

Problem 1

$$(a) \quad a^n u[n] \xleftrightarrow{ZT} \frac{1}{1 - az^{-1}}, \quad \text{ROC: } |z| > |a|$$

$$h[n] = (0.5)^n u[n] + 2(-0.25)^n u[n]$$

$$H(z) = \frac{1}{1 - 0.5z^{-1}} + 2 \frac{1}{1 + 0.25z^{-1}}$$

$$\text{ROC: } |z| > 0.5 \cap |z| > 0.25$$

$$\text{ROC: } |z| > 0.5$$

$$H(z) = \frac{(1 + 0.25z^{-1}) + 2(1 - 0.5z^{-1})}{(1 - 0.5z^{-1})(1 + 0.25z^{-1})}$$

$$H(z) = \frac{3 - 0.75z^{-1}}{1 - 0.25z^{-1} - 0.125z^{-2}}$$

$$H(e^{j2\pi f}) = \frac{3 - 0.75e^{-j2\pi f}}{1 - 0.25e^{-j2\pi f} - 0.125e^{-j4\pi f}}$$

Problem 1 (continued)

$$\frac{Y(z)}{X(z)} = H(z) = \frac{3 - 0.75z^{-1}}{1 - 0.25z^{-1} - 0.125z^{-2}}$$

$$Y(z) - 0.25z^{-1}Y(z) - 0.125z^{-2}Y(z)$$

$$= 3X(z) - 0.75z^{-1}X(z)$$

$$y[n] - 0.25y[n-1] - 0.125y[n-2]$$

$$= 3x[n] - 0.75x[n-1]$$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n] - \frac{3}{4}x[n-1]$$

## Problem 2

(-)

$$a[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$A(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \quad \text{ROC: } |z| > \frac{1}{2}$$

$$b[n] = n \left(\frac{-1}{4}\right)^n u[n]$$

$$\begin{aligned} B(z) &= -z \frac{d}{dz} \frac{1}{1 + \frac{1}{4}z^{-1}} \\ &= -z \frac{d}{dz} \left(1 + \frac{1}{4}z^{-1}\right)^{-1} \\ &= -z \cdot (-1) \left(1 + \frac{1}{4}z^{-1}\right)^{-2} \left(\frac{1}{4}\right) (-1) (z^{-2}) \\ &= \frac{-1}{4} \frac{z^{-1}}{\left(1 + \frac{1}{4}z^{-1}\right)^2}, \quad \text{ROC: } |z| > \frac{1}{4} \end{aligned}$$

$$x[n] = a[n] * b[n]$$

$$X(z) = A(z) B(z)$$

$$= \frac{1}{1 - \frac{1}{2}z^{-1}} \frac{-1}{4} \frac{z^{-1}}{\left(1 + \frac{1}{4}z^{-1}\right)^2},$$

$$\text{ROC: } |z| > \frac{1}{2} \cap |z| > \frac{1}{4}$$

$$\text{ROC: } |z| > \frac{1}{2}$$

# Problem 2 (continued)

(=)

$$a^n u[n] \xleftrightarrow{zT} \frac{1}{1-az^{-1}} \quad |z| > |a|$$

$$-a^n u[-n-1] \xleftrightarrow{zT} \frac{1}{1-az^{-1}} \quad |z| < |a|$$

$$(n+1) a^n u[n] \xleftrightarrow{zT} \frac{1}{(1-az^{-1})^2} \quad |z| > |a|$$

$$-(n+1) a^n u[-n-1] \xleftrightarrow{zT} \frac{1}{(1-az^{-1})^2} \quad |z| < |a|$$

$$X(z) = \left( \frac{1}{1-az^{-1}} \right)^2$$

$$X[n] = \begin{cases} (n+1) a^n u[n], & |z| > |a| \\ -(n+1) a^n u[-n-1], & |z| < |a| \end{cases}$$

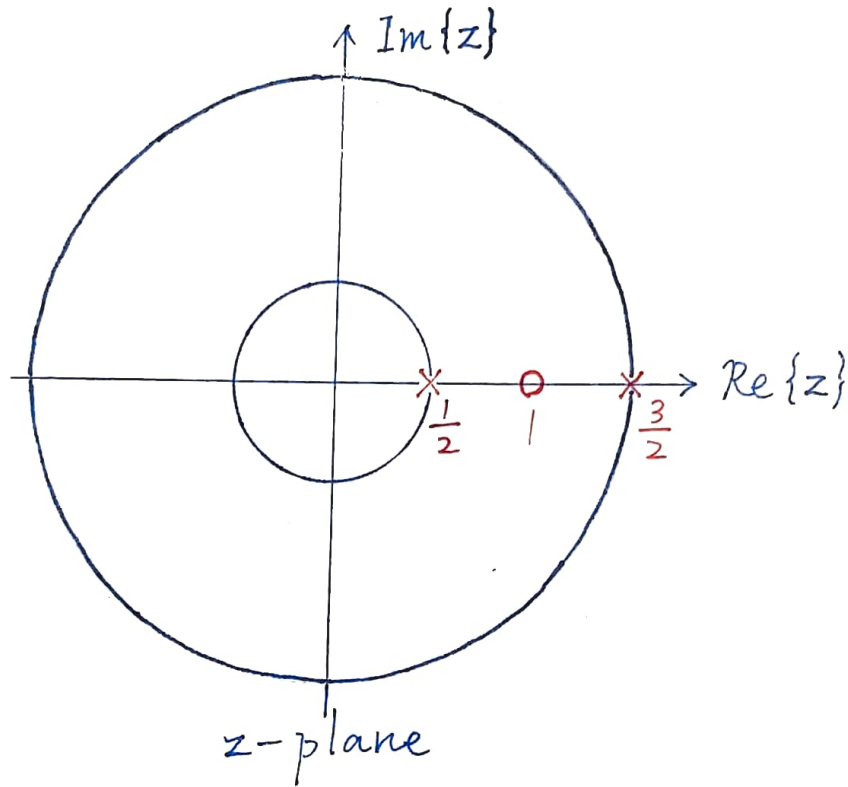
### Problem 3

$$(1) \quad H(z) = \frac{1}{1 - 0.5z^{-1}}, \quad |z| > 0.5$$

$$(2) \quad H(z) = \frac{1}{(1 - 0.5z^{-1})^2}, \quad |z| > 0.5$$

# Problem 4

$$H(z) = \frac{2 - 2z^{-1}}{1 - 2z^{-1} + \frac{3}{4}z^{-2}}$$
$$= \frac{2(1 - z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{3}{2}z^{-1})}$$



$|z| < \frac{1}{2}$       anti-causal      non-stable

$\frac{1}{2} < |z| < \frac{3}{2}$       non-causal      BIBO-stable

$\frac{3}{2} < |z|$       causal      non-stable

Problem 5

$$(-) \quad x[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$$

$$y[n] = \frac{1}{2} \left(\frac{1}{3}\right)^n u[n] + \frac{1}{4} \left(\frac{1}{6}\right)^n u[n]$$

$$Y(z) = \frac{1}{2} \frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{1}{4} \frac{1}{1 - \frac{1}{6}z^{-1}}, \quad |z| > \frac{1}{3}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\frac{1}{2} \frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{1}{4} \frac{1}{1 - \frac{1}{6}z^{-1}}}{\frac{1}{1 - \frac{1}{3}z^{-1}}}$$

$$= \frac{1}{2} + \frac{1}{4} \frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{1}{2} + \frac{1}{4} \frac{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{-\frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{3}{4} - \frac{1}{6} z^{-1} \frac{1}{1 - \frac{1}{6}z^{-1}}$$

$$h[n] = \frac{3}{4} \delta[n] - \frac{1}{6} \left(\frac{1}{6}\right)^{n-1} u[n-1]$$

Problem 5 (continued)

$$\begin{aligned} H(z) &= \frac{1}{2} + \frac{1}{4} \frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{6}z^{-1}} \\ &= \frac{\frac{1}{2}(1 - \frac{1}{6}z^{-1}) + \frac{1}{4}(1 - \frac{1}{3}z^{-1})}{1 - \frac{1}{6}z^{-1}} \\ &= \frac{\frac{3}{4} - \frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1}} \end{aligned}$$

$$H(e^{j2\pi f}) = \frac{\frac{3}{4} - \frac{1}{6}e^{-j2\pi f}}{1 - \frac{1}{6}e^{-j2\pi f}}$$



Problem 5 (continued)

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{4} - \frac{1}{6} z^{-1}}{1 - \frac{1}{6} z^{-1}}$$

$$Y(z) - \frac{1}{6} z^{-1} Y(z) = \frac{3}{4} X(z) - \frac{1}{6} z^{-1} X(z)$$

$$y[n] - \frac{1}{6} y[n-1] = \frac{3}{4} x[n] - \frac{1}{6} x[n-1]$$