103060003

Midterm Exam II

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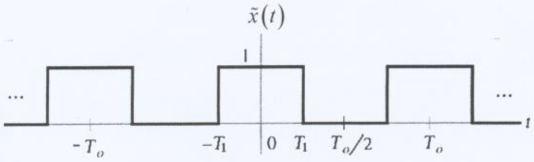
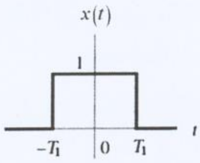
1. Consider a continuous-time linear time-invariant (LTI) system with impulse response $h(t)$. What is the definition of an eigenfunction of this kind of systems? What is the definition the corresponding eigenvalue? Show that complex exponentials e^{st} are eigenfunctions of continuous-time LTI systems. What are the corresponding eigenvalues? (15%)

2. Consider a periodic signal $\tilde{x}(t)$ and an aperiodic signal $x(t)$ as shown below. Determine the Fourier series representation of $\tilde{x}(t)$ with coefficients a_k and the Fourier transform $X(j\omega)$ of $x(t)$. Describe the relationship between a_k and $X(j\omega)$. Also show them in the same figure for the case of $T_0 = 4T_1$. (15%)





$\frac{1}{2}u(t) + \frac{1}{2}$
even odd

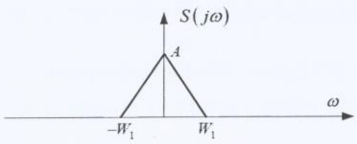
3. Show that the Fourier transform of the unit step function $u(t)$ is $U(j\omega) = 1/(j\omega) + \pi\delta(\omega)$. Also use this result and the convolution theorem to determine the Fourier transform of the following signal:

$$\int_0^t e^{-2t}dt. \text{ (15\%)}$$

4. Consider the uniform impulse train $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ and a continuous-time signal $s(t)$ with Fourier transform $S(j\omega)$ as shown below.

$P(jw) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - \frac{2\pi}{T}k)$

$a_k = \frac{1}{T}\int_{-T}^{T} \delta(t-kT) e^{-jwt} dt$



(a) Determine the Fourier series representation and the corresponding Fourier transform of $p(t)$. (7%)

(b) Plot the Fourier transform (or spectrum) of $r(t)=s(t)p(t)$ for $T=5\pi/(4W_1)$. Is it possible to recover $s(t)$ from $r(t)$? Why ? (4%) $W=\frac{5\pi}{4T}$ Yes

(c) Plot the Fourier transform (or spectrum) of $r(t)=s(t)p(t)$ for $T=3\pi/(4W_1)$. Is it possible to recover $s(t)$ from $r(t)$? Why ? (4%) $W=\frac{3\pi}{4T}$ No

5. Consider a discrete-time periodic signal $x[n]$ with fundamental period N and Fourier series coefficients a_k. Prove each of the following statements:
   (a) If $x[n]$ is real, $a_k=a^*_{-k}$ and a_0 is real. (5%)
   (b) If $x(n)$ is real and even, its Fourier series coefficients are real and even. (5%)
   $x[n]$

6. Consider three continuous-time LTI systems with the following impulse responses:

$$h_1(t)=u(t) \quad H_1=\frac{1}{jw}+\pi\delta(w)$$
$$h_2(t)=-2\delta(t)+5e^{-2t}u(t) \quad -2+\frac{5}{2+jw}$$
$$h_3(t)=2te^{-t}u(t). \quad H_3=\frac{2}{(1+jw)^2}$$

Show that these three LTI systems have the same response to the input $x(t)=\cos(t)$. (9%)

$X(jw)=\pi(\delta(w-w_0)+\delta(w+w_0))$

7. The input $x(t)$ and the output $y(t)$ of a stable and causal continuous-time LTI system are related by the following differential equation:

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=2x(t).$$

$H(jw)=\frac{2}{1+6jw+8}=\frac{1}{2+jw}-\frac{1}{4+jw}$

   (a) Find the frequency response of the system. (5%)
   (b) Find the impulse response of the system. (7%) $h(t)=e^{-2t}u(t)-e^{-4t}u(t)$

8. Consider a discrete-time LTI system formed by a cascade of two LTI subsystems with the following frequency responses:

$H_1\times H_2=\frac{2-e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega}+\frac{1}{4}e^{-j2\Omega})(1+\frac{1}{2}e^{-j\Omega})}=\frac{2-e^{-j\Omega}}{1+\frac{1}{8}e^{-j3\Omega}}$

$$H_1(e^{j\Omega})=(2-e^{-j\Omega})/(1+\frac{1}{2}e^{-j\Omega})$$
$$H_2(e^{j\Omega})=1/(1-\frac{1}{2}e^{-j\Omega}+\frac{1}{4}e^{-j2\Omega}).$$

$\frac{1}{2}e^{-j\Omega}-\frac{1}{4}e^{-j2\Omega}+\frac{1}{8}e^{-j3\Omega}$

   (a) Find the difference equation describing the overall system. (5%) $y[n]+\frac{1}{8}y[n-3]=2x[n]-x[n-1]$
   (b) Determine the impulse response of the overall system. (7%)

$\frac{1}{e^{j\Omega}+\frac{1}{8}e^{-j2\Omega}}$

9. Consider a discrete-time LTI system with impulse response $h[n]=(1/2)^n\cos(\pi n/2)u[n]$. Determine the output of the system when the input signal is $x[n]=\cos(\pi n/2)$. (7%)

$(-\frac{1}{8}e^{-j2\Omega}-\frac{1}{4}e^{-j\Omega}-\frac{1}{2})(2-e^{-j\Omega})=+\frac{1}{8}e^{-j3\Omega}+\frac{1}{4}e^{-j2\Omega}+\frac{1}{2}e^{-j\Omega}-\frac{1}{4}e^{-j2\Omega}-\frac{1}{2}e^{-j\Omega}-1$

$+\frac{2}{1+\frac{1}{2}e^{-j\Omega}+\frac{1}{4}e^{-j2\Omega}}$