Problem 1

$x(t) = rect(t)$

$$X(f) = \int_{-\infty}^{+\infty} rect(t) e^{-j2\pi f t} dt$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} 1 \, e^{-j2\pi f t} dt$$

$$= \frac{1}{-j2\pi f} (e^{-j2\pi f(\frac{1}{2})} - e^{-j2\pi f(-\frac{1}{2})})$$

$$= \frac{1}{-j2\pi f} [e^{j(-\pi f)} - e^{j(\pi f)}]$$

$$= \frac{1}{-j2\pi f} \{[\cos(-\pi f) + j\sin(-\pi f)] - [\cos(\pi f) + j\sin(\pi f)]\}$$

$$= \frac{1}{-j2\pi f} [-2j\sin(\pi f)]$$

$$= \frac{\sin(\pi f)}{\pi f}$$

$$= sinc(f) \quad \blacksquare$$

$x(t) = \delta(t)$

$$X(f) = \int_{-\infty}^{+\infty} \delta(t) e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{+\infty} \delta(t) e^{-j2\pi f \cdot 0} dt$$

$$= \int_{-\infty}^{+\infty} \delta(t) \cdot 1 \, dt$$

$$= 1 \quad \blacksquare$$

$x(t) = e^{-at} u(t)$

$$X(f) = \int_{-\infty}^{+\infty} e^{-at} u(t) e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{+\infty} u(t) e^{(-a-j2\pi f)t} dt$$

$$= \int_{0}^{+\infty} e^{(-a-j2\pi f)t} dt$$

$$= \frac{1}{-a-j2\pi f} [0 - 1]$$

$$= \frac{1}{a+j2\pi f} \quad \blacksquare$$

Problem 2

$$x[n]=\begin{cases}1, & |n|\le M\\ 0, & \text{else}\end{cases}$$

$$X(f)=\sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi fn}$$

$$=\sum_{n=-M}^{+M}\left[e^{-j2\pi f}\right]^n$$

$$=\frac{e^{-j2\pi f(-M)}-e^{-j2\pi f(M+1)}}{1-e^{-j2\pi f}}$$

since
$$\sum_{n=-M}^{+M} r^n=\sum_{n=0}^{+M} r^{+n}+\sum_{n=0}^{+M} r^{-n}-r^0$$

$$=\frac{1-r^{M+1}}{1-r}+\frac{1-r^{-(M+1)}}{1-r^{-1}}-1$$

$$=\frac{1-r^{M+1}}{1-r}+\frac{r^{-M}-r}{1-r}+\frac{r-1}{1-r}$$

$$=\frac{r^{-M}-r^{M+1}}{1-r}$$ ▨

$$x[n]=\delta[n]$$

$$X(f)=\sum_{n=-\infty}^{+\infty}\delta[n]e^{-j2\pi fn}$$

$$=\delta[0]e^{-j2\pi f\cdot 0}$$

$$=1$$ ▨

$$x[n]=a^n u[n]$$

$$X(f)=\sum_{n=-\infty}^{+\infty} a^n u[n] e^{-j2\pi fn}$$

$$=\sum_{n=0}^{+\infty} a^n e^{-j2\pi fn}$$

$$=\sum_{n=0}^{+\infty}\left[ae^{-j2\pi f}\right]^n$$

$$=\frac{1}{1-ae^{-j2\pi f}}$$ ▨

Problem 3

$$x(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

$$X[k]=\int_{0^-}^{T^-}\sum_{n=-\infty}^{+\infty}\delta(t-nT)e^{-j2\pi\frac{k}{T}t}dt$$

$$=\int_{0^-}^{T^-}\delta(t)e^{-j2\pi\frac{k}{T}t}dt$$

$$=\int_{0^-}^{T^-}\delta(t)e^{-j2\pi\frac{k}{T}\cdot 0}dt$$

$$=\int_{0^-}^{T^-}\delta(t)\cdot 1\cdot dt$$

$$=1$$

Problem 4

$$x[n]=\sum_{l=-\infty}^{+\infty}\delta[n-lN]$$

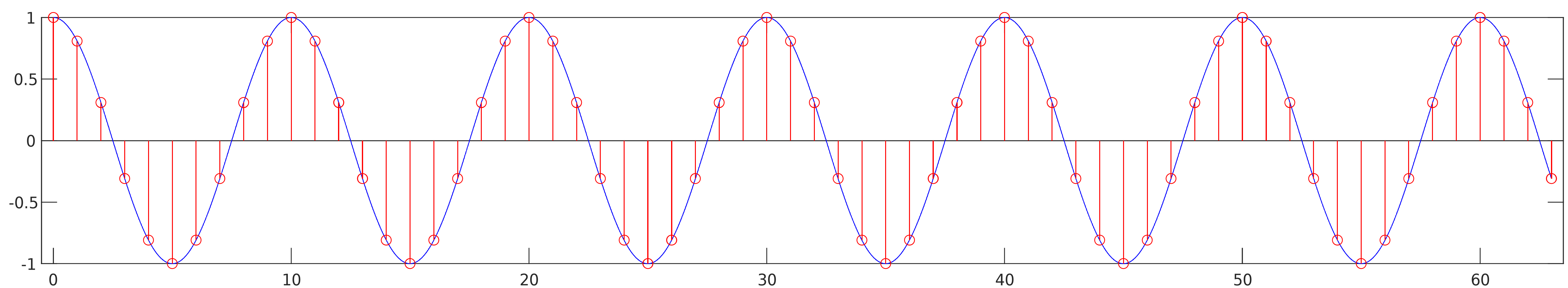
$$X[k]=\sum_{n=0}^{N-1}\sum_{l=-\infty}^{+\infty}\delta[n-lN]e^{-j2\pi\frac{k}{N}n}$$

$$=\sum_{n=0}^{N-1}\delta[n]e^{-j2\pi\frac{k}{N}n}$$

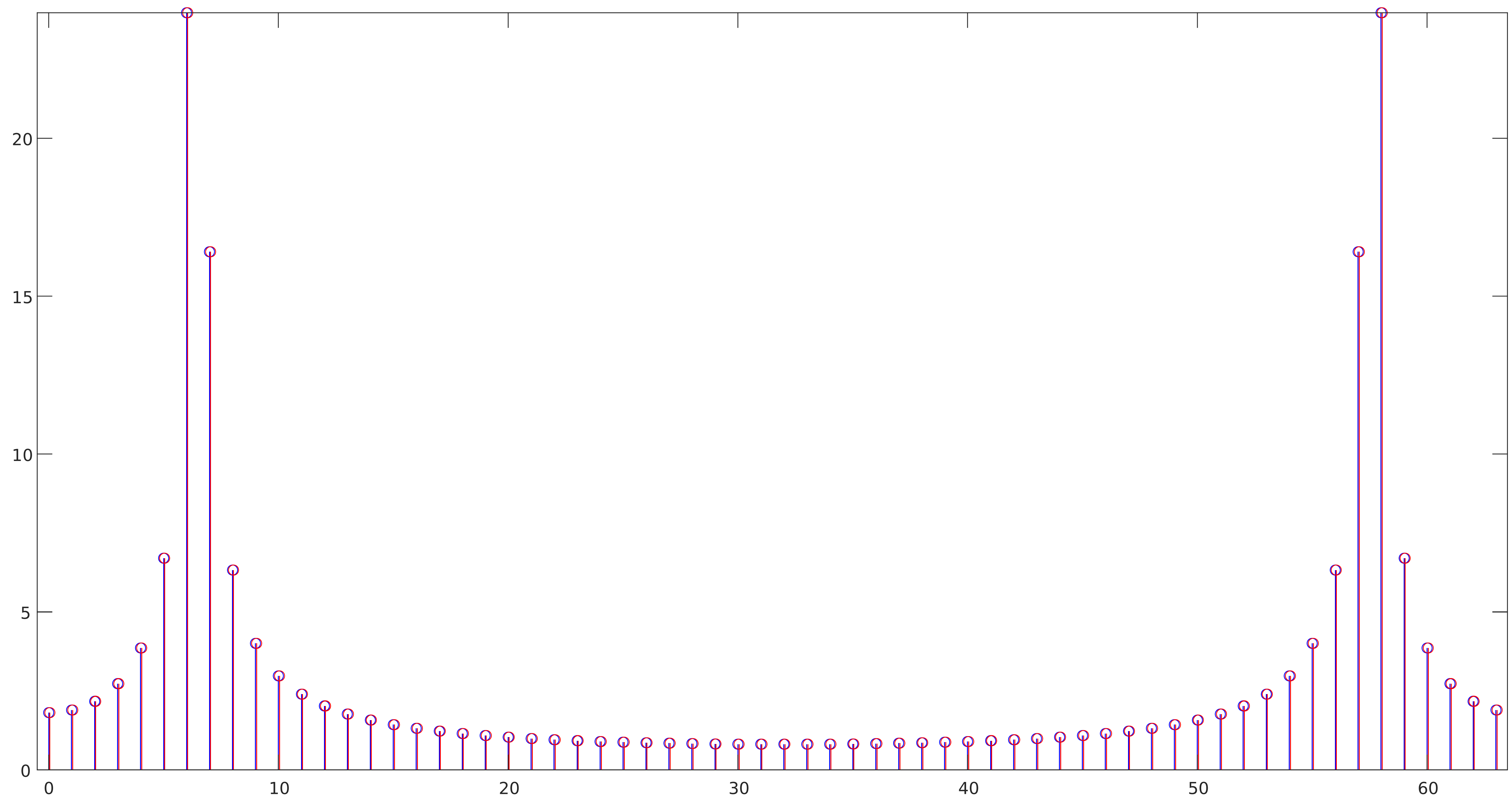
$$=\delta[0]e^{-j2\pi\frac{k}{N}\cdot 0}$$

$$=1$$

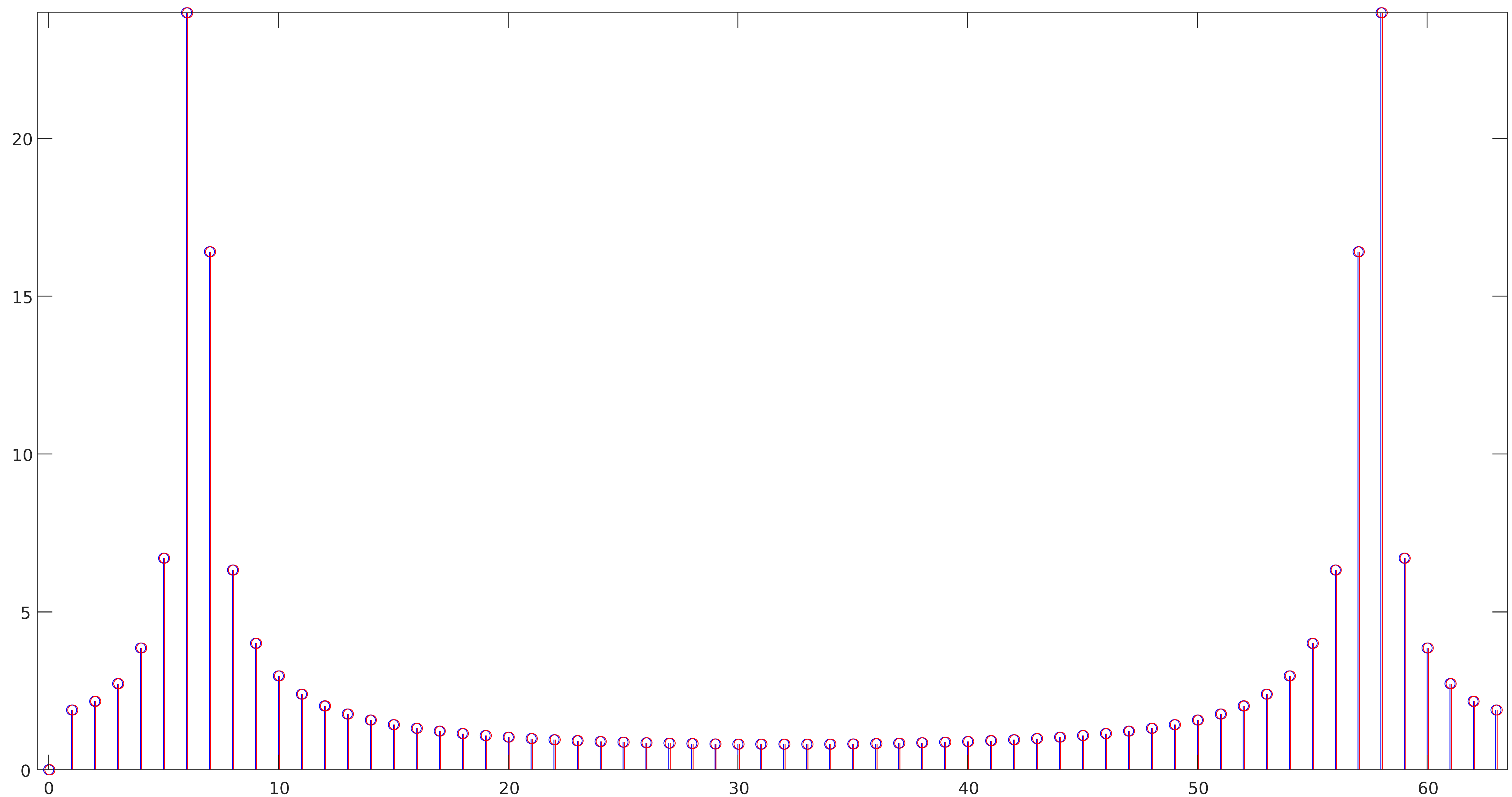
108061112, Homework #03, Problem 5, Part I, (a), f = 0.1, L = 64, (1) 圖壹 x(t) and x[n]



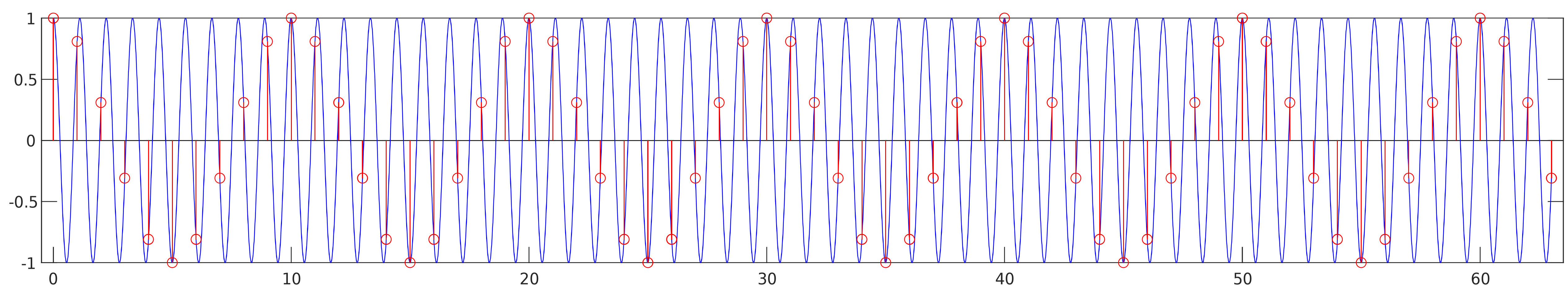
108061112, Homework #03, Problem 5, Part I, (a), f = 0.1, L = 64, (2) 圖貳 DFT of x[n]



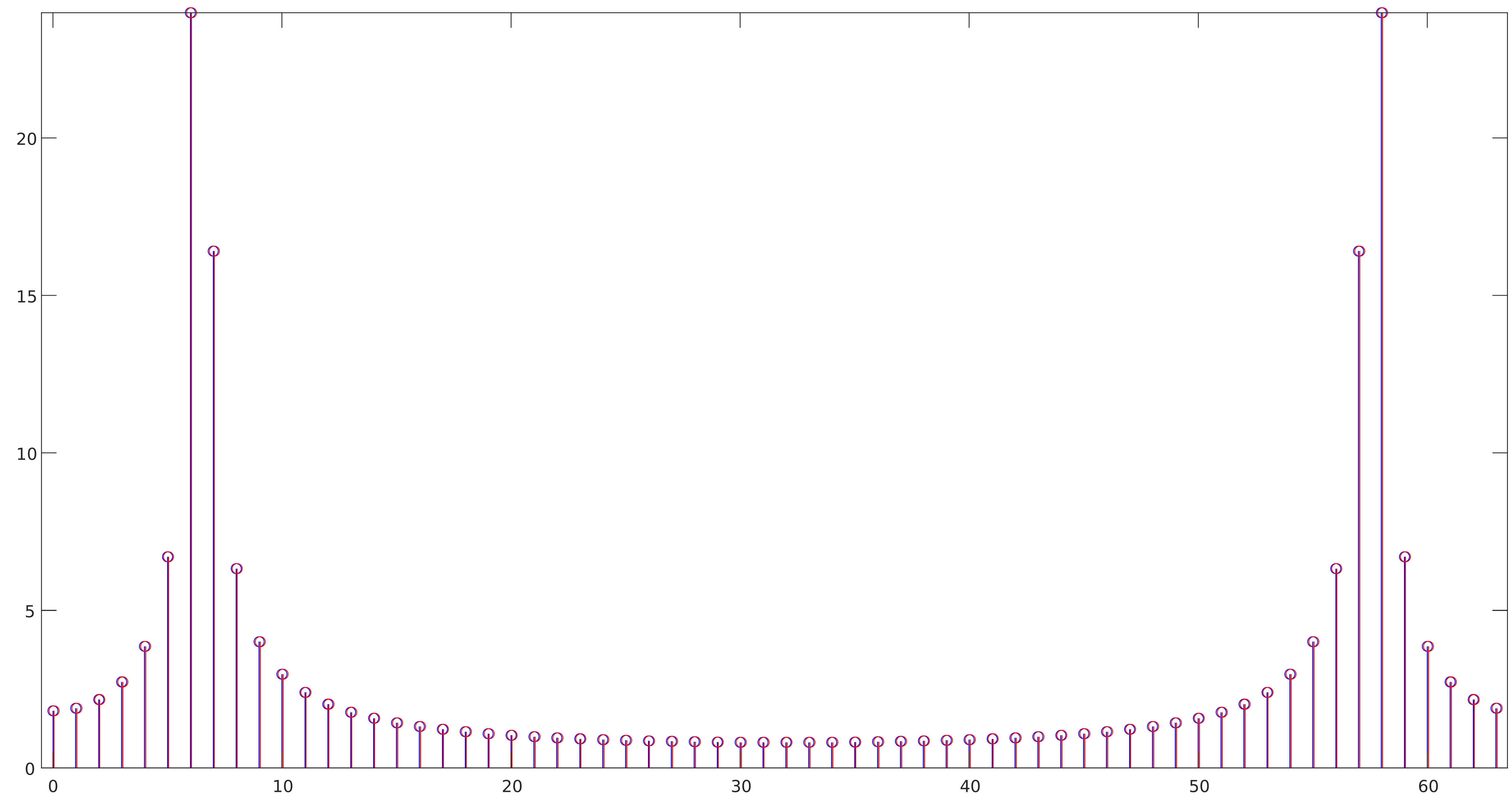
108061112, Homework #03, Problem 5, Part I, (a), f = 0.1, L = 64, (2) 圖參 zero-mean DTFT of x[n]



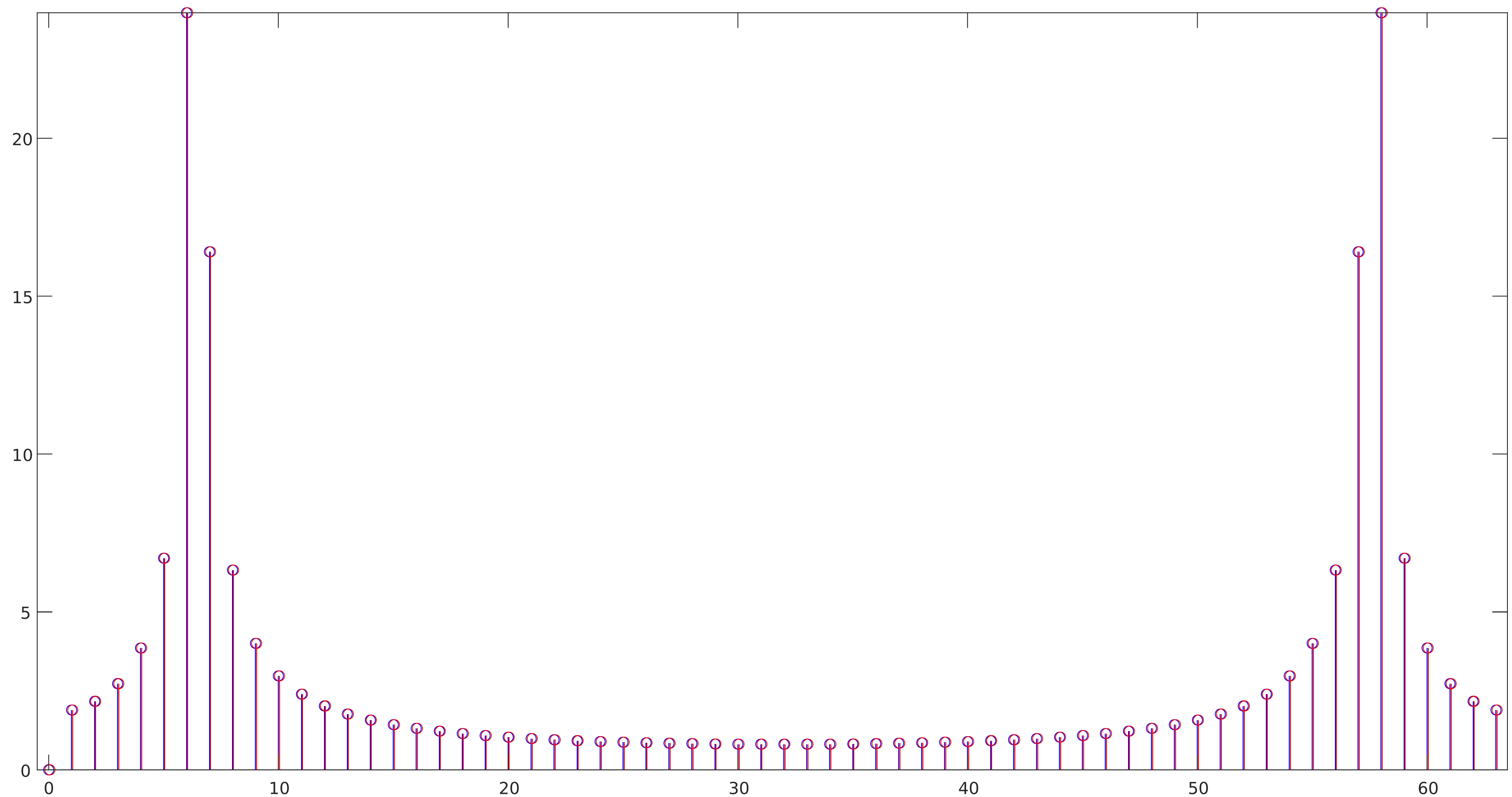
108061112, Homework #03, Problem 5, Part I, (b), f = 0.9, L = 64, (1) 圖肆 x(t) and x[n]



108061112, Homework #03, Problem 5, Part I, (b), f = 0.9, L = 64, (2) 圖伍 DFT of x[n]



108061112, Homework #03, Problem 5, Part I, (b), f = 0.9, L = 64, (2) 圖陸 zero-mean DTFT of x[n]



108061112, Homework #03

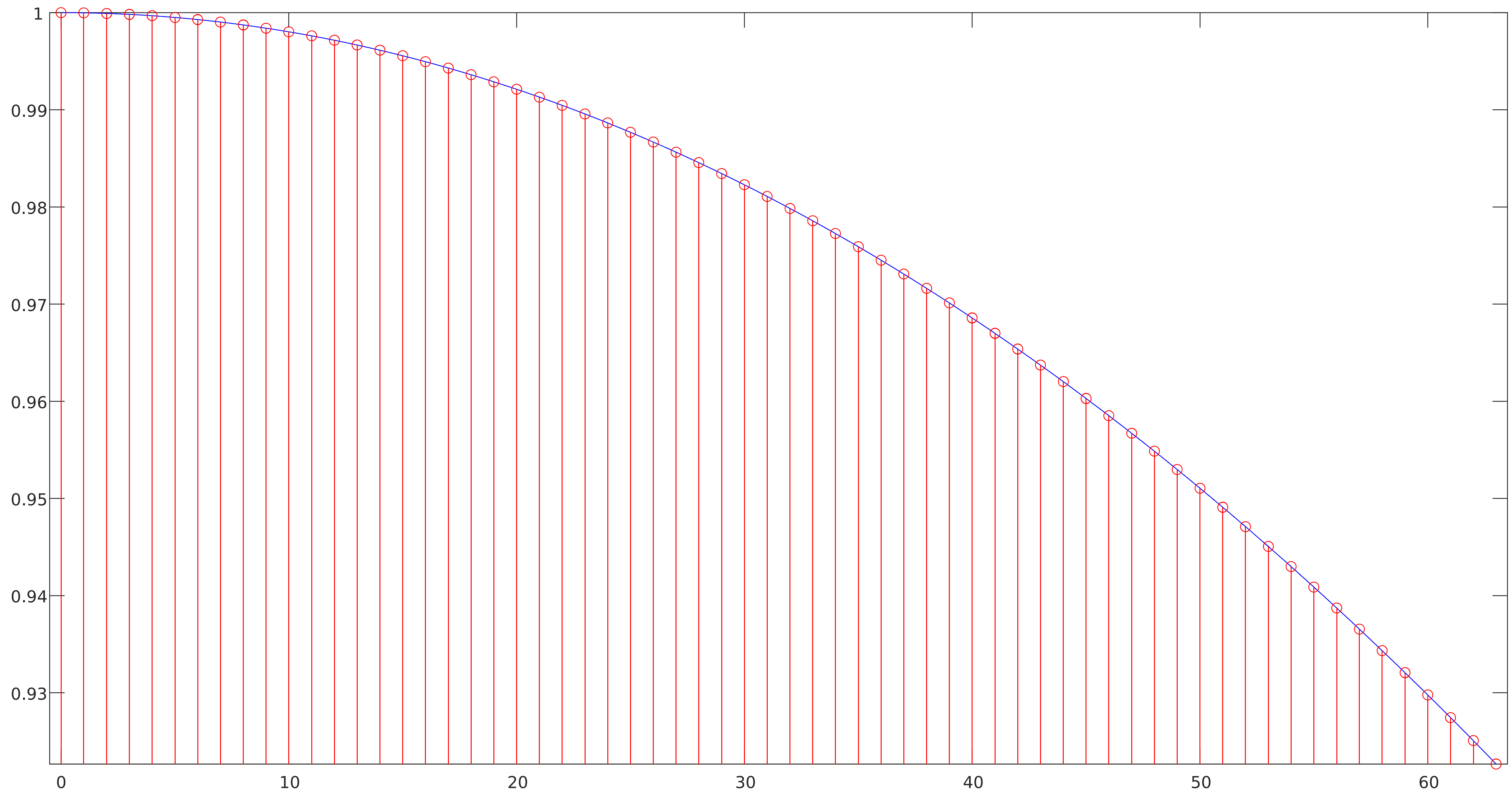
Problem 5, Part I, (c) Compare the results of (a) and (b)

圖壹和圖肆的 x(t) 頻率不同，
但 sample 所得的 x[n] 一模一樣。

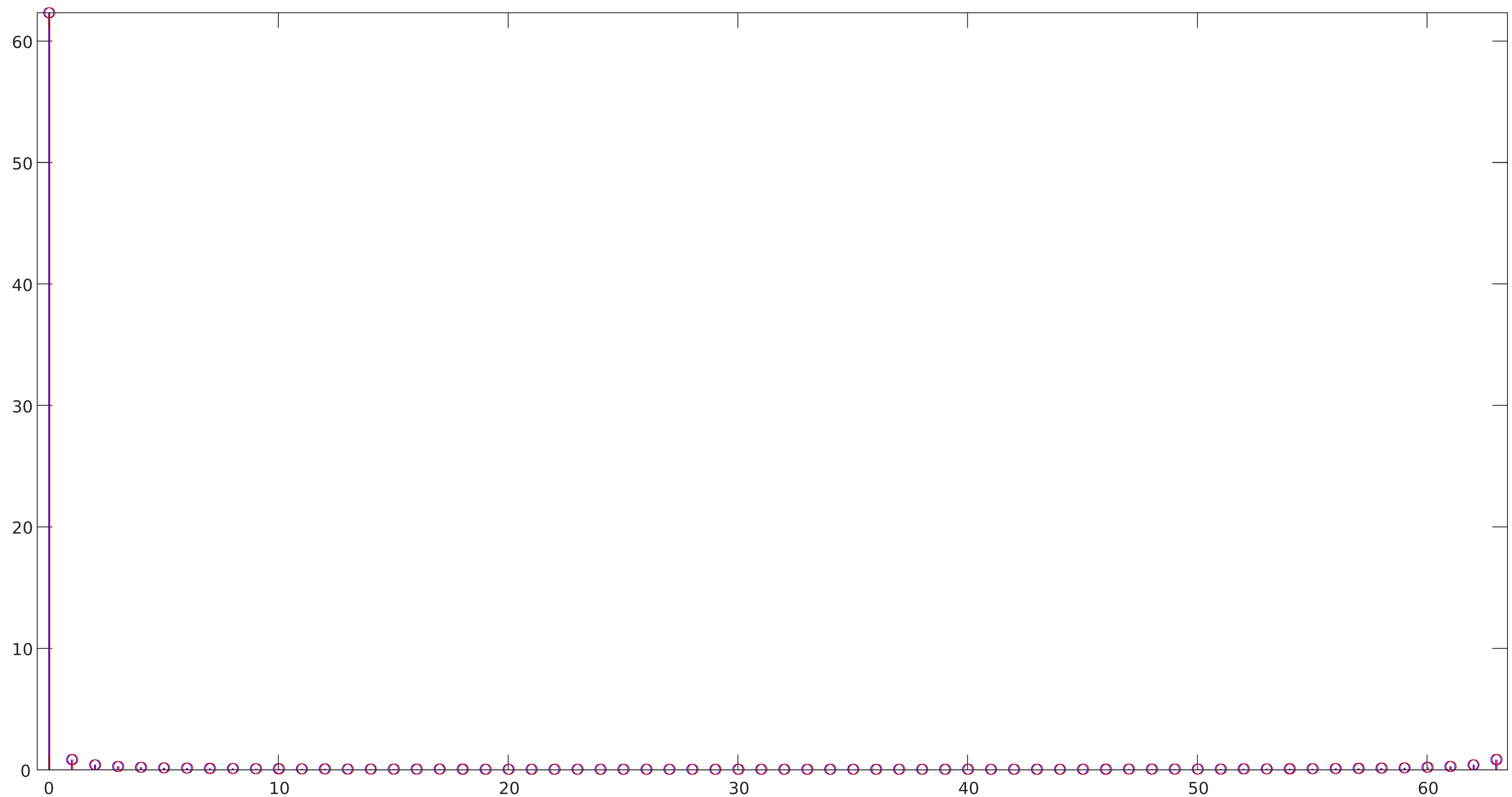
因此圖貳和圖伍一模一樣，
圖參和圖陸也一模一樣。

圖貳和圖參的差異只有 x[0] 。
圖貳的 x[0] 非零，
而圖參的 x[0] = 0 。

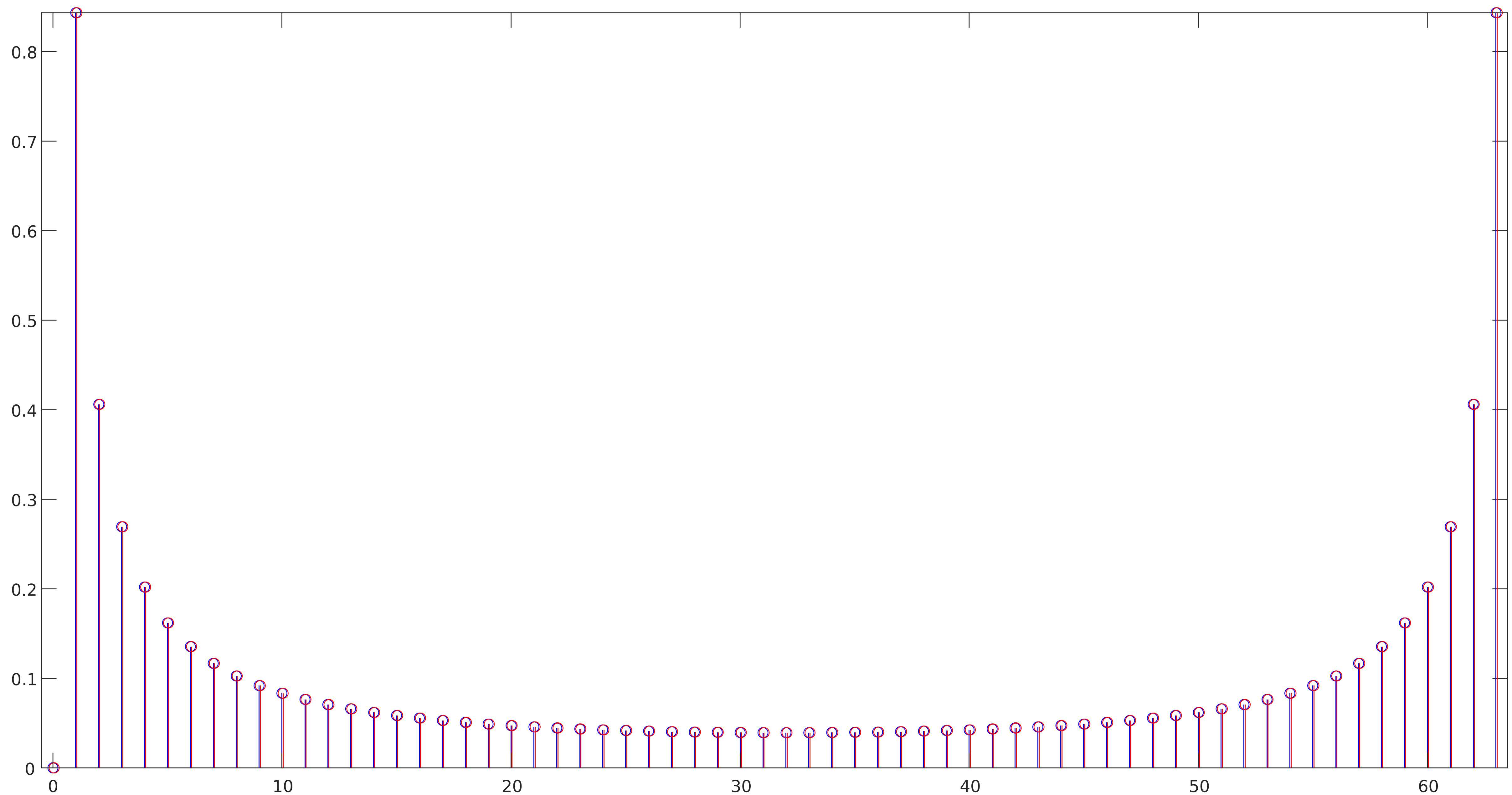
108061112, Homework #03, Problem 5, Part II, (a), f = 0.001, L = 64, (1) 圖柒 x(t) and x[n]



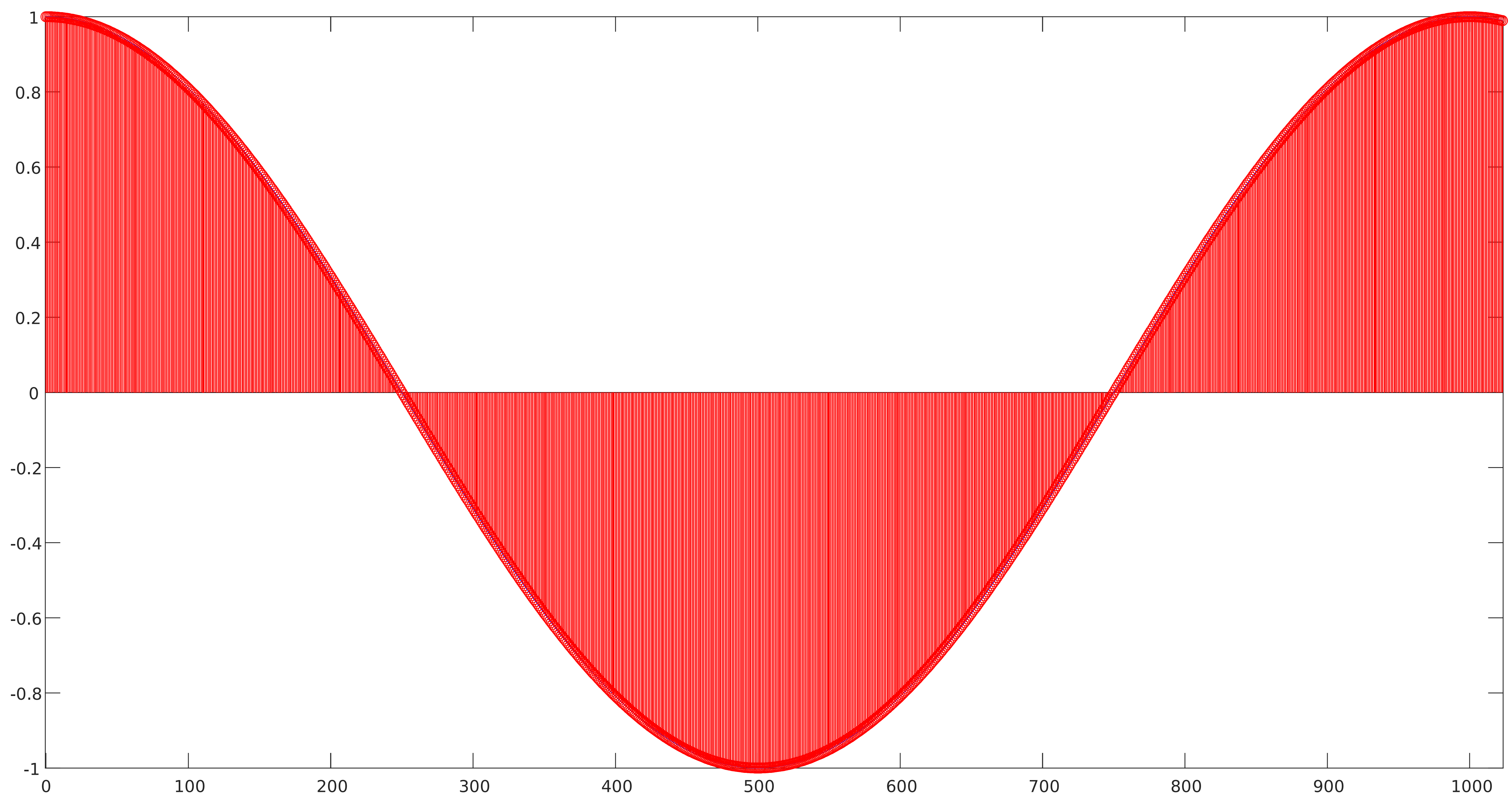
108061112, Homework #03, Problem 5, Part II, (a), f = 0.001, L = 64, (2) 圖捌 DFT of x[n]



108061112, Homework #03, Problem 5, Part II, (a), f = 0.001, L = 64, (2) 圖玖 zero-mean DTFT of x[n]

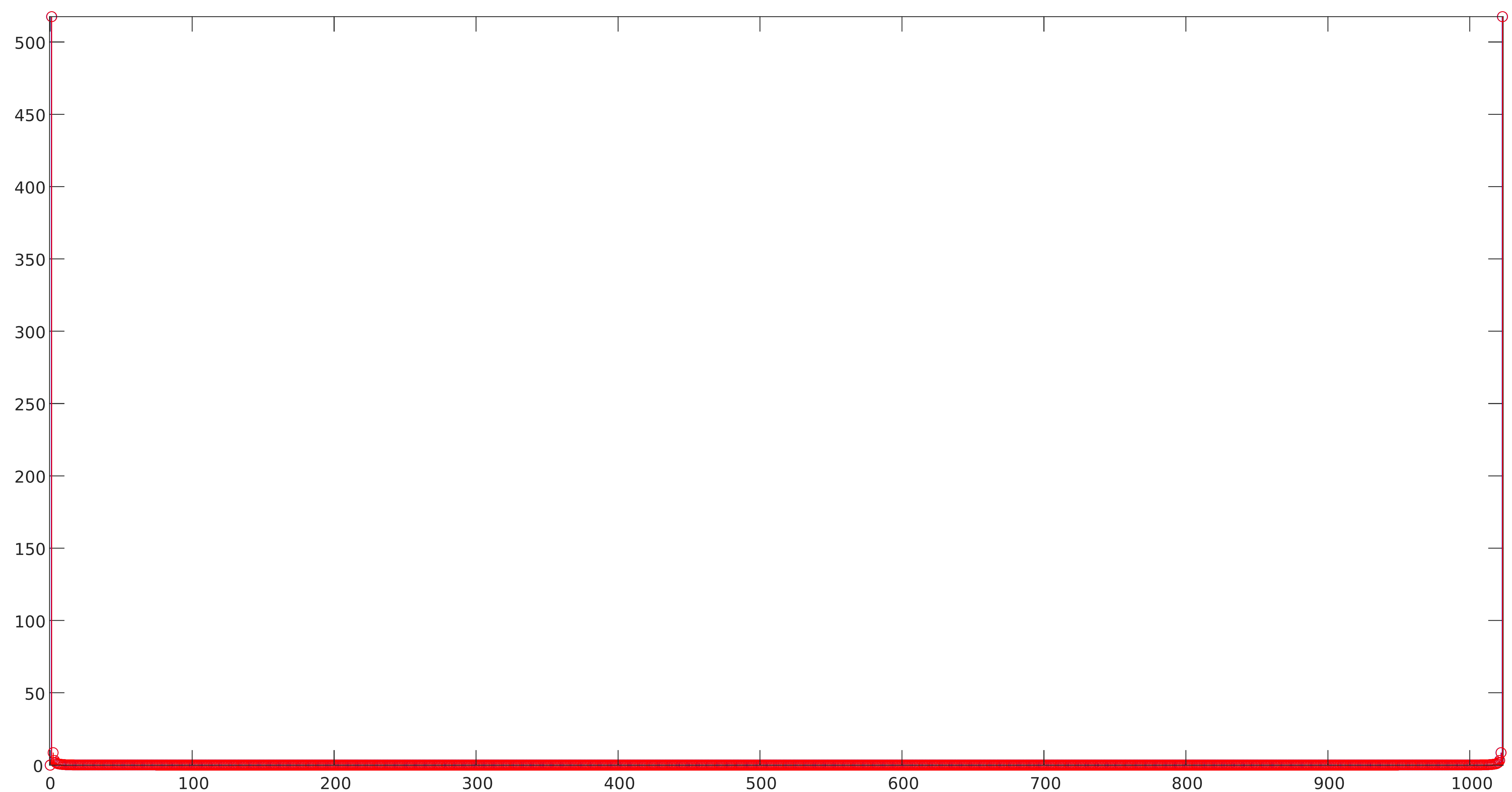


108061112, Homework #03, Problem 5, Part II, (b), f = 0.001, L = 1024, (1) 圖拾 x(t) and x[n]



108061112, Homework #03, Problem 5, Part II, (b), f = 0.001, L = 1024, (2) 圖拾壹 DFT of x[n]

108061112, Homework #03, Problem 5, Part II, (b), f = 0.001, L = 1024, (2) 圖拾貳 zero-mean DTFT of x[n]



108061112, Homework #03

Problem 5, Part II, (c) Compare the results of (a) and (b)

圖柒和圖拾的 x(t) 頻率相同。

圖拾壹和圖拾貳的差異只有 x[0] 。
圖拾壹的 x[0] 非零，
而圖拾貳的 x[0] = 0 。