Homework #18 林靖 108061112

Problem 1

(一)

$$x[n] = u[n]$$

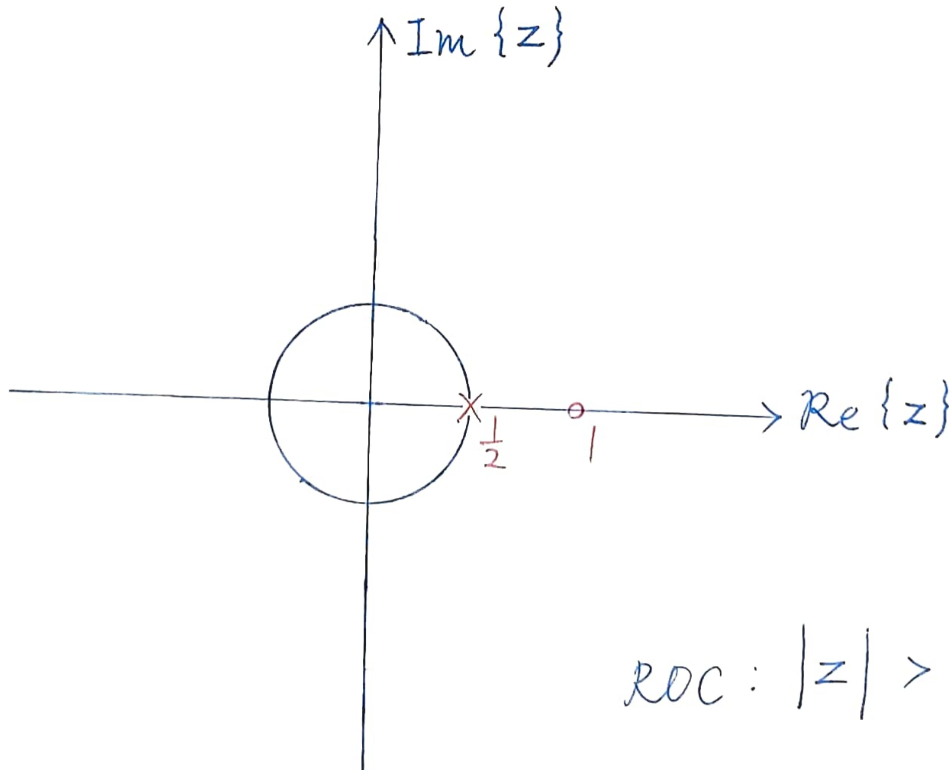
$$X(z) = \frac{1}{1-z^{-1}}, \quad ROC: |z| > 1$$

$$y[n] = 2\left(\frac{1}{2}\right)^n u[n]$$

$$Y(z) = 2\,\frac{1}{1-\frac{1}{2}z^{-1}}, \quad ROC: |z| > \frac{1}{2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2\,\frac{1}{1-\frac{1}{2}z^{-1}}}{\frac{1}{1-z^{-1}}}$$

$$= 2\cdot\frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}}$$



$$ROC: |z| > \frac{1}{2}$$

z plane

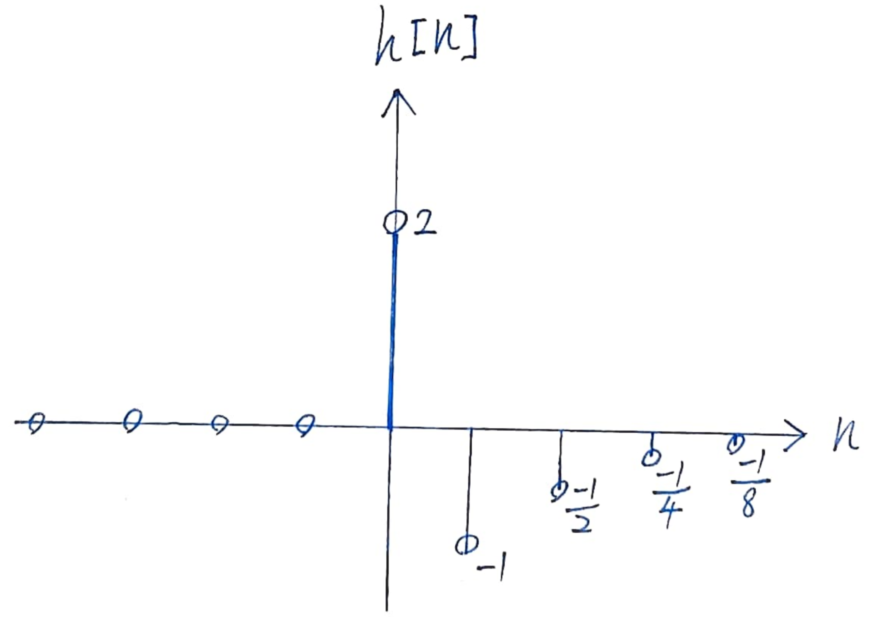
Problem 1 (continued)

(=)

$$H(z) = 2\,\frac{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$= 2 - z^{-1}\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$h[n] = 2\,\delta[n] - \left(\frac{1}{2}\right)^{n-1} u[n-1]$$



Problem 1 (continued)

(三)

Yes, the system is stable.

Yes, the system is causal.

Problem 2

$$a^n u[n] \xleftrightarrow{ZT} \frac{1}{1-az^{-1}}, \quad ROC: |z|>|a|$$

$$(n+1)\,a^n u[n] \xleftrightarrow{ZT} \frac{1}{(1-az^{-1})^2}, \quad ROC: |z|>|a|$$

$$u[n] \xleftrightarrow{ZT} \frac{1}{1-z^{-1}}, \quad ROC: |z|>1$$

$$(n+1)\,u[n] \xleftrightarrow{ZT} \frac{1}{(1-z^{-1})^2}, \quad ROC: |z|>1$$

$$y[n] = u[n+3] * u[n-4] - u[n-1] * u[n-4]$$

$$Y(z) = \frac{z^{+3}}{1-z^{-1}}\frac{z^{-4}}{1-z^{-1}} - \frac{z^{-1}}{1-z^{-1}}\frac{z^{-4}}{1-z^{-1}}$$

$$= \frac{z^{-1}}{(1-z^{-1})^2} - \frac{z^{-5}}{(1-z^{-1})^2}$$

$$y[n] = [(n-1)+1]\,u[(n-1)] - [(n-5)+1]\,u[(n-5)]$$

$$= n\,u[n-1] - (n-4)\,u[n-5]$$

Problem 3

(a)

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=\frac{1}{8}x[n]+\frac{1}{8}x[n-1]$$

$$Y(z)-\frac{1}{4}z^{-1}Y(z)-\frac{1}{8}z^{-2}Y(z)=\frac{1}{8}X(z)+\frac{1}{8}z^{-1}X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\frac{1}{8}+\frac{1}{8}z^{-1}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

$$=\frac{1}{8}\frac{(1+z^{-1})}{(1+\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$H(e^{j2\pi f})=\frac{\frac{1}{8}(1+z^{-j2\pi f})}{(1+\frac{1}{4}z^{-j2\pi f})(1-\frac{1}{2}z^{-j2\pi f})}$$

$$H(z)=\frac{1}{8}\frac{-(1-\frac{1}{2}z^{-1})+2(1+\frac{1}{4}z^{-1})}{(1+\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

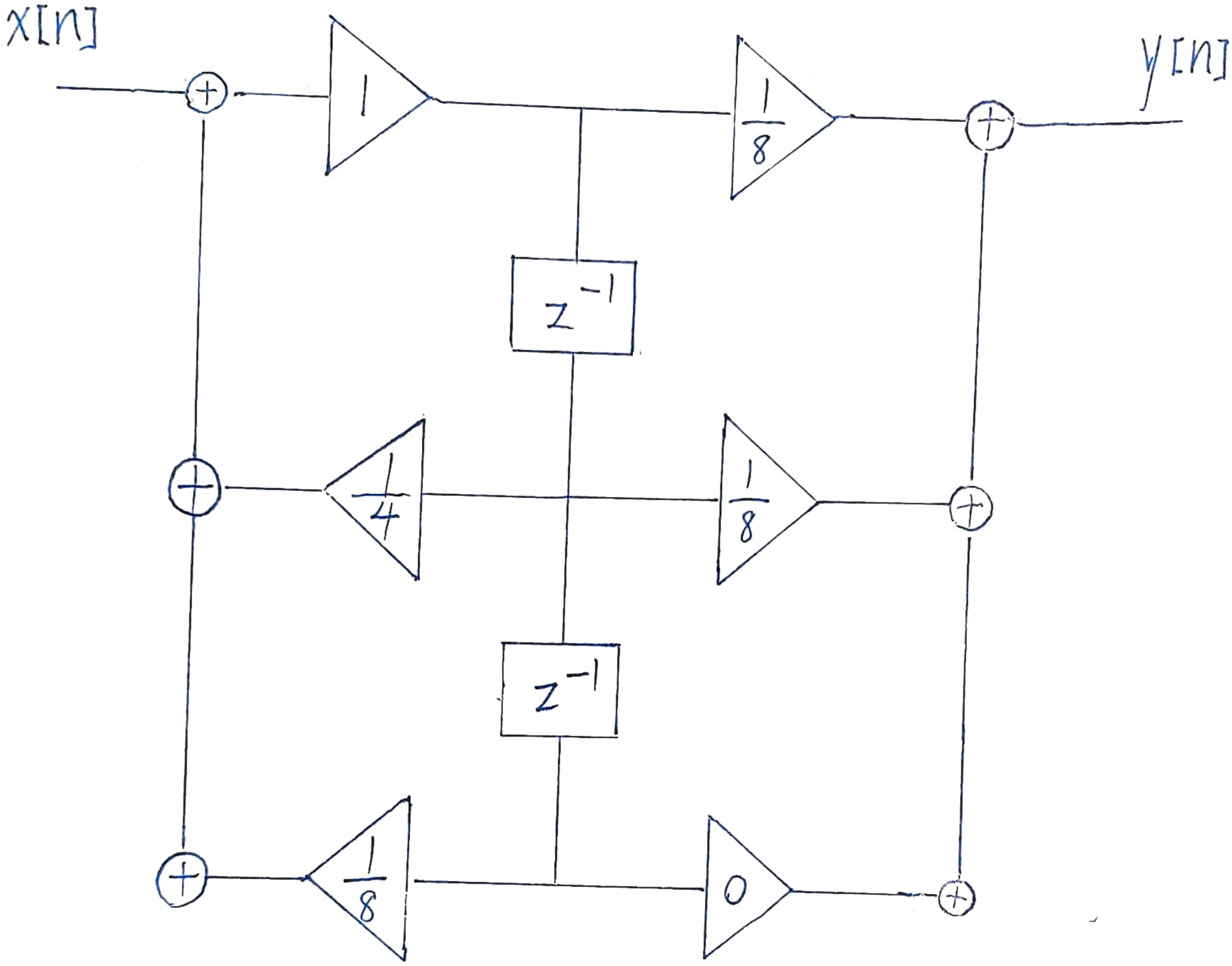
$$=\frac{1}{8}\left(-\frac{1}{1+\frac{1}{4}z^{-1}}+2\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

$$=\frac{-1}{8}\frac{1}{1+\frac{1}{4}z^{-1}}+\frac{1}{4}\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$h[n]=\frac{-1}{8}\left(\frac{-1}{4}\right)^{n}u[n]+\frac{1}{4}\left(\frac{1}{2}\right)^{n}u[n]$$

Problem 3 (continued)

(b)



Problem 4

(a)

$$x[n] = \frac{-1}{3}\left(\frac{1}{3}\right)^n u[n] + \frac{4}{3}\left[-(2)^n u[-n-1]\right]$$

$$X(z) = \frac{-1}{3}\frac{1}{1-\frac{1}{3}z^{-1}} + \frac{4}{3}\frac{1}{1-2z^{-1}}$$

ROC : $|z| > \frac{1}{3}$ $\quad |z| < 2$

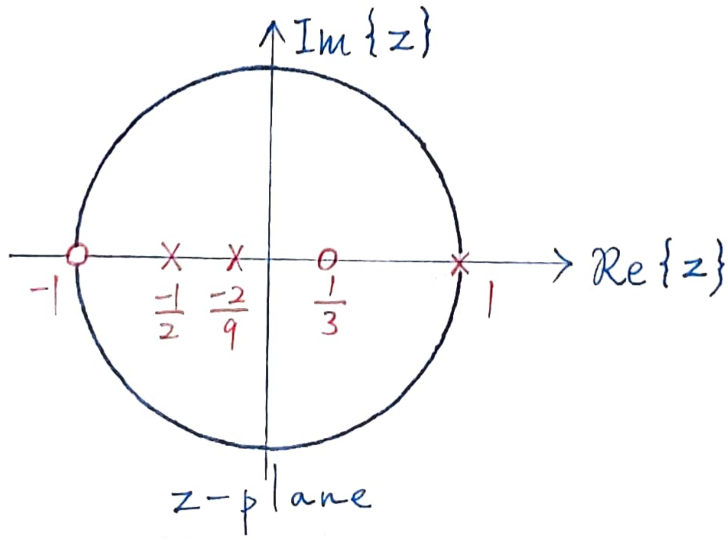
$$X(z) = \frac{\frac{-1}{3}(1-2z^{-1}) + \frac{4}{3}(1-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$$

$$= \frac{\frac{-1}{3}+\frac{4}{3} + \frac{2}{3}z^{-1} - \frac{4}{9}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}$$

(b)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\dfrac{1+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1-2z^{-1})}}{\dfrac{1+\frac{2}{9}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}}$$

$$= \frac{(1+z^{-1})(1-\frac{1}{3}z^{-1})}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1+\frac{2}{9}z^{-1})}$$



ROC : $|z| > 1$

Problem 4 (continued)

(c)

$$H(z) = \frac{(1+z^{-1})(1-\frac{1}{3}z^{-1})}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1+\frac{2}{9}z^{-1})}$$

$$H(x^{-1}) = \frac{6(x-3)(x+1)}{(x-1)(x+2)(2x+9)}$$

$$= \frac{\theta_1}{x-1} + \frac{\theta_2}{x+2} + \frac{\theta_3}{2x+9}$$

$$6(x-3)(x+1) = \theta_1(x+2)(2x+9) + \theta_2(x-1)(2x+9) + \theta_3(x-1)(x+2)$$

$$6x^2-12x-18 = 2\theta_1 x^2 + 13\theta_1 x + 18\theta_1 + 2\theta_2 x^2 + 7\theta_2 x - 9\theta_2 + \theta_3 x^2 + \theta_3 x - 2\theta_3$$

$$\begin{cases} 6 = 2\theta_1 + 2\theta_2 + \theta_3 \\ -12 = 13\theta_1 + 7\theta_2 + \theta_3 \\ -18 = 18\theta_1 - 9\theta_2 - 2\theta_3 \end{cases}$$

$$\begin{pmatrix} 2 & 2 & 1 & 6 \\ 13 & 7 & 1 & -12 \\ 18 & -9 & -2 & -18 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 1 & 6 \\ 0 & -12 & -11 & -102 \\ 0 & -27 & -11 & -72 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 1 & 6 \\ 0 & -12 & -11 & -102 \\ 0 & 0 & -165 & -1890 \end{pmatrix}$$

$-165\theta_3 = -1890, \quad \theta_3 = 126/11$

$-12\theta_2 - 11 \cdot 126/11 = -102, \quad \theta_2 = -2$

$2\theta_1 + 2(-2) + 126/11 = 6, \quad \theta_1 = -8/11$

$$H(x^{-1}) = \frac{-8/11}{x-1} + \frac{-2}{x+2} + \frac{126/11}{2x+9} = \frac{8/11}{1-x} + \frac{-1}{1+\frac{1}{2}x} + \frac{14/11}{1+\frac{2}{9}x}$$

$$H(z) = \frac{8}{11}\frac{1}{1-z^{-1}} - \frac{1}{1+\frac{1}{2}z^{-1}} + \frac{14}{11}\frac{1}{1+\frac{2}{9}z^{-1}}$$

$$h[n] = \frac{8}{11}(1)^n u[n] - \left(\frac{-1}{2}\right)^n u[n] + \frac{14}{11}\left(\frac{-2}{9}\right) u[n]$$

(d)

No, the system is not stable.

Problem 5

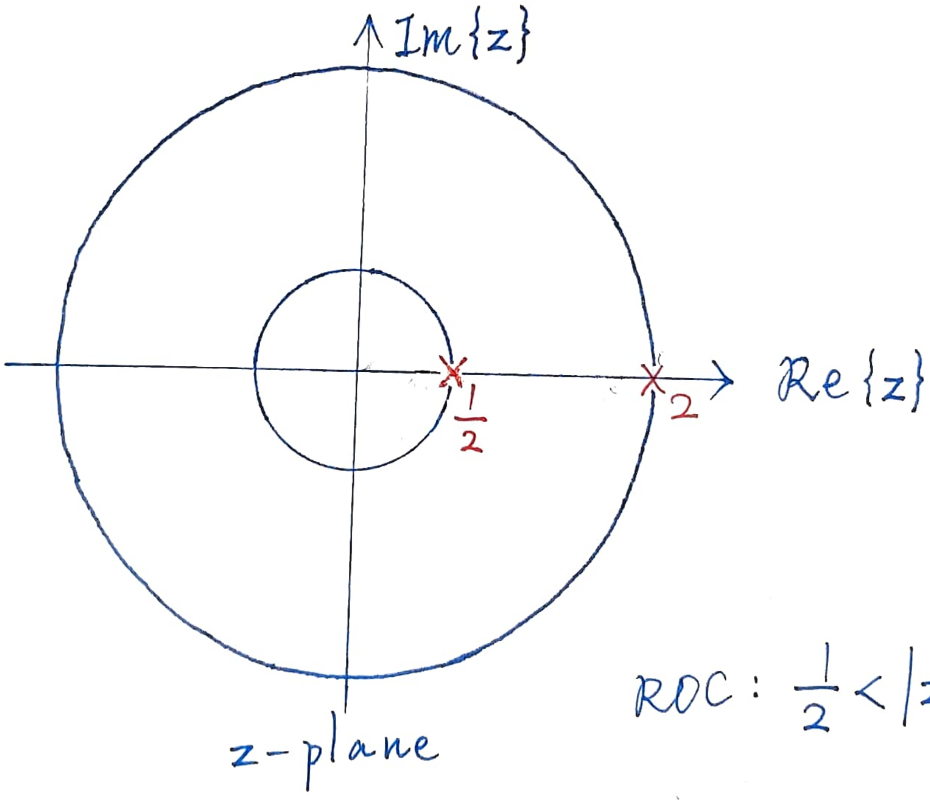
(一) $$H_1(z) = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}} = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$= \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-2z^{-1}}$$

$$A = \left.\frac{z^{-1}}{1-2z^{-1}}\right|_{z^{-1}=2} = \frac{2}{-3}$$

$$B = \left.\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$$H_1(z) = \frac{-2}{3}\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{2}{3}\frac{1}{1-2z^{-1}}$$



ROC: $\frac{1}{2} < |z| < 2$

$$h_1[n] = \frac{-2}{3}\left[\left(\frac{1}{2}\right)^n u[n]\right] + \frac{2}{3}\left[-2^n u[-n-1]\right]$$

Problem 5 (continued)

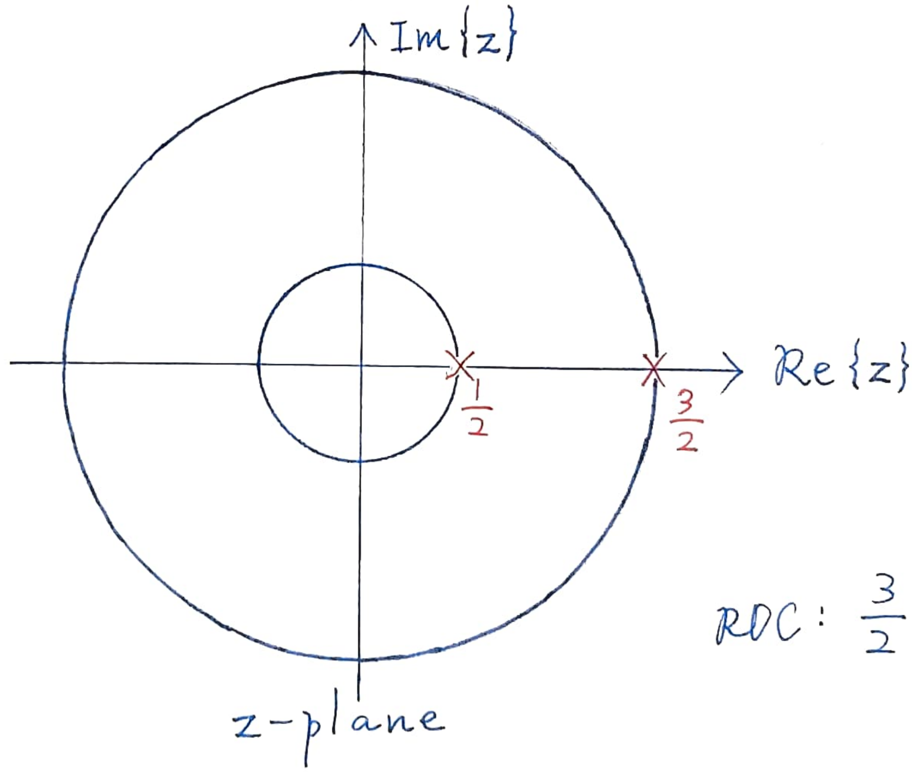
(二) $H_2(z) = \dfrac{\frac{1}{2}z^{-1}}{1-2z^{-1}+\frac{3}{4}z^{-2}} = \dfrac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{2}z^{-1})}$

$= \dfrac{A}{1-\frac{1}{2}z^{-1}} + \dfrac{B}{1-\frac{3}{2}z^{-1}}$

$A = \left.\dfrac{\frac{1}{2}z^{-1}}{1-\frac{3}{2}z^{-1}}\right|_{z^{-1}=2} = \dfrac{1}{-2}$

$B = \left.\dfrac{\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=\frac{2}{3}} = \dfrac{\frac{1}{3}}{\frac{2}{3}} = \dfrac{1}{2}$

$H_2(z) = \dfrac{-1}{2}\dfrac{1}{1-\frac{1}{2}z^{-1}} + \dfrac{1}{2}\dfrac{1}{1-\frac{3}{2}z^{-1}}$



ROC: $\frac{3}{2} < |z|$

$h[n] = \frac{-1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{3}{2}\right)^n u[n]$