Homework #4 108061112

Problem 1

(1)

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_1(t-nT)$$

$$= \sum_{n=-\infty}^{+\infty} \left[x_1(t) * \delta(t-nT)\right]$$

$$= x_1(t) * \sum_{n=-\infty}^{+\infty} \delta(t-nT)$$

where $T = 2$.

(2)

$$X_1(f) = \int_{-\infty}^{+\infty} rect(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} (1)\, e^{-j2\pi ft}\, dt$$

$$= \frac{1}{-j2\pi f} \cdot \left(e^{-j2\pi f(\frac{1}{2})} - e^{-j2\pi f(-\frac{1}{2})}\right)$$

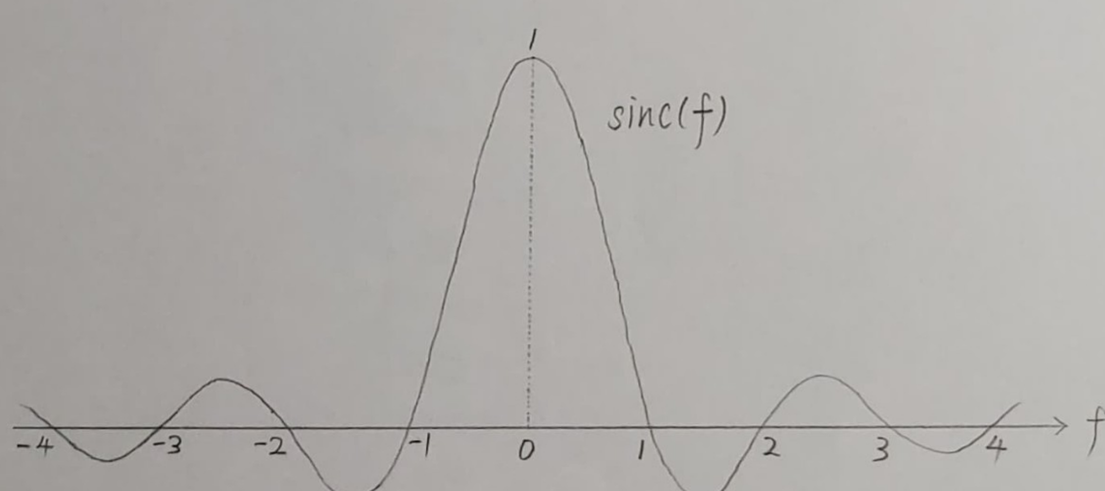
$$= \frac{1}{-j2\pi f}\left[e^{j(-\pi f)} - e^{j(\pi f)}\right]$$

$$= \frac{1}{-j2\pi f}\left\{\left[\cos(-\pi f) + j\sin(-\pi f)\right] - \left[\cos(\pi f) + j\sin(\pi f)\right]\right\}$$

$$= \frac{1}{-j2\pi f}\left[-j2\sin(\pi f)\right]$$

$$= \frac{\sin(\pi f)}{\pi f}$$

$$= sinc(f)$$

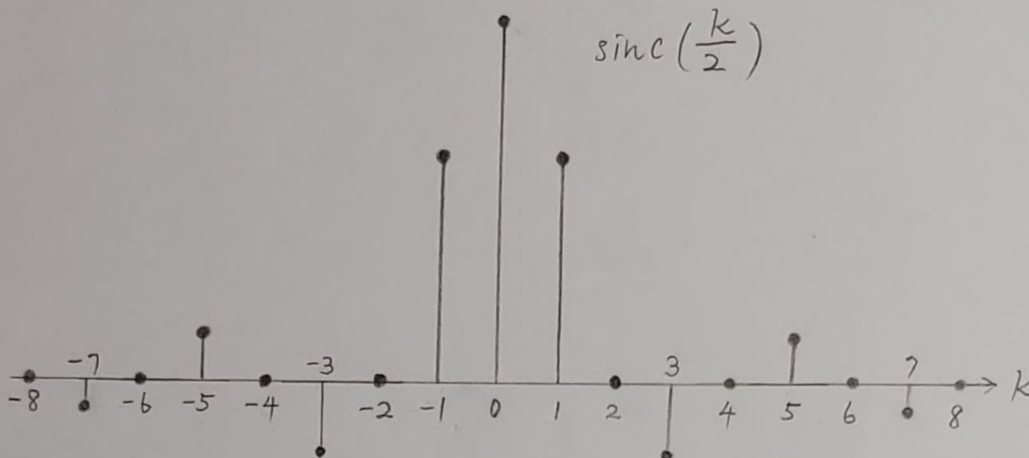


Problem 1 (continued)

(3) $X_p[k] = X_1\left(\frac{k}{T}\right)$

$$= \text{sinc}\left(\frac{k}{T}\right)$$

where $T = 2$



(4)

$X_1(f)=\text{sinc}(f)$ 是連續的訊號，而 $X_p[k]=\text{sinc}\left(\frac{k}{2}\right)$ 是離散的。

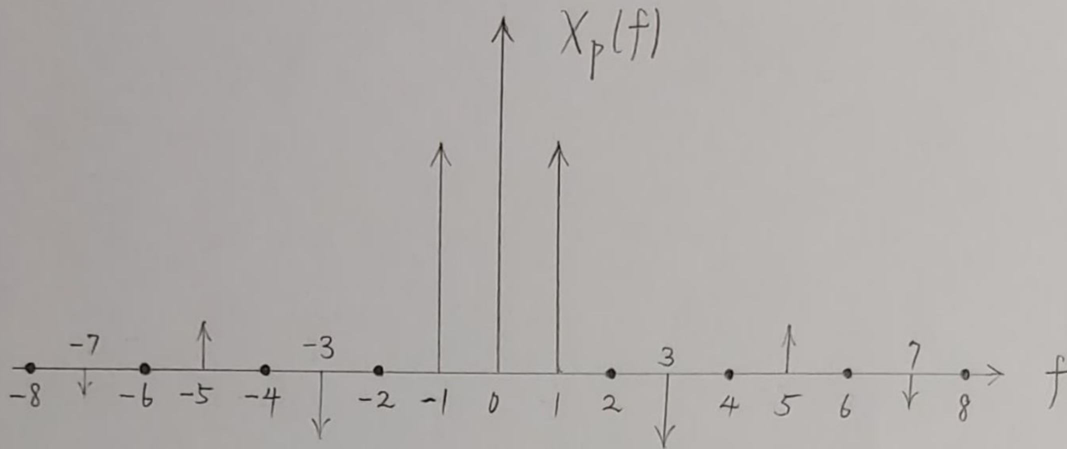
將 $X_1(f)$ 沿橫軸拉伸兩倍，再 point sampling 後得 $X_p[k]$。

Problem 1 (continued)

(5) $x_p(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} X_p[k] \, e^{+j2\pi \frac{k}{T} t}$

$$X_p(f) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} X_p[k] \, \delta(f - \frac{k}{T})$$

$$= \sum_{k=-\infty}^{+\infty} \frac{1}{2} \operatorname{sinc}(\frac{k}{2}) \, \delta(f - \frac{k}{2})$$



(6)

$X_p[k]$ 是離散的，沒有面積。

$X_p(f)$ 是連續的，有面積。

將 $X_1(f)$ 沿橫軸拉伸兩倍，再 impulse sampling 得 $X_p(f)$。

Problem 2

(1) for $t \neq 0$, $\delta(t) = 0$,

$$\delta(t)\,x(t) = 0 \times x(t) = 0,$$

$$\delta(t)\,x(0) = 0 \times x(0) = 0.$$

for $t = 0$, $\delta(t)\,x(t) = \delta(t)\,x(0)$.

$\blacksquare$

(2) for $t-a \neq 0$, $\delta(t-a) = 0$,

$$\delta(t-a)\,x(t) = 0 \times x(t) = 0,$$

$$\delta(t-a)\,x(a) = 0 \times x(a) = 0.$$

for $t-a = 0$, $\delta(t-a)\,x(t) = \delta(t-a)\,x(a)$.

$\blacksquare$

Problem 2 (continued)

$$(3)\ x(t) * \delta(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{+\infty} x(\tau)\,\delta(\tau-t)\,d\tau$$

$$= \int_{-\infty}^{+\infty} x(t)\,\delta(\tau-t)\,d\tau$$

$$= x(t) \int_{-\infty}^{+\infty} \delta(\tau-t)\,d\tau$$

$$= x(t) \cdot 1$$

$$(4)\ x(t) * \delta(t-a) = \int_{-\infty}^{+\infty} x(\tau)\,\delta((t-a)-\tau)\,d\tau$$

$$= \int_{-\infty}^{+\infty} x(\tau)\,\delta(\tau-(t-a))\,d\tau$$

$$= \int_{-\infty}^{+\infty} x(t-a)\,\delta(\tau-(t-a))\,d\tau$$

$$= x(t-a) \int_{-\infty}^{+\infty} \delta(\tau-(t-a))\,d\tau$$

$$= x(t-a) \cdot 1$$

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Problem 3

$x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)$

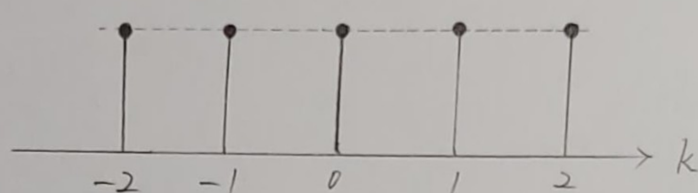
(1) $X[k] = \int_{0^-}^{T^-} \sum_{n=-\infty}^{+\infty} \delta(t-nT) e^{-j2\pi\frac{k}{T}t} dt$

$= \int_{0^-}^{T^-} \delta(t) e^{-j2\pi\frac{k}{T}t} dt$

$= \int_{0^-}^{T^-} \delta(t) e^{-j2\pi\frac{k}{T}\cdot 0} dt$

$= \int_{0^-}^{T^-} \delta(t)\, dt$

$= 1$



(2) $x(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{T}(1) e^{+j2\pi\frac{k}{T}t}$

$X(f) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} \delta(f - \frac{k}{T})$