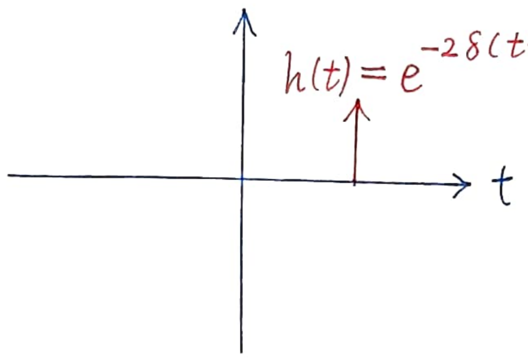
Homework #11 林靖 108061112

Problem 1

$$y(t) = e^{-t}x(t-2)$$

$$h(t) = e^{-t}\delta(t-2)$$

$$h(t) = e^{-2}\delta(t-2)$$



$$\int_{-\infty}^{+\infty} \left| e^{-2}\delta(t-2) \right| dt = e^{-2} < \infty$$

(a) Yes.

(b) Yes.

(c) Yes.

(d) Yes.

Problem 2

(1)

$$e^{-t}u(t) \xleftrightarrow{LT} \frac{1}{s+1}, \quad Re\{s\} > -1$$

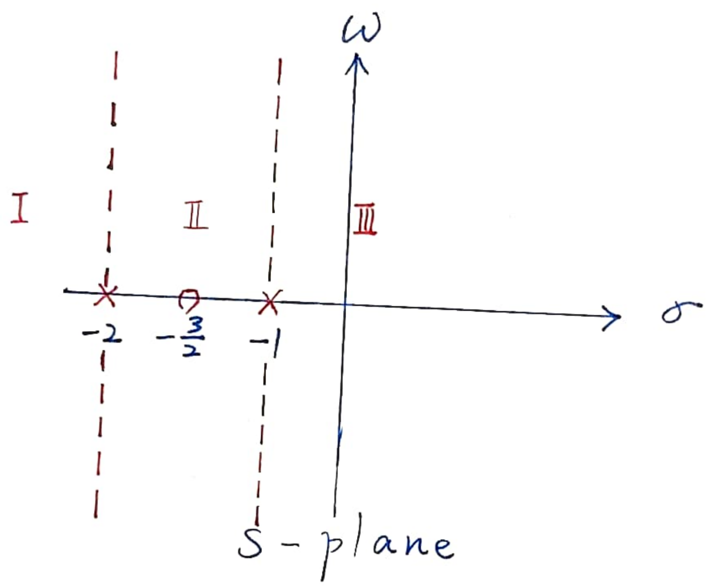
(2)

$$t\,e^{-t}u(t) \xleftrightarrow{LT} \frac{1}{(s+1)^2}, \quad Re\{s\} > -1$$

Problem 3

$$H(s) = \frac{2s+3}{s^2+3s+2}$$

$$= \frac{2\left(s+\frac{3}{2}\right)}{(s+1)(s+2)}$$



region I	region II	region III
anti-causal	non-causal	causal
not BIBO stability	not BIBO stability	BIBO stability

Problem 4

$$h(t) = \left[h_1(t) + h_2(t) \right] * h_3(t)$$

$$H(s) = \left[H_1(s) + H_2(s) \right] \times H_3(s)$$

$$\delta(t) \xleftrightarrow{LT} 1 \quad , \quad \text{R.O.C.} : \text{all s-plane}$$

$$e^{-t} u(t) \xleftrightarrow{LT} \frac{1}{s+1} \quad , \quad \text{R.O.C.} : Re\{s\} > -1$$

$$H(s) = \left(1 + \frac{1}{s+1} \right) \times \frac{1}{s+1}$$

$$H(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2} \quad , \quad \text{R.O.C.} : Re\{s\} > -1$$

$$h(t) = e^{-t} u(t) + t e^{-t} u(t)$$

Problem 5

(一) $$h(t) = 2e^{-t}u(t) - 3e^{-2t}u(t)$$

$$H(s) = 2\frac{1}{s+1} - 3\frac{1}{s+2}$$

$$H(s) = \frac{-s+1}{s^2+3s+2}$$

$$H(j2\pi f) = \frac{-j2\pi f+1}{-4\pi^2 f^2 + j6\pi f + 2}$$

(二) $$Y(j2\pi f) = \frac{1}{2+j2\pi f-3j} + \frac{1}{2+j2\pi f+3j}$$

$$Y(j2\pi f) = \frac{1}{(2-3j)+j2\pi f} + \frac{1}{(2+3j)+j2\pi f}$$

$$y(t) = e^{-(2-3j)t}u(t) + e^{-(2+3j)t}u(t)$$

$$y(t) = e^{-2t}u(t)\left[e^{3jt} + e^{-3jt}\right]$$

$$y(t) = e^{-2t}u(t)\ 2\ \cos 3t$$

Problem 6

(一)

$$\frac{d}{dt} h(t) + 3h(t) = e^{-4t} u(t) + c\, e^{-5t} u(t)$$

$$s H(s) + 3 H(s) = \frac{1}{s+4} + c \frac{1}{s+5}$$

$$H(s) = \frac{1}{s+3} \left(\frac{1}{s+4} + \frac{c}{s+5} \right)$$

$$e^{st} \longrightarrow \boxed{\begin{matrix} \text{C T} \\ \text{L T I} \end{matrix}} \longrightarrow H(s)\, e^{st}$$

$$e^{1t} \longrightarrow \boxed{\begin{matrix} \text{C T} \\ \text{L T I} \end{matrix}} \longrightarrow \frac{2}{15}\, e^{1t}$$

$$H(1) = \frac{2}{15}$$

$$\frac{1}{4} \left(\frac{1}{5} + \frac{c}{6} \right) = \frac{2}{15}$$

$$c = 2$$

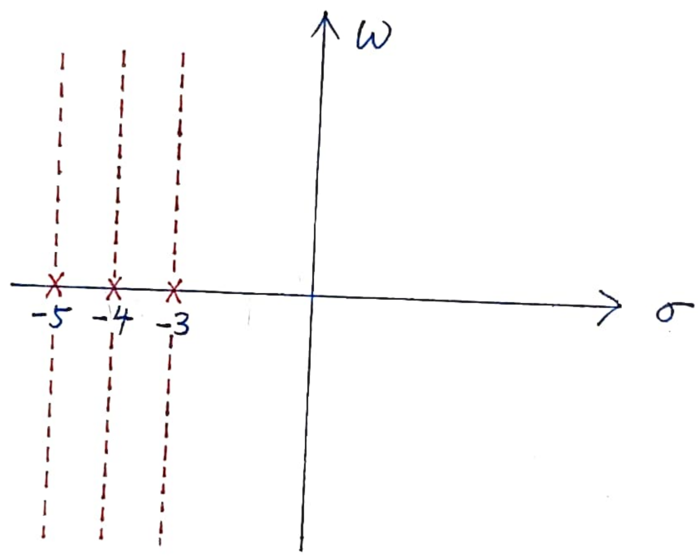
Problem 6 (continued)

(=)

$$H(s) = \frac{1}{s+3}\left(\frac{1}{s+4} + \frac{2}{s+5}\right)$$

$$= \frac{1}{s+3} \frac{(s+5)+2(s+4)}{(s+4)(s+5)}$$

$$= \frac{3s+13}{(s+3)(s+4)(s+5)}$$



s-plane

Problem 6 (continued)

(三)

R.O.C. : $Re\{s\} > -3$

The system is stable

because the R.O.C. contains the imaginary axis.

Problem 7

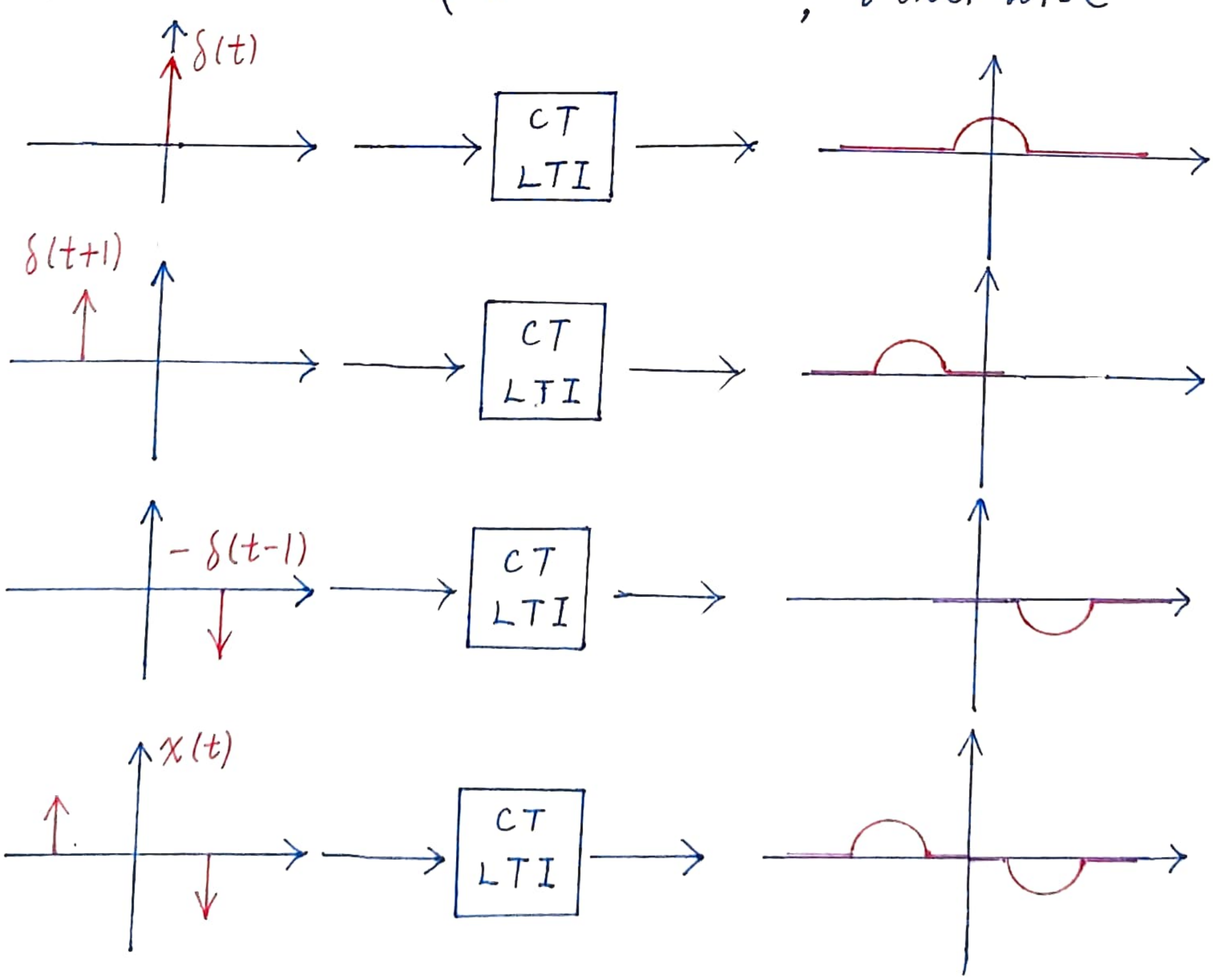
$$y(t) = h(t) * x(t)$$

$$= h(t) * \delta(t+1) - h(t) * \delta(t-1)$$

$$= h(t+1) - h(t-1)$$

$$= \begin{cases} \cos[\pi(t+1)], & |t+1| < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

$$- \begin{cases} \cos[\pi(t-1)], & |t-1| < 0.5 \\ 0, & \text{otherwise} \end{cases}$$



Problem 8

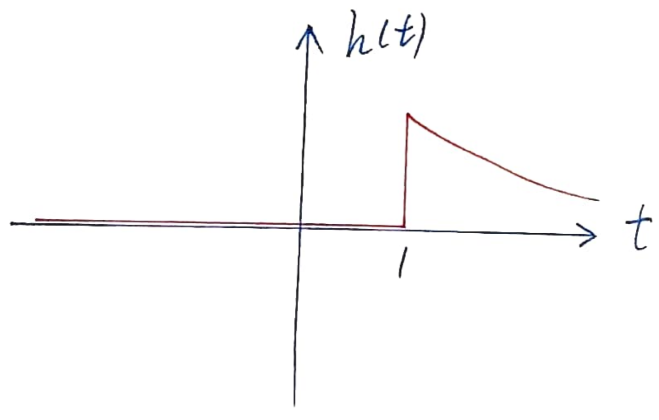
$$h(t) = \int_{-\infty}^{t} e^{\tau - t}\, \delta(\tau - 1)\, d\tau$$

$$= \int_{-\infty}^{t} e^{1-t}\, \delta(\tau - 1)\, d\tau$$

$$= e^{1-t} \int_{-\infty}^{t} \delta(\tau - 1)\, d\tau$$

$$= e^{1-t}\, u(t-1)$$

Yes, the syetem is causal.



Problem 9

(a) $$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

$$Y(s) = \frac{2}{3}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}$$

$$Re\{s\} < 2 \qquad Re\{s\} > -1$$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$= \frac{\frac{2}{3}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}}{\frac{s+2}{s-2}}$$

$$= \frac{s-2}{s+2}\,\frac{\frac{2}{3}(s+1)+\frac{1}{3}(s-2)}{(s-2)(s+1)}$$

$$= \frac{s}{(s+2)(s+1)}, \quad Re\{s\} > -1$$

$$= \frac{2(s+1)-(s+2)}{(s+2)(s+1)}$$

$$= \frac{2}{s+2} - \frac{1}{s+1}$$

$$h(t) = 2e^{-2t}u(t) - e^{-t}u(t)$$

Problem 9

$$e^{st} \longrightarrow \boxed{\begin{matrix}\text{CT}\\ \text{LTI}\end{matrix}} \longrightarrow H(s)\, e^{st}$$

$$e^{-3t} \longrightarrow \boxed{\begin{matrix}\text{CT}\\ \text{LTI}\end{matrix}} \longrightarrow H(-3)\, e^{-3t}$$

$$x(t) = e^{-3t}$$

$$\begin{aligned} y(t) &= H(-3)\, e^{-3t} \\ &= \frac{(-3)}{(-1)(-2)}\, e^{-3t} \\ &= \frac{-3}{2}\, e^{-3t} \end{aligned}$$

Problem 10

not linear

because frequency changed.