Homework #12　林靖　108061112

Problem 1

$$e^{-at}u(t) \xleftrightarrow{LT} \frac{1}{s+a}, \quad \text{ROC: } Re\{s\} > Re\{a\}$$

$$h(t-a) \xleftrightarrow{LT} e^{-as}H(s), \quad \text{ROC: } R_h$$

$$\frac{d}{dt}h(t) \xleftrightarrow{LT} sH(s), \quad \text{ROC: } R_h$$

$$e^{-3(t)}u(t) \xleftrightarrow{LT} \frac{1}{s+3}, \quad Re\{s\} > -3$$

$$e^{-3(t-2)}u(t-2) \xleftrightarrow{LT} e^{-2s}\frac{1}{s+3}, \quad Re\{s\} > -3$$

$$\frac{d^2}{dt^2}\left(e^{-3(t-2)}u(t-2)\right) \xleftrightarrow{LT} s^2e^{-2s}\frac{1}{s+3}, \quad Re\{s\} > -3$$

Problem 2

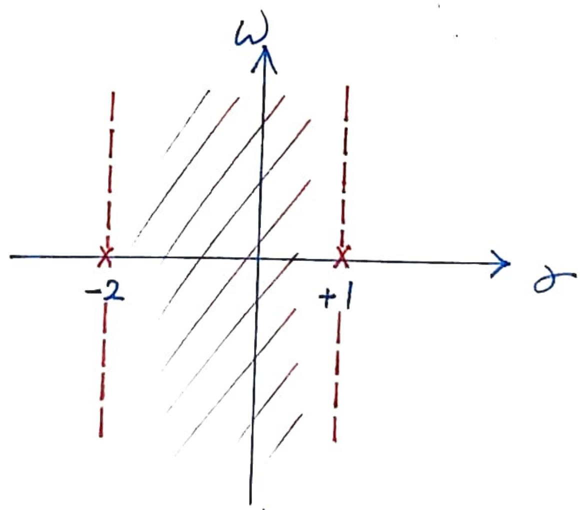
$$y''(t) + y'(t) - 2y(t) = x(t)$$

$$s^2 Y(s) + s\,Y(s) - 2\,Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + s - 2}$$

$$= \frac{1}{(s+2)(s-1)}$$

ROC: $\text{Re}\{s\} > -2$, $\text{Re}\{s\} < 1$



s-plane

Problem 2 (continued)

$$x(t) = \sum_{n=1}^{\infty} x_n(t) \qquad y(t) = \sum_{n=1}^{\infty} y_n(t)$$

$$x_n(t) = u(t-n) \qquad y_n(t) = ?$$

$$X_n(s) = e^{-ns}\frac{1}{s} \qquad Y_n(s) = H(s)\,X_n(s)$$

Problem 2 (continued)

$$Y_n(s) = H(s)\,X_n(s)$$

$$= \left[\frac{1}{(s+2)(s-1)}\,\frac{1}{s}\right] e^{-ns}$$

$$= \left[\frac{A}{s+2} + \frac{B}{s-1} + \frac{C}{s}\right] e^{-ns}$$

$$A = \frac{1}{6}$$

$$B = \frac{1}{3}$$

$$C = \frac{-1}{2}$$

$$Y_n(s) = \left[\frac{\frac{1}{6}}{s+2} + \frac{\frac{1}{3}}{s-1} + \frac{\frac{-1}{2}}{s}\right] e^{-ns}$$

Problem 2 (continued)

$$Y_n(s) = \frac{1}{6} e^{-ns} \frac{1}{s+2} + \frac{1}{3} e^{-ns} \frac{1}{s-1} + \frac{-1}{2} e^{-ns} \frac{1}{s}$$

$$y_n(t) = \frac{1}{6} e^{-2(t-n)} u(t-n) - \frac{1}{3} e^{t-n} u(-(t-n)) + \frac{-1}{2} u(t-n)$$

$$y(t) = \sum_{n=1}^{\infty} y_n(t)$$

Problem 3

$$H(s) = \frac{A(s)}{1 + A(s)\,B(s)}$$

$$= \frac{\dfrac{s+2}{s^2+2s+4}}{1 + K\,\dfrac{s+2}{s^2+2s+4}}$$

$$= \frac{s+2}{(s^2+2s+4) + K(s+2)}$$

$$= \frac{s+2}{s^2 + (K+2)s + (2K+4)}$$

$$s^2 + (K+2)s + (2K+4) = 0$$

$$s = \frac{-(K+2) \pm \sqrt{(K+2)(K-6)}}{2}$$

Problem 3 (continued)

case $K < -2$: $-(K+2) > 0$

$+\sqrt{(K+2)(K-6)} > 0$

$\Rightarrow \dfrac{-(K+2)+\sqrt{(K+2)(K-6)}}{2} > 0$

unstable

case $-2 < K < 6$: $-(K+2) < 0$

$Re\{\pm\sqrt{(K+2)(K-6)}\} = 0$

$\Rightarrow Re\left\{\dfrac{-(K+2)\pm\sqrt{(K+2)(K-6)}}{2}\right\} < 0$

stable

case $6 < K$:

$$
\begin{aligned}
& S \\
&= \frac{-(K+2)\pm\sqrt{(K+2)(K-6)}}{2} \\
&\leq \frac{-(K+2)+\sqrt{(K+2)(K-6)}}{2} \\
&< \frac{-(K+2)+\sqrt{(K+2)(K+2)}}{2} \\
&= 0
\end{aligned}
$$

stable

$\therefore$ smallest value of K is -2

Problem 4

一、

$$x \longrightarrow \boxed{} \longrightarrow y \qquad\qquad ax \longrightarrow \boxed{} \longrightarrow ay$$

$$x_1 \longrightarrow \boxed{} \longrightarrow y_1$$

$$x_2 \longrightarrow \boxed{} \longrightarrow y_2 \qquad\qquad x_1 + x_2 \longrightarrow \boxed{} \longrightarrow y_1 + y_2$$

二、

causal : ROC 在最右邊 pole 的右邊

BIBO stability : ROC 包含虛軸

三、

$$x \longrightarrow \boxed{[A]} \longrightarrow y$$

$$y = Ax = \lambda x$$

x = eigenvector

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = h(t) * x(t) = H(s)\,x(t)$$

$$x(t) = e^{st}$$

Problem 5

$$s^2 Y(s) + 2\beta s Y(s) + \omega_0^2 Y(s) = \alpha X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\alpha}{s^2 + 2\beta s + \omega_0^2}$$

$$H(j\omega) = \frac{\alpha}{-\omega^2 + 2\beta j\omega + \omega_0^2}$$

$$= \frac{\alpha}{(\omega_0^2 - \omega^2) + j(2\beta\omega)}$$

Problem 6

$$h(t) = e^{-t} u(t)$$

$$H(s) = \frac{1}{s+1}$$

$$x(t) = u(t+1) - u(t-1)$$

$$X(s) = \frac{1}{s} e^{+s} - \frac{1}{s} e^{-s}$$

$$y(t) = h(t) * x(t)$$

$$Y(s) = H(s) \times X(s)$$

$$= \frac{1}{s+1} \left(\frac{1}{s} e^{s} - \frac{1}{s} e^{-s} \right)$$

$$= \frac{1}{(s+1)\,s} e^{s} - \frac{1}{(s+1)\,s} e^{-s}$$

$$= \left(\frac{1}{s} - \frac{1}{s+1} \right) e^{s} - \left(\frac{1}{s} - \frac{1}{s+1} \right) e^{-s}$$

$$= \frac{1}{s} e^{s} - \frac{1}{s+1} e^{s} - \frac{1}{s} e^{-s} + \frac{1}{s+1} e^{-s}$$

$$y(t) = u(t+1) - e^{-(t+1)} u(t+1) - u(t-1) + e^{-(t-1)} u(t-1)$$

$$= u(t+1)\left[1 - e^{-(t+1)}\right] - u(t-1)\left[1 - e^{-(t-1)}\right]$$

Problem 7

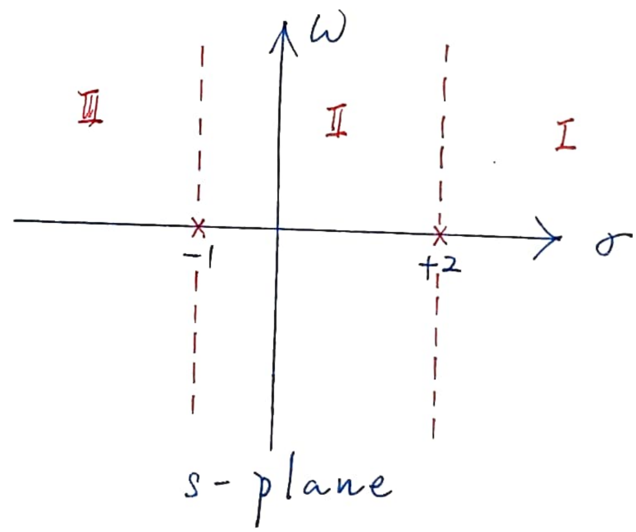
$$H(s) = \frac{s-1}{(s+1)(s-2)}$$

$$= \frac{A}{s+1} + \frac{B}{s-2}$$

$$A = \frac{2}{3}$$

$$B = \frac{1}{3}$$

$$H(s) = \frac{2}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}$$



region III	region II	region I
$h(t) = -\frac{2}{3}e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$	$h(t) = \frac{2}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(-t)$	$h(t) = \frac{2}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$
anti-causal unstable	noncausal stable	causal unstable

Problem 8

$$\sin(\omega_0 t)\, u(t) \xleftrightarrow{LT} \frac{\omega_0}{s^2+\omega_0}, \quad Re\{s\} > 0$$

$$\sin(5t)\, u(t) \xleftrightarrow{LT} \frac{5}{s^2+25}$$

$$\frac{1}{5}\sin(5t)\, u(t) \xleftrightarrow{LT} \frac{1}{s^2+25}$$

$$t^2 \frac{1}{5}\sin(5t)\, u(t) \xleftrightarrow{LT} \frac{d^2}{dt^2}\frac{1}{s^2+25}$$

$$t \cdot \frac{2}{5}\sin(5t)\, u(t)$$
$$+$$
$$t^2 \cdot 1 \cos(5t)\, u(t) \xleftrightarrow{LT} s\, \frac{d^2}{dt^2}\frac{1}{s^2+25}, \quad Re\{s\} > 0$$
$$+$$
$$t^2 \cdot \frac{1}{5}\sin(5t)\, \delta(t)$$

Problem 9

$$h(t) = e \cdot e^{-(t+1)} u(t+1)$$

$$H(s) = e \cdot \frac{1}{s+1} \cdot e^{s}$$

$$= \frac{1}{s+1} e^{s+1}$$

$$x(t) = \sin^2 t$$

$$= \frac{1}{2} - \frac{1}{2} \cos 2t$$

$$= \frac{1}{2} - \frac{1}{2} \frac{e^{j2t} + e^{-j2t}}{2}$$

$$= \frac{1}{2} e^{0t} - \frac{1}{4} e^{(j2)t} - \frac{1}{4} e^{(-j2)t}$$

$$y(t) = \frac{1}{2} H(0) e^{0t} - \frac{1}{4} H(j2) e^{j2t} - \frac{1}{4} H(-j2) e^{-j2t}$$

$$= \frac{1}{2} \frac{1}{1} e^{1} - \frac{1}{4} \frac{1}{j2+1} e^{j2+1} e^{j2t} - \frac{1}{4} \frac{1}{-j2+1} e^{-j2+1} e^{-j2t}$$

Problem 10

$$X(s) = \frac{(s+4)}{(s+2)(s^2+6s+13)}$$

$$= \frac{A}{s+2} + \frac{Bs+C}{s^2+6s+13}$$

$$s+4 = (s^2+6s+13)A + (s+2)(Bs+C)$$

$$= As^2+6As+13A + Bs^2+(2B+C)s+2C$$

$$0s^2+1s+4 = (A+B)s^2 + (6A+2B+C)s + 13A+2C$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 6 & 2 & 1 \\ 13 & 0 & 2 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 6 & 2 & 1 & 1 \\ 13 & 0 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -4 & 1 & 1 \\ 0 & -13 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -4 & 1 & 1 \\ 0 & 0 & 5 & -3 \end{pmatrix}$$

$$5C = -3$$

$$C = -\frac{3}{5}$$

$$-4B + \frac{-3}{5} = 1$$

$$B = \frac{-2}{5}$$

$$A + \frac{-2}{5} = 0$$

$$A = \frac{2}{5}$$

$$X(s) = \frac{2}{5}\frac{1}{s+2} + \frac{-2s-3}{5(s^2+6s+13)}$$

$$x(t) = \frac{2}{5}\left(-e^{-2t}u(-t)\right) + \left(-\frac{2}{5}e^{-3t}\cos(2t)u(t)\right) + \left(\frac{3}{10}e^{-3t}\sin(2t)u(t)\right)$$

選(B)