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Problem 1

(一)

$$\begin{cases}\left(1+\frac{-5}{4}z^{-1}\right)Y+\left(1+\frac{1}{4}z^{-1}\right)W=\left(\frac{1}{10}\right)X\\ \left(1+\frac{-3}{2}z^{-1}\right)Y+\left(2\right)W=\left(\frac{-2}{5}\right)X\end{cases}$$

$$W=\left(\frac{-1}{2}+\frac{3}{4}z^{-1}\right)Y+\left(\frac{-1}{5}\right)X$$

$$\left(\frac{1}{10}\right)X=\left(1+\frac{-5}{4}z^{-1}\right)Y+\left(1+\frac{1}{4}z^{-1}\right)\left[\left(\frac{-1}{2}+\frac{3}{4}z^{-1}\right)Y+\left(\frac{-1}{5}\right)X\right]$$

$$\left(\frac{1}{10}\right)X=\left(1+\frac{-5}{4}z^{-1}\right)Y+\left(\frac{-1}{2}+\frac{5}{8}z^{-1}+\frac{3}{16}z^{-2}\right)Y+\left(\frac{-1}{5}+\frac{-1}{20}z^{-1}\right)X$$

$$\left(\frac{1}{2}+\frac{-5}{8}z^{-1}+\frac{3}{16}z^{-2}\right)Y=\left(\frac{3}{10}+\frac{1}{20}z^{-1}\right)X$$

$$H=\frac{Y}{X}=\frac{\frac{3}{10}+\frac{1}{20}z^{-1}}{\frac{1}{2}+\frac{-5}{8}z^{-1}+\frac{3}{16}z^{-2}}$$

$$H(e^{j2\pi f})=\frac{\frac{3}{10}+\frac{1}{20}e^{-j2\pi f}}{\frac{1}{2}+\frac{-5}{8}e^{-j2\pi f}+\frac{3}{16}e^{-j4\pi f}}$$

Problem 1 (continued)

(二)

$$H(z) = \frac{\frac{3}{10} + \frac{1}{20} z^{-1}}{\frac{1}{2} + \frac{-5}{8} z^{-1} + \frac{3}{16} z^{-2}}$$

$$= \frac{\frac{3}{5} + \frac{1}{10} z^{-1}}{1 + \frac{-5}{4} z^{-1} + \frac{3}{8} z^{-2}}$$

$$= \frac{\frac{3}{5}\left(1 + \frac{1}{6} z^{-1}\right)}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{3}{4} z^{-1}\right)}$$

$$= \frac{a}{1 - \frac{1}{2} z^{-1}} + \frac{b}{1 - \frac{3}{4} z^{-1}}$$

$$a = \left.\frac{\frac{3}{5}\left(1 + \frac{1}{6} z^{-1}\right)}{1 - \frac{3}{4} z^{-1}}\right|_{z^{-1} = 2} = \frac{\frac{3}{5}\left(1 + \frac{1}{6} \cdot 2\right)}{1 - \frac{3}{4} \cdot 2} = \frac{\frac{3}{5}\left(1 + \frac{1}{3}\right)}{1 - \frac{3}{2}}$$

$$b = \left.\frac{\frac{3}{5}\left(1 + \frac{1}{6} z^{-1}\right)}{1 - \frac{1}{2} z^{-1}}\right|_{z^{-1} = \frac{4}{3}} = \frac{\frac{3}{5}\left(1 + \frac{1}{6} \cdot \frac{4}{3}\right)}{1 - \frac{1}{2} \cdot \frac{4}{3}} = \frac{\frac{3}{5}\left(1 + \frac{2}{9}\right)}{1 - \frac{2}{3}}$$

$$H(z) = \frac{\frac{3}{5} \cdot \frac{4}{3}}{\frac{-1}{2}} \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{\frac{3}{5} \cdot \frac{11}{9}}{\frac{1}{3}} \frac{1}{1 - \frac{3}{4} z^{-1}}, \quad ROC: \frac{3}{4} < |z|$$

$$h[n] = \frac{-8}{5} \left(\frac{1}{2}\right)^n u[n] + \frac{11}{5} \left(\frac{3}{4}\right)^n u[n]$$

Problem 1 (continued)

(三)

$$\frac{Y}{X} = H = \frac{\frac{3}{5} + \frac{1}{10} z^{-1}}{1 + \frac{-5}{4} z^{-1} + \frac{3}{8} z^{-2}}$$

$$Y + \frac{-5}{4} z^{-1} Y + \frac{3}{8} z^{-2} Y = \frac{3}{5} X + \frac{1}{10} z^{-1} X$$

$$y[n] + \frac{-5}{4} y[n-1] + \frac{3}{8} y[n-2] = \frac{3}{5} x[n] + \frac{1}{10} x[n-1]$$

Problem 2

(一)

$$y[n]+\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]+4x[n-1]-2x[n-2]$$

$$Y+\frac{3}{4}z^{-1}Y+\frac{1}{8}z^{-2}Y=X+4z^{-1}X-2z^{-2}X$$

$$H(z)=\frac{Y}{X}=\frac{1+4z^{-1}-2z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}=\frac{1+4z^{-1}-2z^{-2}}{\left(1+\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$$

$$=\frac{A}{1+\frac{1}{4}z^{-1}}+\frac{B}{1+\frac{1}{2}z^{-1}}+C$$

$$A=\left.\frac{1+4z^{-1}-2z^{-2}}{1+\frac{1}{2}z^{-1}}\right|_{z^{-1}=-4}=\frac{1+4(-4)-2(-4)^2}{1+\frac{1}{2}(-4)}=\frac{1-16-32}{1-2}=\frac{-47}{-1}$$

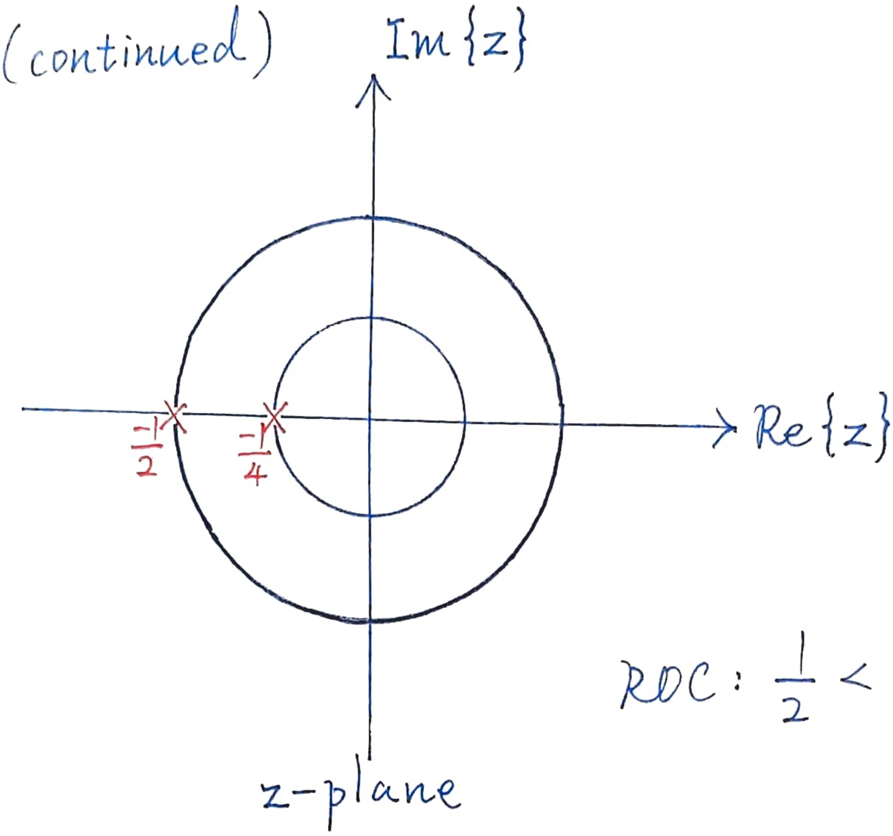
$$B=\left.\frac{1+4z^{-1}-2z^{-2}}{1+\frac{1}{4}z^{-1}}\right|_{z^{-1}=-2}=\frac{1+4(-2)-2(-2)^2}{1+\frac{1}{4}(-2)}=\frac{1-8-8}{1-\frac{1}{2}}=\frac{-15}{\frac{1}{2}}$$

$$C=H(z)\Big|_{z^{-1}=0}-A-B=\frac{1+4\cdot 0-2\cdot 0^2}{1+\frac{3}{4}\cdot 0+\frac{1}{8}\cdot 0^2}-47+30=1-47+30$$

$$H(z)=47\frac{1}{1-\frac{-1}{4}z^{-1}}-30\frac{1}{1-\frac{-1}{2}z^{-1}}-16$$

Problem 2 (continued)

(二)



ROC: $\frac{1}{2} < |z|$

$$H(z) = -16 + 47 \frac{1}{1 - \frac{-1}{4} z^{-1}} - 30 \frac{1}{1 - \frac{-1}{2} z^{-1}}$$

$$h[n] = -16\,\delta[n] + 47 \left(\frac{-1}{4}\right)^n u[n] - 30 \left(\frac{-1}{2}\right)^n u[n]$$

(三)

∵ ROC 包含單位圓

∴ The system is stable

Problem 3

(a)

$$y[n]+\frac{k}{3}y[n-1]=x[n]-\frac{k}{4}x[n-1]$$

$$Y+\frac{k}{3}z^{-1}Y = X+\frac{-k}{4}z^{-1}X$$

$$H(z)=\frac{Y}{X}=\frac{1-\frac{k}{4}z^{-1}}{1-\frac{-k}{3}z^{-1}}$$

pole: $\frac{-k}{3}$

ROC: $\left|\frac{-k}{3}\right|<|z|$

(b)

ROC 包含單位圓: $\left|\frac{-k}{3}\right|<1$

$$|k|<3$$

(c)

$$y[n]=c\cdot\left(\frac{2}{3}\right)^n$$

$$c\left(\frac{2}{3}\right)^n+\frac{1}{3}c\left(\frac{2}{3}\right)^{n-1}=\left(\frac{2}{3}\right)^n-\frac{1}{4}\left(\frac{2}{3}\right)^{n-1}$$

$$c\left(\frac{2}{3}\right)^n+\frac{1}{3}c\frac{3}{2}\left(\frac{2}{3}\right)^n=\left(\frac{2}{3}\right)^n-\frac{1}{4}\frac{3}{2}\left(\frac{2}{3}\right)^n$$

$$c+\frac{1}{2}c=1-\frac{3}{8}$$

$$c=\frac{5}{12}$$

$$y[n]=\frac{5}{12}\left(\frac{2}{3}\right)^n$$

Problem 4

(a)

$$\begin{cases} Y = W + \frac{-7}{4} z^{-1} W + \frac{-1}{2} z^{-2} W \\ W = X + \frac{-1}{4} z^{-1} W + \frac{1}{8} z^{-2} W \end{cases}$$

$$\begin{cases} Y = \left(1 - \frac{7}{4} z^{-1} - \frac{1}{2} z^{-2}\right) W \\ X = \left(1 + \frac{1}{4} z^{-1} - \frac{1}{8} z^{-2}\right) W \end{cases}$$

$$H(z) = \frac{Y}{X} = \frac{1 - \frac{7}{4} z^{-1} - \frac{1}{2} z^{-2}}{1 + \frac{1}{4} z^{-1} - \frac{1}{8} z^{-2}}$$

Problem 4 (continued)

(b)

$$Y(z) + \frac{1}{4} z^{-1} Y(z) + \frac{-1}{8} z^{-2} Y(z) = X(z) + \frac{-7}{4} z^{-1} X(z) + \frac{-1}{2} z^{-2} X(z)$$

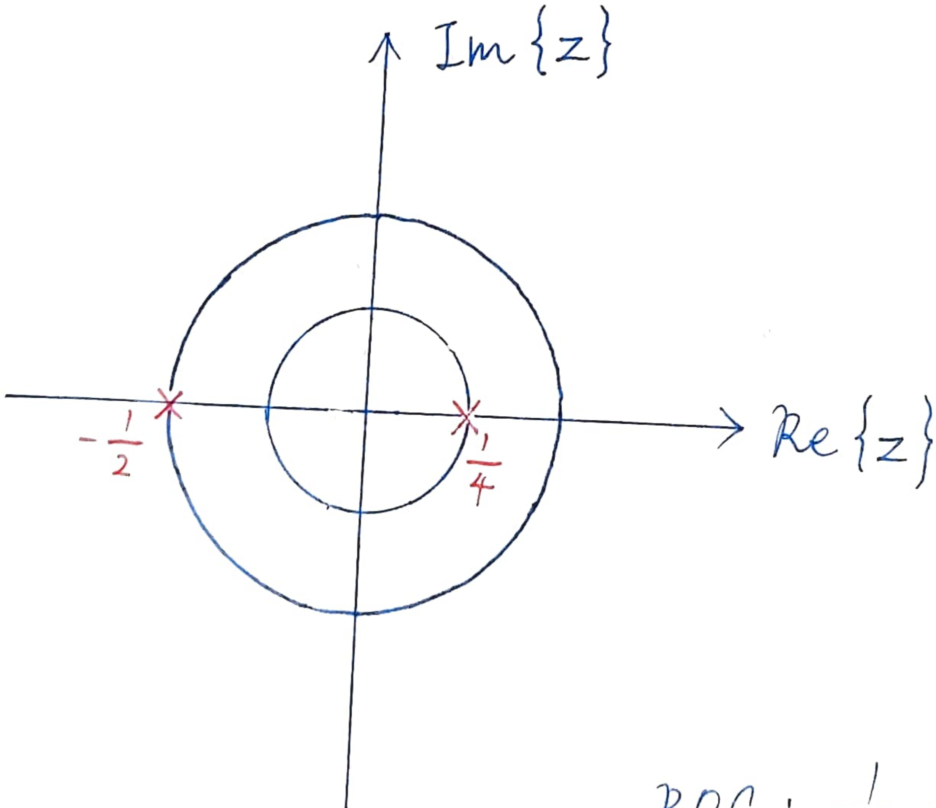
$$y[n] + \frac{1}{4} y[n-1] - \frac{1}{8} y[n-2] = x[n] - \frac{7}{4} x[n-1] - \frac{1}{2} x[n-2]$$

Problem 4 (continued)

(c)

$$H(z) = \frac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$= \frac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}$$



z-plane

ROC: $\frac{1}{2} < |z|$

Problem 4 (continued)

(d)

$$H(x^{-1}) = \frac{1-\frac{7}{4}x-\frac{1}{2}x^2}{(1-\frac{1}{4}x)(1+\frac{1}{2}x)} = \frac{8-14x-4x^2}{(4-x)(2+X)}$$

$$= \frac{a}{4-X} + \frac{b}{2+X} + c$$

$$8-14x-4x^2 = 2a + aX + 4b - bx + 8c + 2cX - cX^2$$

$$\begin{cases} 8 = 2a + 4b + 8c \\ -14 = a - b + 2c \\ -4 = -c \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 4 & 4 \\ 1 & -1 & 2 & -14 \\ 0 & 0 & 1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 & 4 \\ 0 & -3 & -2 & -18 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$c = 4$

$-3b - 2\cdot 4 = -18$, $-3b - 8 = -18$, $-3b = -10$, $b = \frac{10}{3}$

$a + 2\frac{10}{3} + 4\cdot 4 = 4$, $a + \frac{20}{3} + 16 = 4$, $a = \frac{12}{3} - \frac{20}{3} - \frac{48}{3} = \frac{-56}{3}$

$$H(x^{-1}) = \frac{-56}{3}\frac{1}{4-X} + \frac{10}{3}\frac{1}{2+X} + 4$$

$$H(z) = \frac{-14}{3}\frac{1}{1-\frac{1}{4}z^{-1}} + \frac{5}{3}\frac{1}{1+\frac{1}{2}z^{-1}} + 4$$

$$h[n] = \frac{-14}{3}\left(\frac{1}{4}\right)^n u[n] + \frac{5}{3}\left(\frac{-1}{2}\right)^n u[n] + 4\delta[n]$$

Problem 4 (continued)

(e)

Yes, the system is BIBO stable.

Problem 5

$$Y = X + z^{-1}Y - \frac{1}{4}z^{-2}Y$$

$$X = \left(1 - z^{-1} + \frac{1}{4}z^{-2}\right)Y$$

$$H(z) = \frac{Y}{X} = \frac{1}{1 - z^{-1} + \frac{1}{4}z^{-2}} = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

$$h[n] = (n+1)\left(\frac{1}{2}\right)^n u[n]$$