Final Exam

Jan. 11, 2018

Instructor: Chin-Liang Wang

1. Consider a continuous-time linear time-invariant (LTI) system with systems function

$$H(s)=\frac{s^2-2s+1}{s^2-s-2}.$$

(a) Plot the poles and zeros of $H(s)$, and indicate all possible ROCs. (3%)

(b) For each ROC identified in part (1), specify whether the associated system is stable and/or causal. (3%)

(c) Determine the impulse response of the corresponding stable inverse system. (6%)

2. Let $B(s)$ be the transfer function of a causal and stable Butterworth filter of order 3 and

$$B(s)B(-s)=\frac{1}{1+(s/(j\cdot 3))^6}.$$

$W_c = 3$ $\quad$ $s = W_c\, e^{j\left(\frac{(2k+1)\pi}{2N}+\frac{\pi}{2}\right)}$

$2N=6$

(a) Plot the poles and zeros of $B(s)B(-s)$. (4%)

(b) Determine the transfer function $B(s)$. (5%)

(c) Plot the frequency response of the filter and indicate the 3-dB frequency. (3%)

3. Consider the following Laplace transform pair:

$$x(t)\xleftrightarrow{\mathcal{L}}\frac{s}{(s+2)^2},\quad \mathrm{Re}\{s\}>-2.$$

(a) Determine the time-domain signal $x(t)$. (7%)

(b) Determine the Laplace transform of $\int_0^t x(3\tau)\,d\tau$. (7%)

4. Suppose we have two three-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n]=h[n]=\begin{cases}1, & 0\le n\le 2\\ 0, & \text{otherwise}\end{cases}$$

Two periodic sequences $\tilde{x}[n]$ and $\tilde{h}[n]$ are constructed from $x[n]$ and $h[n]$ in the following way:

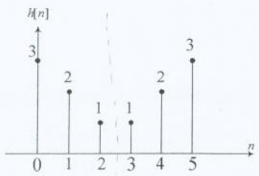
$$\tilde{x}[n]=\sum_{r=-\infty}^{\infty}x[n+Nr],\ \tilde{h}[n]=\sum_{r=-\infty}^{\infty}h[n+Nr].$$

(a) Let $\tilde{y}[n]=\tilde{x}[n]\circledast\tilde{h}[n]$ (periodic convolution). How should we choose N such that $y[n]=x[n]*h[n]$ (linear convolution) is equal to $\tilde{y}[n]$ for $0\le n\le N-1$. (3%)

(b) Compute $y[n]$ by using periodic convolution. (6%)

(c) Describe how to compute $y[n]$ via the discrete Fourier transform (DFT)? (5%)

5. Consider a discrete-time system with the following impulse response:



(a) Determine the frequency response $H(e^{j\Omega})$ of the system. (6%)

(b) Is this a linear-phase system? Justify your answer. (6%)

6. Consider a causal LTI system with impulse response $h[n]$ and input $x[n]$ given by

$$x[n] = (-1/3)^n u[n] - (1/2)^n u[-n-1].$$

(a) Determine the z-transform of $x[n]$. (5%)

(b) If the output corresponding to the input $x[n]$ is $y[n] = (-1/3)^n u[n]$, determine $h[n]$. (7%)

7. Consider the following z-transform pair:

$$x[n] \xleftrightarrow{z} X(z) = \frac{z^{-2} + 2z^{-1} + 2}{z^{-1} + 1}, \quad \text{ROC}.$$

(a) Determine the right-sided sequence $x[n]$ for this z-transform. (6%)

(b) Determine the left-sided sequence $x[n]$ for this z-transform. (6%)

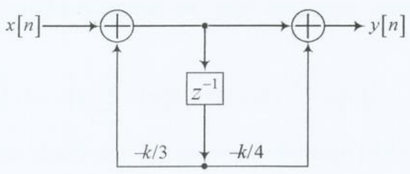
8. Consider a discrete-time LTI system for which the input and the output $x[n]$ and output $y[n]$ are related by the linear constant-coefficient difference equation

$$y[n] + 3y[n-1] = x[n], \quad \text{where } x[n] = (1/2)^n u[n] \text{ and } y[-1] = 1.$$

Determine the output $y[n]$ by using the unilateral z-transform. (10%)

9. Consider a causal discrete-time system with input $x[n]$ and output $y[n]$, as shown below.

$\frac{15}{14} + \frac{6}{14} = \frac{21}{14}$



(a) Find the corresponding system function and draw the pole-zero plot with ROC. (4%)

(b) For what value of k is the system stable? (4%)

(c) Determine $y[n]$ for the case of $k = 1$ and $x[n] = (2/3)^n$ for all n. (4%)

$\frac{-14}{36} + \frac{5}{36} = \frac{-9}{36} = \frac{-1}{4}$