Midterm Exam II

Dec. 17, 2013

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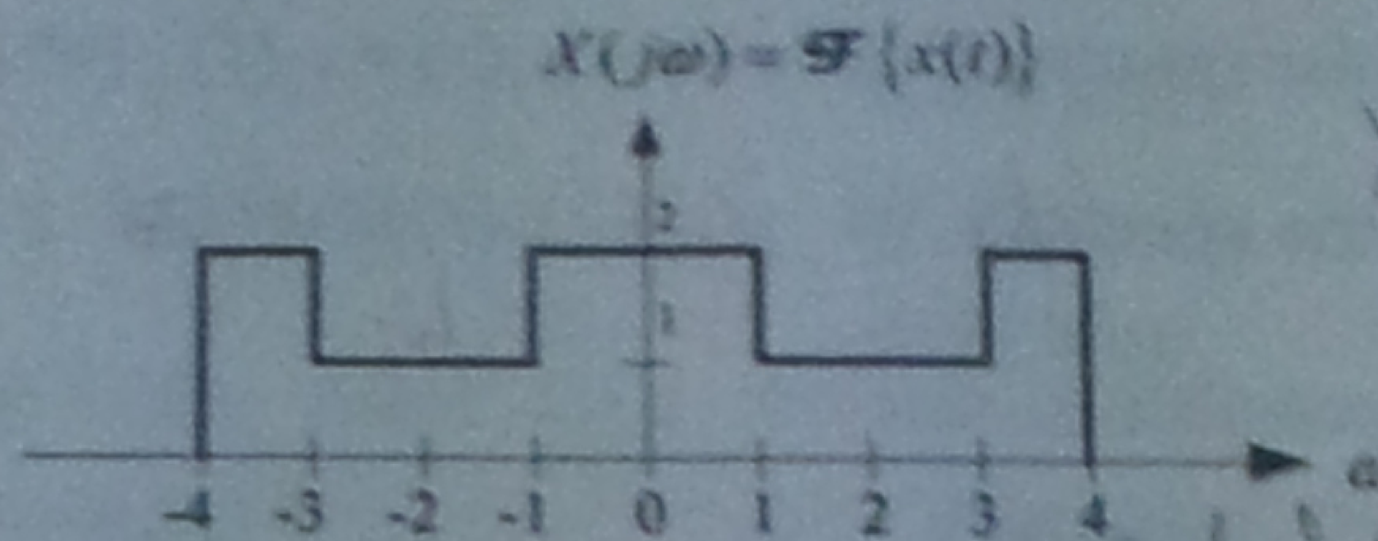
1. Evaluate the quantities for the following signal:



(1) $\int_{-\infty}^{\infty} x(t)dt$ (2%) (2) $\int_{-\infty}^{\infty} |x(t)|^2 dt$ (3%) (3) $\int_{-\infty}^{\infty} x(t)e^{j2t}dt$ (2%)

(4) $x(0)$ (2%) (5) $\left.\frac{dx(t)}{dt}\right|_{t=0}$ (3%)

2. Given $x[n]=|n|(1/3)^{|n|} \xleftrightarrow{DTFT} X(e^{j\Omega})$. Without evaluating $X(e^{j\Omega})$, find $y[n]$ if

(1) $Y(e^{j\Omega})=\mathrm{Im}\{X(e^{j\Omega})\}$, (4%)

(2) $Y(e^{j\Omega})=e^{-j4\Omega}X(e^{j\Omega})$, (4%)

(3) $Y(e^{j\Omega})=X\left(e^{j\left(\Omega+\frac{\pi}{4}\right)}\right)+X\left(e^{j\left(\Omega-\frac{\pi}{4}\right)}\right)$. (4%)

3. Consider a discrete-time linear time-invariant (LTI) system described by the following difference equation:

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n-1].$$

(1) Determine the frequency response $H(e^{j\Omega})$. (6%)

(2) Determine $y[n]$ when $x[n]=(\frac{1}{2})^n u[n]$. (6%)

4. Determine the Fourier transform or the inverse Fourier transform of the following signals:

(1) $x(t)=\sum_{k=0}^{2}(-1)^k \sin\left(\frac{2\pi k}{3}t\right)$, (5%)

(2) $X(j\omega)=\frac{1}{(a+j\omega)^3}$, $a>0$. (5%)

5. Consider the uniform impulse train $p(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$ and the two signals $x_1(t)=\dfrac{\sin(\omega_1 t)}{\omega_1 t}$ and $x_2(t)=\dfrac{\sin(\omega_2 t)}{\omega_2 t}$, where $2\omega_1<\omega_2$.

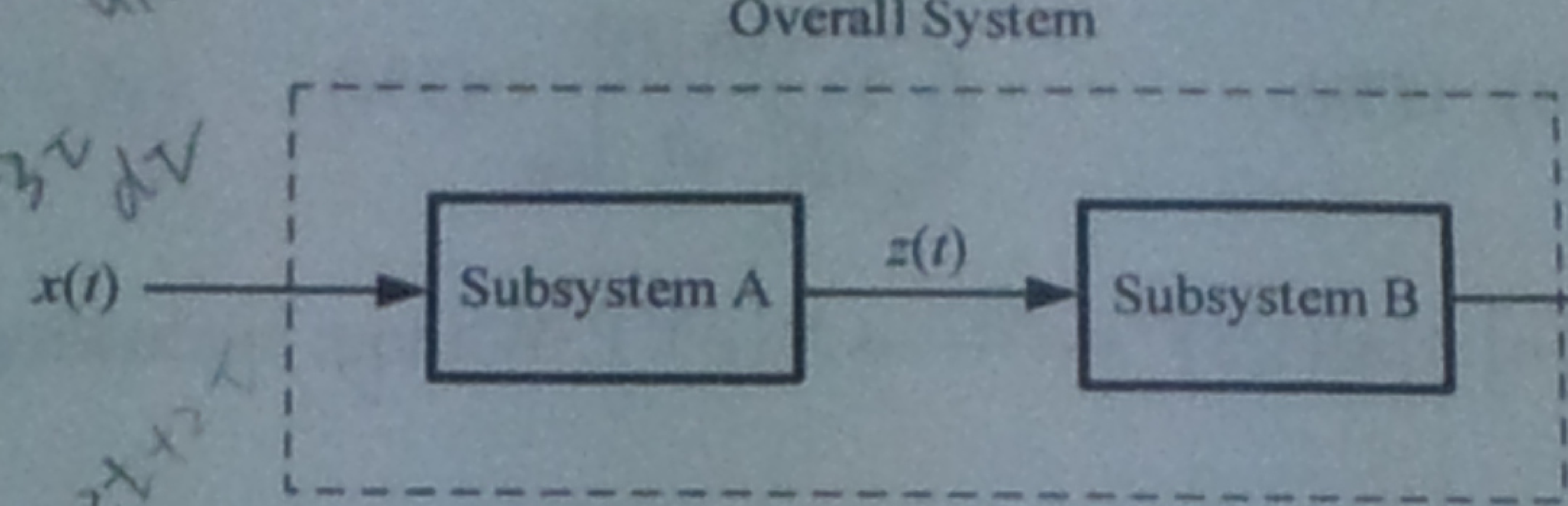
(1) Plot the spectrum of $x(t)=x_1(t)*x_2(t)$. (4%)

(2) Plot the spectrum of $y(t)=x_1(t)p(t)$ for $T=\dfrac{2\pi}{\omega_2}$. (4%)

(3) Is it possible to recover $x_1(t)$ from $y(t)$? Why? (4%)

(*Hint:* $x(t)=\dfrac{\sin(Wt)}{\pi t}\xleftrightarrow{\mathcal{F}}X(j\omega)=\begin{cases}1, & |\omega|<W\\ 0, & |\omega|>W\end{cases}$.)

6. Consider the following system:



The input-output relation of Subsystem A is given by

$$\frac{d^2z(t)}{dt^2}-\frac{dz(t)}{dt}-6z(t)=x(t),$$

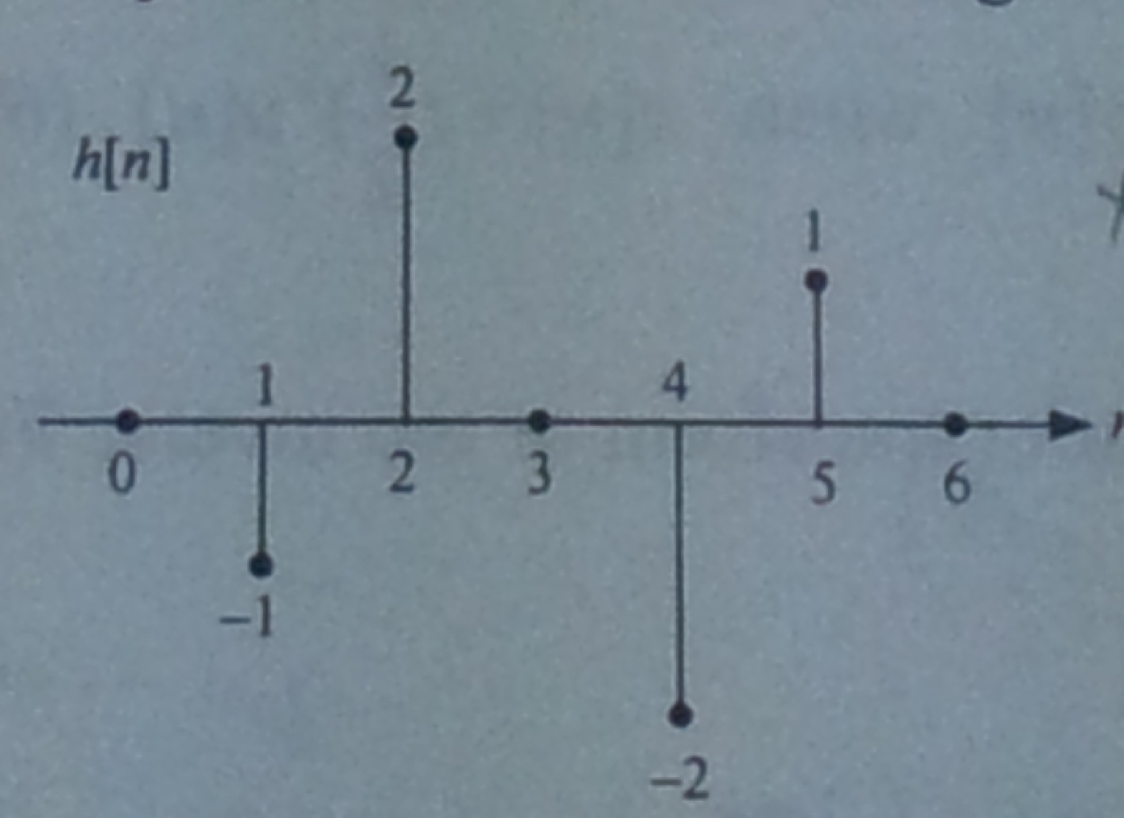
and the input-output relation of Subsystem B is given by

$$\frac{dz(t)}{dt}+6z(t)=\frac{dy(t)}{dt}+by(t).$$

(1) Determine the frequency response and the impulse response of Subsystem A. (7%)

(2) Determine b such that the overall system is causal. Justify your answer. (5%)

7. Consider a discrete-time system with the following impulse response:



(1) Determine the frequency response $H(e^{j\Omega})$ of the system. (4%)

(2) Does this system have linear phase? Justify your answer. (4%)

(*Hint*: Express the frequency response as $H(e^{j\Omega})=R(\Omega)e^{-j(\Omega\alpha+\beta)}$.)

8. The input signal to a discrete-time LTI system is given by $x[n]=\sum_{k=-\infty}^{\infty}\delta[n-4k]$

(1) Determine the discrete-time Fourier series of $x[n]$. (5%)

(2) Consider the discrete-time LTI system with impulse response given by

$$h[n]=\begin{cases}1, & 0\le n\le 2\\ -1, & -2\le n\le -1.\\ 0, & \text{otherwise}\end{cases}$$

Determine the Fourier series coefficients of the output signal $y[n]$. (6%)

9. Suppose we have two three-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n]=h[n]=\begin{cases}1, & 0\le n\le 2\\ 0, & \text{otherwise}\end{cases}.$$

Two periodic sequences $\tilde{x}[n]$ and $\tilde{h}[n]$ are constructed from $x[n]$ and $h[n]$ in the following way:

$$\tilde{x}[n]=\sum_{r=-\infty}^{\infty}x[n+Nr],\ \tilde{h}[n]=\sum_{r=-\infty}^{\infty}h[n+Nr].$$

(1) Let $\tilde{y}[n]=\tilde{x}[n]\circledast\tilde{h}[n]$ (periodic convolution). How should we choose N such that $y[n]=x[n]*h[n]$ (linear convolution) is equal to $\tilde{y}[n]$ for $0\le n\le N-1$. (3%)

(2) Compute $y[n]$ by using periodic convolution. (5%)

(3) Describe how to compute $y[n]$ via the discrete Fourier transform (DFT)? (3%)

10. An ideal low-pass filter (LPF) with zero delay has impulse response $h_{lp}[n]$ and frequency response $H_{lp}(e^{j\Omega})=\begin{cases}1, & |\Omega|<0.2\pi,\\ 0, & 0.2\pi\le|\Omega|\le\pi.\end{cases}$

(1) A new filter is defined by the equation $h_1[n]=e^{j\pi n}h_{lp}[n]$. Determine and plot $H_1(e^{j\Omega})$. What kind of filter is it? (3%)

(2) A second filter is defined by the equation $h_2[n]=2h_{lp}[n]\cos(\pi n/2)$. Determine and plot $H_2(e^{j\Omega})$. What kind of filter is it? (4%)

(3) Is the ideal LPF physically realizable? Why? (3%)