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Problem 1

(1)

$$h[n] = a^n u[n]$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u[n]\, z^{-n}$$

$$= \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= (az^{-1})^0 + (az^{-1})^1 + (az^{-1})^2 + \cdots$$

$$= \frac{1}{1 - az^{-1}}, \quad |az^{-1}| < 1$$

ROC : $|z| > |a|$

(2)

$$h[n] = -a^n u[-n-1]$$

$$H(z) = \sum_{n=\infty}^{-\infty} h[n] z^{-n}$$

$$= \sum_{n=\infty}^{-\infty} -a^n u[-n-1] z^{-n}$$

$$= -\sum_{n=\infty}^{-\infty} u[-n-1] (az^{-1})^n$$

$$= -\sum_{n=-1}^{-\infty} (az^{-1})^n$$

$$= -\left[(az^{-1})^{-1} + (az^{-1})^{-2} + (az^{-1})^{-3} + \cdots \right]$$

$$= -\left[(a^{-1}z)^1 + (a^{-1}z)^2 + (a^{-1}z)^3 + \cdots \right]$$

$$= -\frac{a^{-1}z}{1-a^{-1}z}, \quad |a^{-1}z| > 1$$

$$= -\frac{1}{az^{-1}-1}$$

$$= \frac{1}{1-az^{-1}}$$

ROC: $|z| < |a|$

(3) $$h[n] = (n+1)\, a^n u[n]$$

$$= n\, a^n u[n] + a^n u[n]$$

$$a^n u[n] \overset{ZT}{\longleftrightarrow} \frac{1}{1-az^{-1}}, \quad ROC: |z| > |a|$$

$$n\, a^n u[n] \overset{ZT}{\longleftrightarrow} -z \frac{d}{dz} \frac{1}{1-az^{-1}}$$

$$= -z \frac{d}{dz} (1-az^{-1})^{-1}$$

$$= -z \left[-(1-az^{-1})^{-2} (-a)(-z^{-2}) \right]$$

$$= \frac{az^{-1}}{(1-az^{-1})^2}$$

$$(n+1)\, a^n u[n] \overset{ZT}{\longleftrightarrow} \frac{1}{1-az^{-1}} + \frac{az^{-1}}{(1-az^{-1})^2}$$

$$= \frac{1}{(1-az^{-1})^2}, \quad ROC: |z| > |a|$$

(4) $h[n] = -(n+1)\,a^n u[-n-1]$

$$= n\left(-a^n u[-n-1]\right) + \left(-a^n u[-n-1]\right)$$

$$-a^n u[-n-1] \xleftrightarrow{ZT} \frac{1}{1-az^{-1}}, \quad ROC: |z| < |a|$$

$$n\left(-a^n u[-n-1]\right) \xleftrightarrow{ZT} -z \frac{d}{dz}\frac{1}{1-az^{-1}}$$

$$= \frac{az^{-1}}{(1-az^{-1})^2}$$

$$-(n+1)\,a^n u[-n-1] \xleftrightarrow{ZT} \frac{1}{1-az^{-1}} + \frac{az^{-1}}{(1-az^{-1})^2}$$

$$= \frac{1}{(1-az^{-1})^2}, \quad ROC: |z| < |a|$$

(5) $h[n] = \delta[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n]\, z^{-0}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n]$$

$$= \delta[0]$$

$$= 1$$

ROC : all z-plane

(6) $$h[n] = \delta[n-n_0]$$

$$H(z) = \sum_{n=\infty}^{-\infty} h[n]\, z^{-n}$$

$$= \sum_{n=\infty}^{-\infty} \delta[n-n_0]\, z^{-n}$$

$$= \sum_{n=\infty}^{-\infty} \delta[n-n_0]\, z^{-n_0}$$

$$= \delta[n_0-n_0]\, z^{-n_0}$$

$$= z^{-n_0}$$

ROC: all z-plane

(7) $h[n] = u[n]$

$$H(z) = \sum_{n=\infty}^{-\infty} h[n] \, z^{-n}$$

$$= \sum_{n=\infty}^{-\infty} u[n] \, z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n}$$

$$= z^{-0} + z^{-1} + z^{-2} + \cdots$$

$$= (z^{-1})^{0} + (z^{-1})^{1} + (z^{-1})^{2} + \cdots$$

$$= \frac{1}{1 - z^{-1}}, \quad |z^{-1}| < 1$$

ROC : $|z| > 1$

(8) (9) (10) (11)

$$y[n] = \alpha^n \cos(\omega_0 n) u[n] + j \alpha^n \sin(\omega_0 n) u[n]$$

$$= \alpha^n [\cos(\omega_0 n) + j \sin(\omega_0 n)] u[n]$$

$$= \alpha^n e^{j(\omega_0 n)} u[n]$$

$$= (\alpha e^{j\omega_0})^n u[n]$$

$$a^n u[n] \overset{ZT}{\longleftrightarrow} \frac{1}{1 - a z^{-1}}, \quad ROC: |z| > |a|$$

$$(\alpha e^{j\omega_0})^n u[n] \overset{ZT}{\longleftrightarrow} \frac{1}{1 - (\alpha e^{j\omega_0}) z^{-1}}, \quad ROC: |z| > \alpha$$

$$G(z) = \frac{1}{1 - \alpha e^{j\omega_0} z^{-1}}$$

$$= \frac{1}{1 - \alpha(\cos\omega_0 + j \sin\omega_0) z^{-1}}$$

$$= \frac{1}{(1 - \alpha \cos\omega_0 z^{-1}) + j(-\alpha \sin\omega_0 z^{-1})}$$

$$= \frac{(1 - \alpha \cos\omega_0 z^{-1}) - j(-\alpha \sin\omega_0 z^{-1})}{(1 - \alpha \cos\omega_0 z^{-1})^2 + (\alpha \sin\omega_0 z^{-1})^2}$$

$$= \frac{(1 - \alpha \cos\omega_0 z^{-1}) + j(\alpha \sin\omega_0 z^{-1})}{1 - 2\alpha \cos\omega_0 z^{-1} + \alpha^2 \cos^2\omega_0 z^{-2} + \alpha^2 \sin^2\omega_0 z^{-2}}$$

$$= \frac{1 - \alpha \cos\omega_0 z^{-1}}{1 - 2\alpha \cos\omega_0 z^{-1} + \alpha^2 z^{-2}} + j \frac{\alpha \sin\omega_0 z^{-1}}{1 - 2\alpha \cos\omega_0 z^{-1} + \alpha^2 z^{-2}}$$

ROC: $|z| > \alpha$