{-t}u(t) - 3e^{-2t}u(t)$

$H(s) = 2\frac{1}{s+1} - 3\frac{1}{s+2}$

$H(s) = \frac{2s+4-3s-3}{(s+1)(s+2)}$

$H(s) = \frac{-s+1}{s^2+3s+2}$

$e^{-at}u(t) \overset{CTFT}{\longleftrightarrow} \frac{1}{a+j2\pi f}$

$$\boxed{e^{-at}u(t) \overset{CT}{\longleftrightarrow} \frac{1}{s+a} \quad Re\{s\} > Re\{-a\}}$$

$$H(j2\pi f) = \frac{-j2\pi f + 1}{(2 - 4\pi^2 f^2) + j6\pi f}$$ (7)

$$\Rightarrow \quad Y(j2\pi f) = \frac{1}{2 + j2\pi f - 3j} + \frac{1}{2 + j2\pi f + 3j}$$

$$= \frac{1}{(2-3j) + j2\pi f} + \frac{1}{(2+3j) + j2\pi f}$$

$$y(t) = e^{-(2-3j)t} u(t) + e^{-(2+3j)t} u(t)$$

$$y(t) = e^{-2t} \left(e^{+3jt} + e^{-3jt} \right) u(t)$$

$$y(t) = 2 e^{-2t} \cos 3t \; u(t)$$

Problem 6
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七、(10%)

A causal LTI system has an impulse response $h(t)$ that satisfies the differential equation

CT

$$\boxed{\frac{dh(t)}{dt}+3h(t)=e^{-4t}u(t)+ce^{-5t}u(t),}$$

where c is a constant. Moreover, the system output is $(2/15)e^{t}$ when the input to the system is e^{t}.

(一) (3%) Determine the constant c.

(二) (3%) If the transfer function of this system is $H(s)$, find its poles.

(三) (4%) Specify the region of convergence of $H(s)$ and tell whether or not the system is stable.

ROC

$$e^{st} \rightarrow \boxed{H(s)} \rightarrow H(s)e^{st}$$

$$e^{1t} \qquad H(1)e^{1t}$$

$$sH(s)+3H(s)=\frac{1}{s+4}+c\frac{1}{s+5}$$

$$H(s)=\frac{1}{s+3}\left(\frac{1}{s+4}+c\frac{1}{s+5}\right)$$

$$\frac{2}{15}e^{t}=H(1)e^{1t}$$

$$H(1)=\frac{1}{4}\times\left(\frac{1}{5}+\frac{c}{6}\right)$$

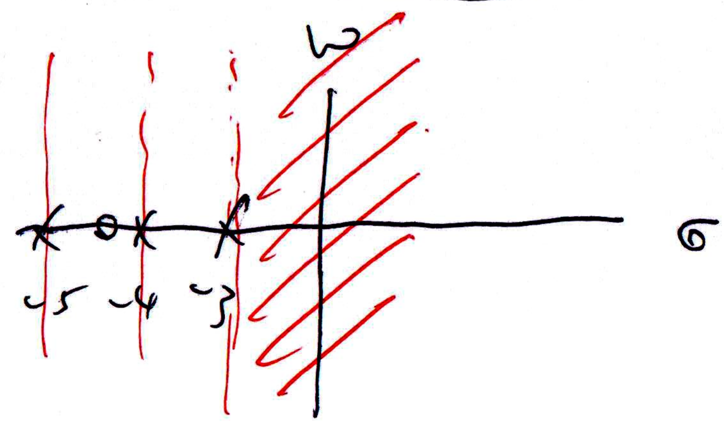
$$\frac{2}{15} \qquad \frac{8}{15}=\frac{6+5c}{30}=\frac{16}{30} \qquad \boxed{c=2}$$

9

$$H(s) = \frac{1}{s+3}\left(\frac{1}{s+4} + \frac{2}{s+5}\right)$$

$$H(s) = \frac{(s+5) + 2(s+4)}{(s+3)(s+4)(s+5)}$$

$$\boxed{H(s) = \frac{3s+13}{(s+3)(s+4)(s+5)}}$$



s plane

The system is stable because the ROC contains Imaginary axis.

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Problem 7

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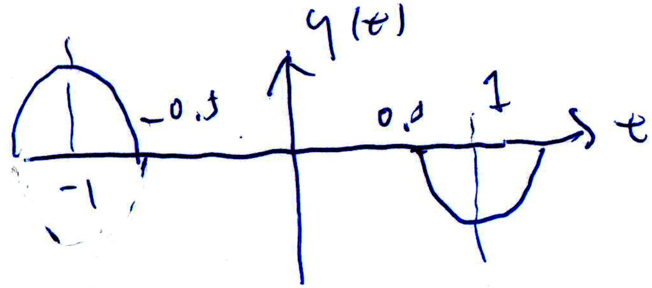
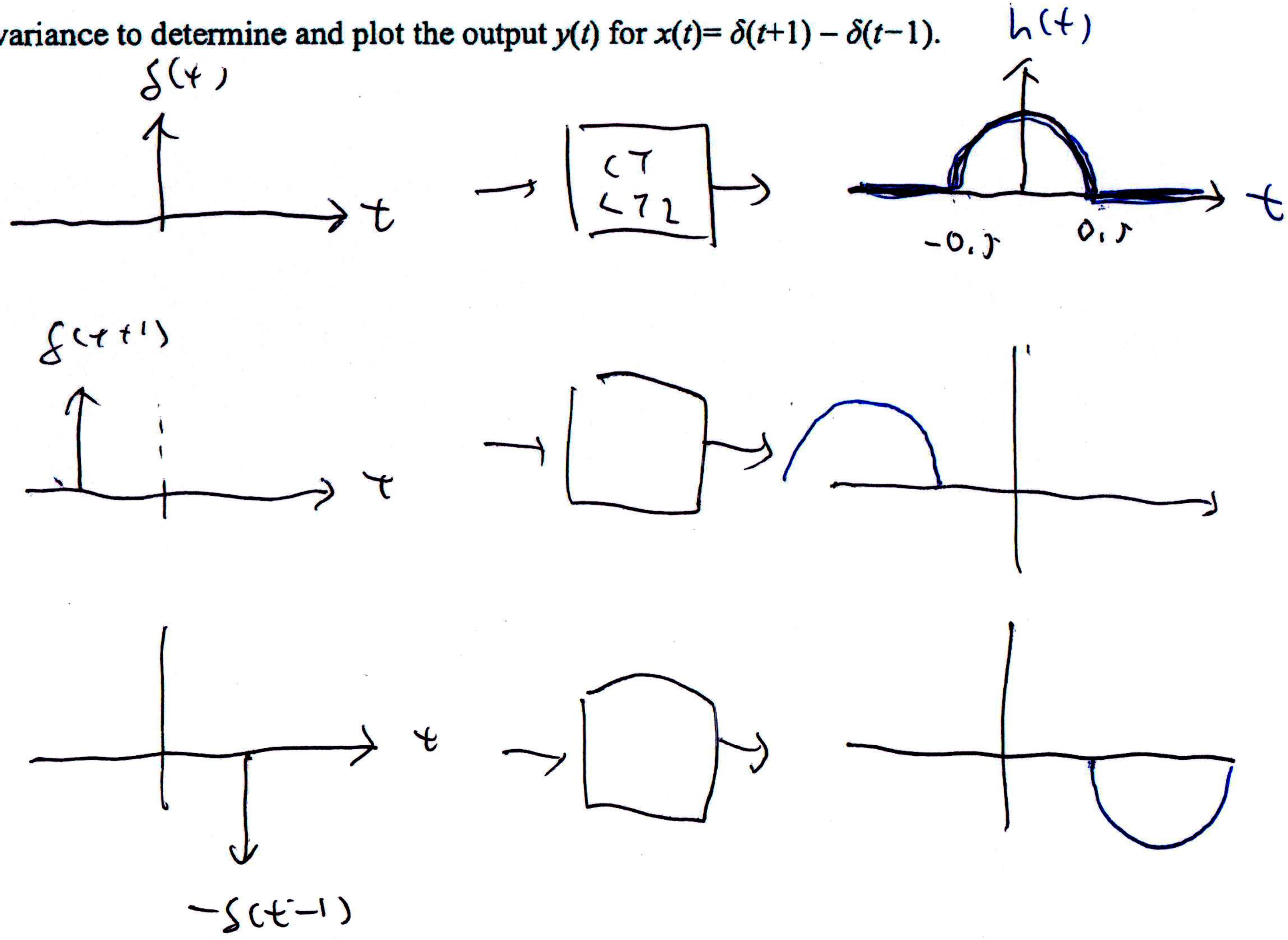
二、(10%)



(a) (5%) The impulse response of an LTI system is $h(t) = \begin{cases} \cos(\pi t), |t| < 0.5 \\ 0, \quad \text{otherwise} \end{cases}$. Use linearity and time invariance to determine and plot the output $y(t)$ for $x(t) = \delta(t+1) - \delta(t-1)$.



Problem 8

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二、(10%)

(a) (5%) Consider an LTI system with input and output related through the equation:

$$y(t) = \int_{-\infty}^{t} e^{\tau-t} x(\tau-1)\,d\tau$$

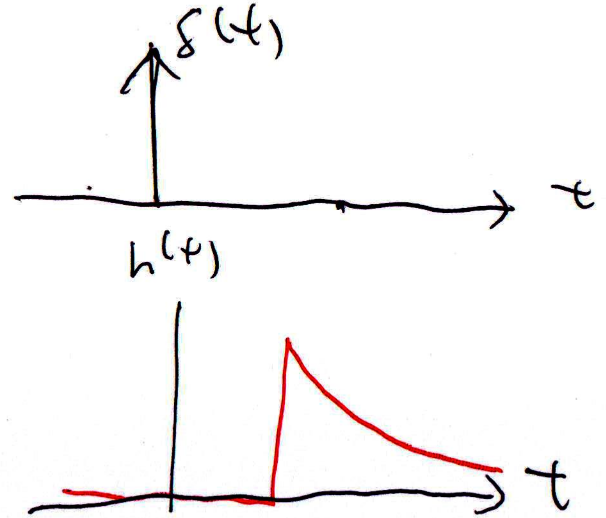
The impulse response $h(t)$ for this system = ________. Is the system causal? __yes__ (simply answer yes or no)

$$h(t) = \int_{-\infty}^{t} e^{\tau-t}\,\delta(\tau-1)\,d\tau$$

$$= \int_{-\infty}^{t} e^{1-t}\,\delta(\tau-1)\,d\tau$$

$$= e^{-(t-1)} \boxed{\int_{-\infty}^{t} \delta(\tau-1)\,d\tau} \rightarrow \begin{cases} 0, & t<1 \\ 1, & t>1 \end{cases} \rightarrow u(t-1)$$

$$= e^{-(t-1)}\,u(t-1)$$



δ(t)
t
h(t)
t

Problem 9

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七、(15%) Consider the continuous-time LTI system with input $x(t)$, output $y(t)$ and impulse response $h(t)$, for which we are given the following information:

$x(t) = 0,\ t > 0$ and $X(s) = (s+2)/(s-2)$, and $y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$.

(a) (10%) Determine the transfer function, $H(s)$, of the system (3%), its region of convergence (2%), and the impulse response $h(t)$ of the system (5%).

(b) (5%) What is the output $y(t)$ if the input to the LTI system is $x(t) = e^{-3t}$, $-\infty < t < \infty$?

$$H(s) = \frac{Y(s)}{X(s)}$$

$$y(t) = -\frac{2}{3}e^{-(-2)t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

$$Y(s) = -\frac{2}{3}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}$$

$Re\{s\} < 2$ $\qquad$ $Re(s) > -1$

$$X(s) = \frac{s+2}{s-2} = \frac{s-2+4}{s-2} = 1 + 4\frac{1}{s+(-2)}$$

$$x(s) = \delta(t) + 4(-1)e^{2t}u(-t)$$

$Re\{s\} < 2$

(14)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{2}{3}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}}{\frac{s+2}{s-2}}$$

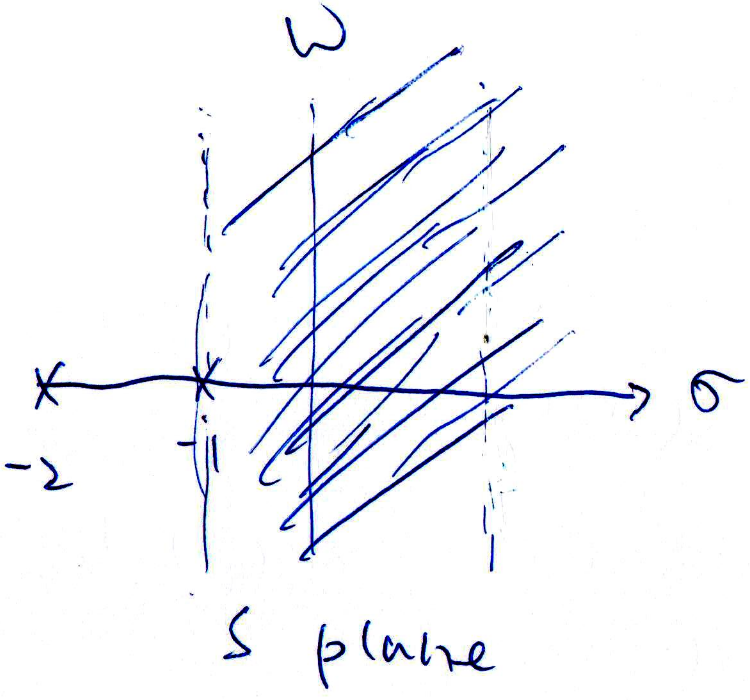
$$= \frac{\cancel{s-2}}{s+2} \; \frac{\frac{2}{3}(s+1) + \frac{1}{3}(s-2)}{(\cancel{s-2})(s+1)}$$

$$H(s) = \frac{s}{(s+2)(s+1)} = \frac{a}{s+2} + \frac{b}{s+1}$$

$$H(s) = \frac{2(s+1) - (s+2)}{(s+2)(s+1)}$$

$$H(s) = \frac{2}{s+2} - \frac{1}{s+1}$$

$$h(t) = 2e^{-2t}u(t) - e^{-t}u(t)$$



$$x(t) = e^{+st}$$

$$y(t) = H(s)\, e^{+st}$$

$$x(t) = e^{-3t}$$

$$y(t) = H(-3)\, e^{-3t}$$

$$\downarrow$$

$$-\frac{3}{2}$$

$$y(t) = -\frac{3}{2}\, e^{-3t}$$

$$H(s) = \frac{s}{(s+1)(s+2)}$$

$$H(-3) = \frac{(-3)}{(-2)\times(-1)}$$

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Problem 10

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五、(5 pts) A continuous-time ~~linear system~~ S with input $x(t)$ and output $y(t)$ yields the following input-output pairs:

$$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$$
$$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$$

(一)(2 pts) Is this system linear time-invariant? Just simply answer yes or no. Not linear system

$$e^{j2t} \to \boxed{} \to e^{j3t}$$