Final Exam

January 16, 2014

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1. Determine the bilateral Laplace transform and the corresponding region of convergence (ROC) or the inverse Laplace transform for the following signals:

   (1) $x(t)=e^{-t}(\sin t)u(t)$. (5%)

   (2) $X(s)=\dfrac{d}{ds}\left(\dfrac{e^{-3s}}{s}\right)$ with ROC $\mathrm{Re}\{s\}>0$. (5%)

2. Consider a continuous-time linear time-invariant (LTI) system with system function

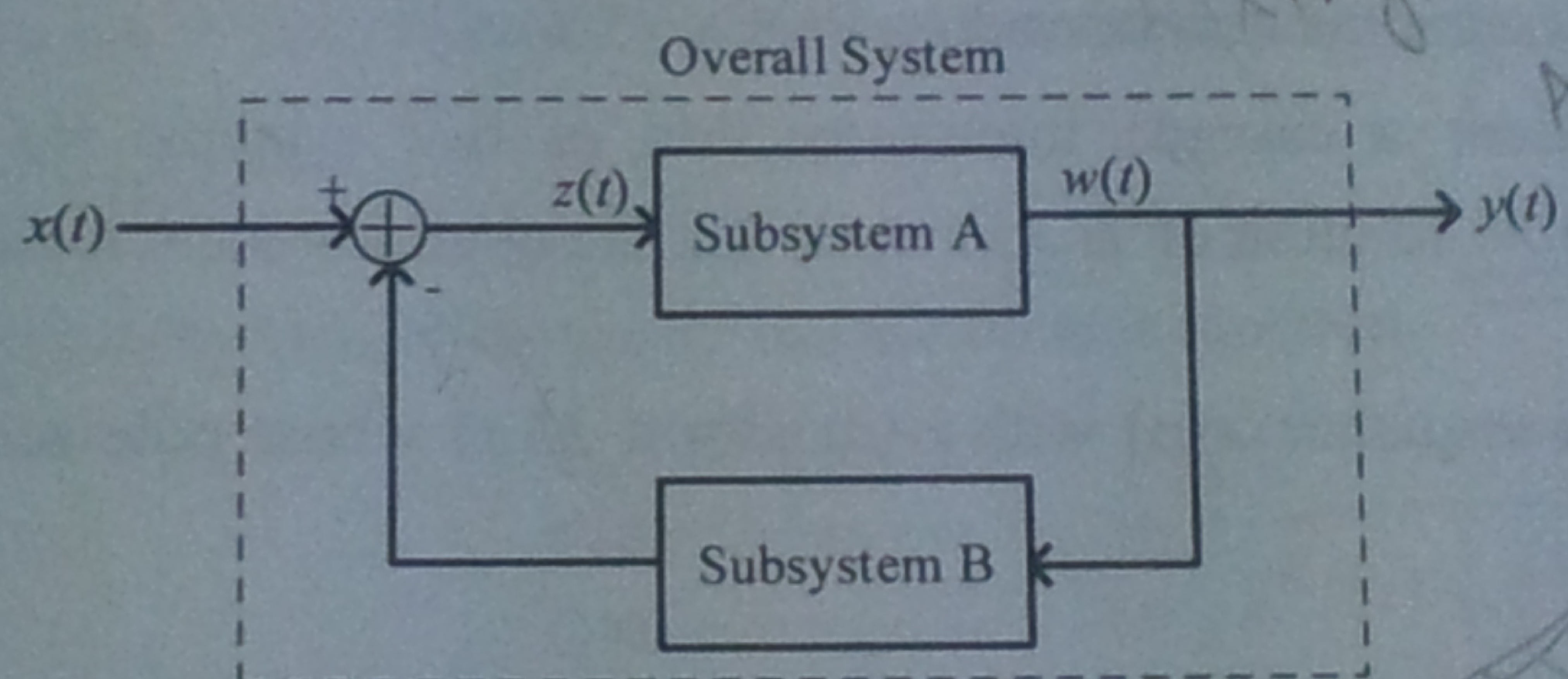
$$H(s)=\frac{s^2-2s+1}{s^2-s-2}.$$

   (1) Plot the poles and zeros of $H(s)$, and indicate all possible ROCs. (4%)

   (2) For each ROC identified in part (1), specify whether the associated system is stable and/or causal. (4%)

   (3) Determine the impulse response $h_{inv}(t)$ of the corresponding stable inverse system. (6%)

3. Consider the following system:



The input-output relation of the causal Subsystem A is given by

$$\frac{dw(t)}{dt}+aw(t)=\frac{dz(t)}{dt}-z(t),$$

and the causal Subsystem B has the impulse response $h_B=e^{-t}u(t)$.

   (1) Show that the overall system function can be written as

$$H(s)=\frac{Y(s)}{X(s)}=\frac{H_A(s)}{1+H_A(s)H_B(s)}.\quad (3\%)$$

   (2) Determine a such that the overall impulse response is $h(t)=\delta(t)-2e^{-t}u(t)$. (4%)

   (3) Find the causal input $x(t)$ that could produce the output $y(t)=e^{-2t}u(t)$. (4%)

4. Let $B(s)$ be the transfer function of a causal and stable Butterworth filter of order 2 and

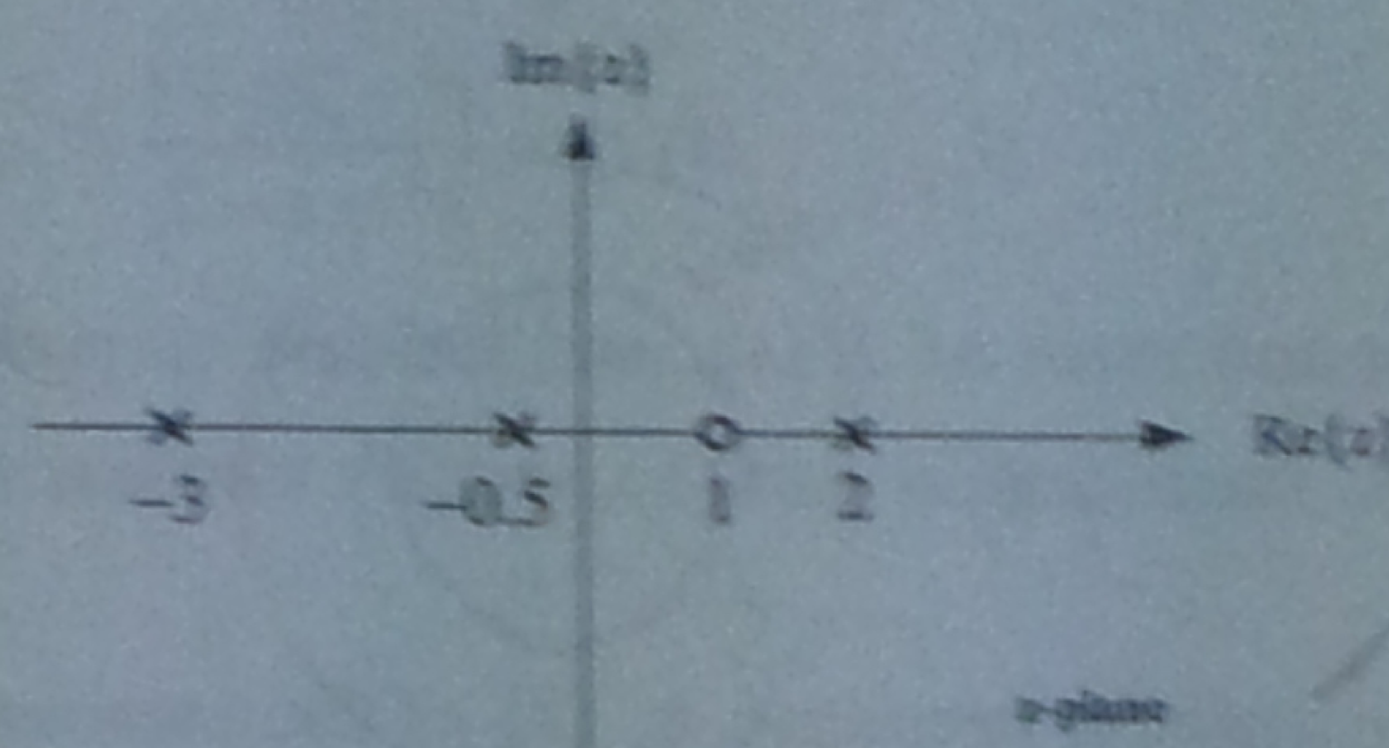
$$B(s)B(-s)=\frac{1}{1+(s/2j)^4}.$$

(1) Plot the poles and zeros of $B(s)B(-s)$. (3%)

(2) Determine the transfer function $B(s)$. (3%)

(3) Plot the frequency response of the filter and indicate the 3-dB frequency. (4%)

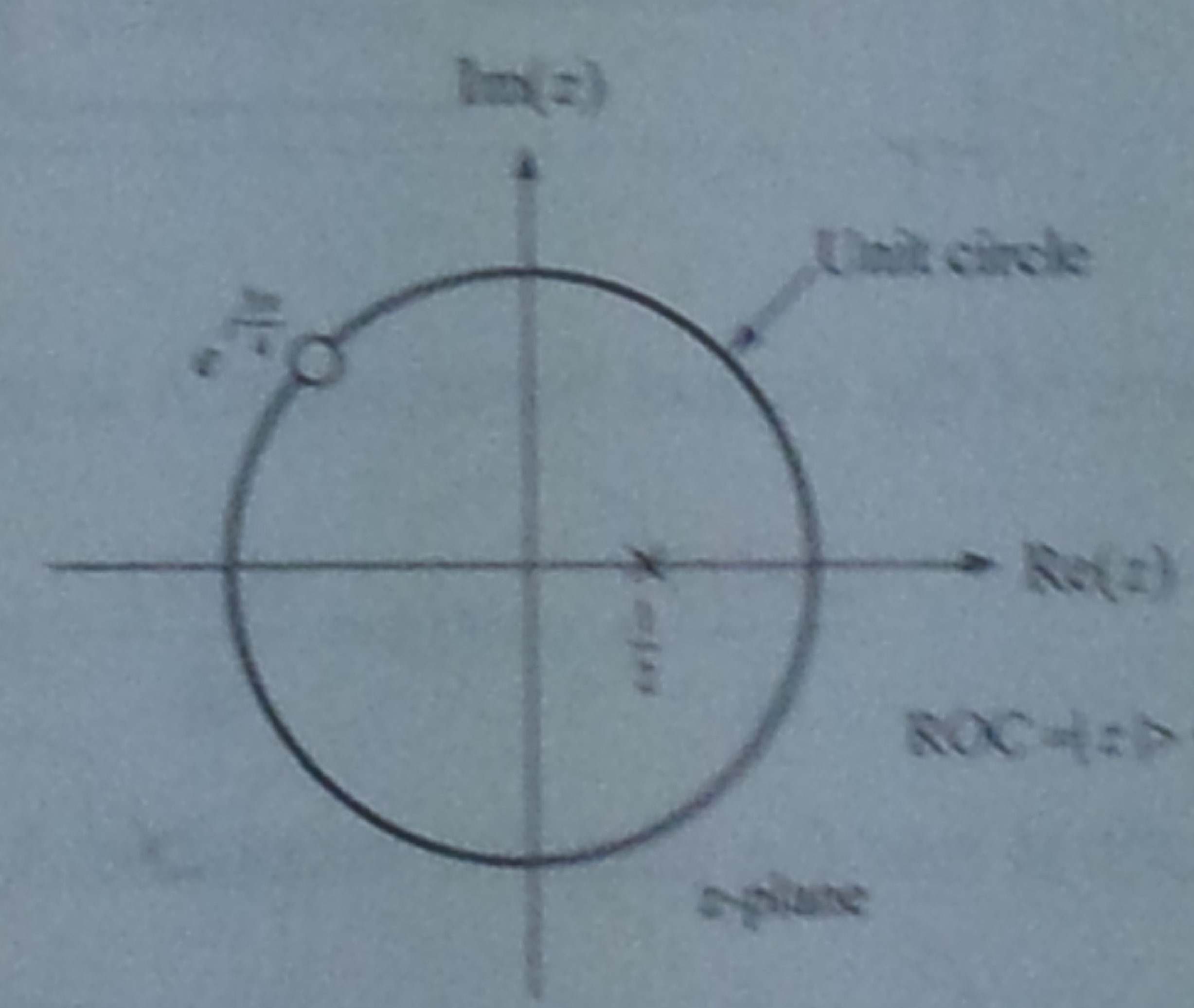
5. Consider a discrete-time LTI system $H_1(z)$ whose pole-zero plot is shown in the following figure:



(1) How many two-sided impulse responses can be associated with this pole-zero plot? Determine the corresponding ROCs. (5%)

(2) Consider a cascade interconnection of two systems $H_1(z)$ and $H_2(z)$. Determine a possible solution of $H_2(z)$ such that the overall system is causal and stable. (5%)

6. Consider a sequence $x[n]$ with z-transform $X(z)$ whose pole-zero plot is shown as follows:



Determine the z-transform of each of the following signals in terms of $X(z)$. Sketch th pole-zero plot and indicate the ROC for each case.

(1) $x_1[n]=x[-n+4]$. (5%)

(2) $x_2[n]=x[n]\cdot(2e^{-j\frac{\pi}{4}})^n$. (5%)

7. Indicate whether each of the following statements is true or false. If true, give a *brief explanation*; if false, give a *counterexample*.

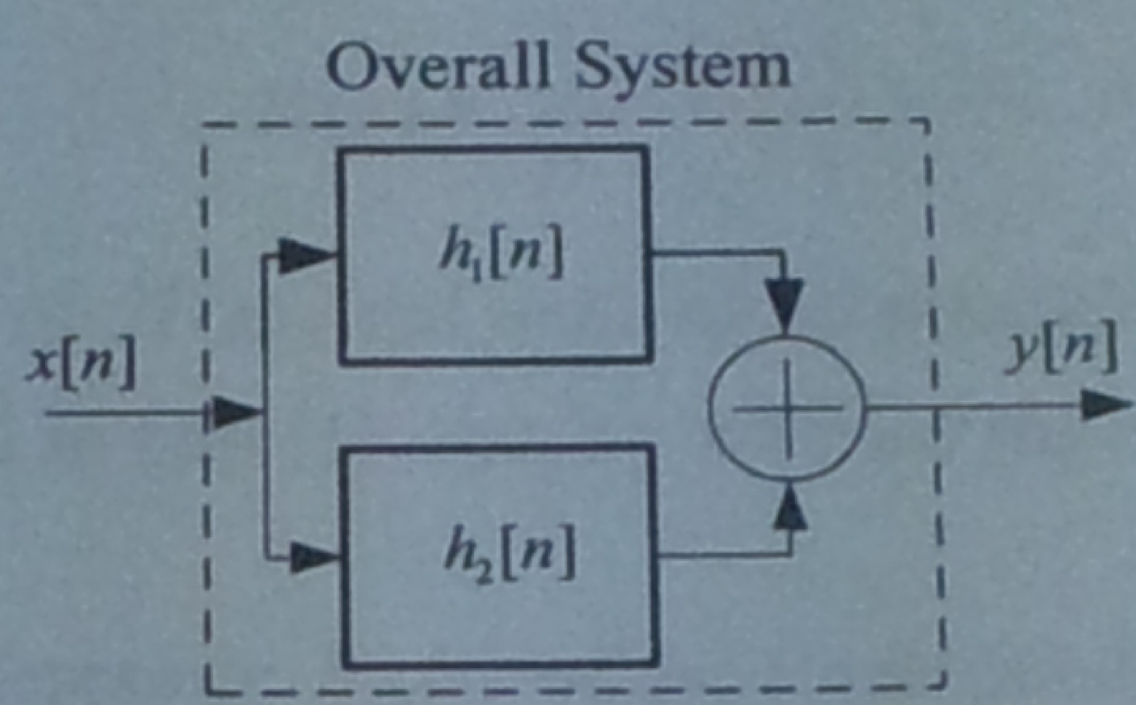
   (1) The ROC for the z-transform of a linear combination of signals is the same as the intersection of the ROCs for the z-transforms of the individual signals. (3%)

   (2) If a continuous-time LTI system and its inverse system are both causal and stable, then all poles and zeros must lie in the left half of the *s*-plane. (3%)

8.

   (1) Describe the linear-phase property of a discrete-time LTI system. (3%)

   (2) Consider a parallel interconnection of two causal and stable discrete-time LTI systems as follows:



   where $h_1[n]=3\delta[n]+2\delta[n-1]+\delta[n-2]$ and $h_2[n]=-\delta[n]-\delta[n-1]+a\delta[n-2]$ with $a>0$. Determine a such that the overall system is linear-phase. Justify your answer. (6%)

9. A causal and stable discrete-time LTI system has the system function

$$H(z)=\frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}}.$$

   (1) Find the impulse response $h[n]$ of the system. (4%)

   (2) Find the output $y[n]$ of the system when the input is $x[n]=e^{-j(\pi/2)n}$. (4%)

   (3) Is there a causal and stable inverse system of $H(z)$? Justify your answer. (4%)

10. Consider a second-order system function

$$H(z)=\frac{1+2z^{-1}+z^{-2}}{1-1.2(\cos\frac{\pi}{4})z^{-1}+0.36z^{-2}},\quad |z|>0.6.$$

   (1) Draw the corresponding pole-zero plot with ROC. (4%)

   (2) Draw the magnitude response roughly and determine the frequency response type (lowpass, highpass, bandpass, or band-reject filter) of the system. (4%)