Homework#15 林靖 108061112

Problem 1

(1)

$$\frac{X(z)}{Y(z)} = H(z) = \frac{1}{1-\frac{6}{8}z^{-1}+\frac{1}{8}z^{-2}}$$

$$Y(z)-\frac{6}{8}z^{-1}Y(z)+\frac{1}{8}z^{-2}Y(z) = X(z)$$

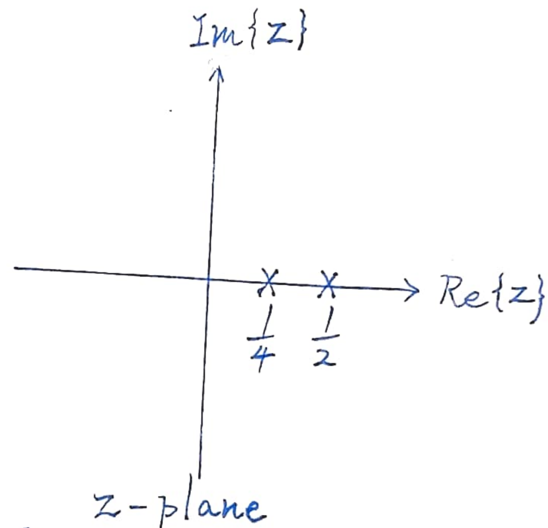
$$y[n]-\frac{6}{8}y[n-1]+\frac{1}{8}y[n-2] = x[n]$$

(2)

$$H(z) = \frac{1}{1-\frac{6}{8}z^{-1}+\frac{1}{8}z^{-2}}$$

$$= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

pole : $\frac{1}{4}$, $\frac{1}{2}$



(3)

$$y[n]-\frac{6}{8}y[n-1]+\frac{1}{8}y[n-2] = 0$$

$$y[n] = r^n$$

$$r^n-\frac{6}{8}r^{n-1}+\frac{1}{8}r^{n-2} = 0$$

$$r^n(1-\frac{1}{4}r^{-1})(1-\frac{1}{2}r^{-1}) = 0$$

$$r = \frac{1}{4} \text{ or } \frac{1}{2}$$

$$y_h[n] = c_1\left(\frac{1}{4}\right)^n + c_2\left(\frac{1}{2}\right)^n$$

(4) All of the poles are located inside the unit circle.

The system is stable.

Problem 2

(1)
$$\frac{X(z)}{Y(z)} = H(z) = \frac{1}{1-\frac{5}{2}z^{-1}+z^{-2}}$$

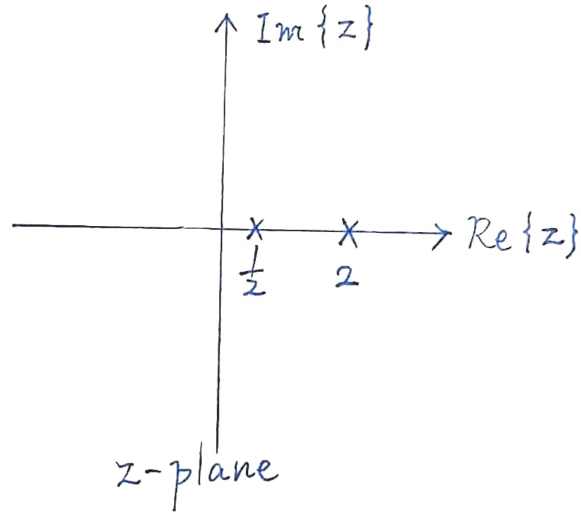
$$Y(z) - \frac{5}{2}z^{-1}Y(z) + z^{-2}Y(z) = X(z)$$

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n]$$

(2)
$$H(z) = \frac{1}{1-\frac{5}{2}z^{-1}+z^{-2}}$$

$$= \frac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

pole : $\frac{1}{2}$, 2



(3)
$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = 0$$

$$y[n] = r^n$$

$$r^n - \frac{5}{2}r^{n-1} + r^{n-2} = 0$$

$$r^n\left(1-\frac{1}{2}r^{-1}\right)\left(1-2r^{-1}\right) = 0$$

$$r = \frac{1}{2} \text{ or } 2$$

$$y_h[n] = c_1\left(\frac{1}{2}\right)^n + c_2\, 2^n$$

(4)

Not all of the poles are located inside the unit circle.

The system is not stable.

Problem 3

(1) $$\frac{X(z)}{Y(z)} = H(z) = \frac{1}{1-5z^{-1}+6z^{-2}}$$

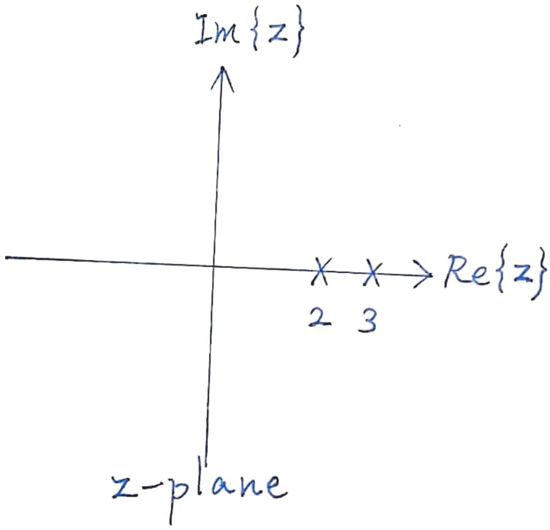
$$Y(z) - 5z^{-1}Y(z) + 6z^{-2}Y(z) = X(z)$$

$$y[n] - 5y[n-1] + 6y[n-2] = x[n]$$

(2) $$H(z) = \frac{1}{1-5z^{-1}+6z^{-2}}$$

$$= \frac{1}{(1-2z^{-1})(1-3z^{-1})}$$

pole : 2 or 3



(3) $$y[n] - 5y[n-1] + 6y[n-2] = 0$$

$$y[n] = r^n$$

$$r^n - 5r^{n-1} + 6r^{n-2} = 0$$

$$r^n(1-2r^{-1})(1-3r^{-1}) = 0$$

$r = 2$ or 3

$$y_h[n] = c_1 2^n + c_2 3^n$$

(4)

Not all of the poles are located inside the unit circle.

The system is not stable.