6、 $x(t) = \cos(t) \quad \therefore \quad X(jw) = \pi\,(\delta(w-1) + \delta(w+1))$

$H_1(jw) = \frac{1}{jw} + \pi\,\delta(w) \quad \therefore \quad Y(jw) = H_1(jw)\,X(jw) = \left[\frac{1}{jw} + \pi\delta(w)\right]\left[\pi\,(\delta(w-1)+\delta(w+1))\right]$

$= \pi\,\frac{1}{j}\,\delta(w-1) + \pi\,\frac{1}{-j}\,\delta(w+1) = \frac{\pi}{j}\,(\delta(w-1) - \delta(w+1)) \quad \therefore \quad y(t) = \sin t$

$H_2(jw) = -2 + \frac{5}{2+jw} \quad \therefore \quad Y(jw) = H_2(jw)\,X(jw) = \left[-2 + \frac{5}{2+jw}\right]\left[\pi\,(\delta(w-1)+\delta(w+1))\right]$

$= \pi\,\frac{1-2j}{2+j}\,\delta(w-1) + \pi\,\frac{1+2j}{2-j}\,\delta(w+1) = \frac{\pi}{j}\,\delta(w-1) - \frac{\pi}{j}\,\delta(w+1) \quad \therefore \quad y(t) = \sin t$

$H_3(jw) = \frac{2}{(1+jw)^2} \quad \therefore \quad Y(jw) = H_3(jw)\,X(jw) = \frac{2}{(1+jw)^2}\left[\pi\,(\delta(w-1)+\delta(w+1))\right]$

+9

$= \frac{2}{(1+j)^2}\,\pi\,\delta(w-1) + \frac{2}{(1-j)^2}\,\pi\,\delta(w+1) = \frac{\pi}{j}\,\delta(w-1) - \frac{\pi}{j}\,\delta(w+1) \quad \therefore \quad y(t) = \sin t$

They have the same response to the input $x(t) = \cos(t)$

7、
(a) $H(jw) = \frac{Y(jw)}{X(jw)} = \frac{2}{-w^2 + 6jw + 8} \quad \therefore$ frequency response $= H(jw) = \frac{2}{-w^2 + 6jw + 8}$

+10

(b) $H(jw) = \frac{2}{1 + 6jw + 8} = \frac{1}{2+jw} - \frac{1}{4+jw} \quad \therefore \quad h(t) = e^{-2t}u(t) - e^{-4t}u(t)$

$\therefore$ impulse response $= h(t) = e^{-2t}u(t) - e^{-4t}u(t)$

8、 $H(e^{j\Omega}) = H_1(e^{j\Omega})\,H_2(e^{j\Omega}) = \frac{2 - e^{-j\Omega}}{1 + \frac{1}{2}e^{-j\Omega}} \times \frac{1}{1 - \frac{1}{2}e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega}} = \frac{2 - e^{-j\Omega}}{1 + \frac{1}{8}e^{-j3\Omega}}$

+5

(a) $H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} \quad \therefore \quad y[n] + \frac{1}{8}\,y[n-3] = 2x[n] - x[n-1]$

(b) by part (a) $H(e^{j\Omega}) = \frac{2 - e^{-j\Omega}}{1 + \frac{1}{8}e^{-j3\Omega}} \quad H(e^{j\Omega}) = \frac{2}{1 + \frac{1}{8}e^{-j3\Omega}} - \frac{1}{8}e^{-j2\Omega} - \frac{1}{4}e^{-j\Omega} - \frac{1}{2}$

+2 ✗ $\therefore h(t) = 2\left(-\frac{1}{8}\right)^{n-3}u[n-3] - \frac{1}{8}\,\delta[n-2] - \frac{1}{4}\,\delta[n-1] - \frac{1}{2}\,\delta[n]$

9. $y[n] = h[n] * x[n] \quad \therefore Y(e^{j\Omega}) = H(e^{j\Omega})X(e^{j\Omega})$

$$H(e^{j\Omega}) = \frac{1}{2\pi}\left(\frac{1}{1-\frac{1}{2}e^{-j\Omega}}\right) * \left(\sum_{k=-\infty}^{\infty}\left[\pi\,\delta(\Omega-\tfrac{\pi}{2}-2\pi k) + \pi\,\delta(\Omega+\tfrac{\pi}{2}-2\pi k)\right]\right.$$

$$X(e^{j\Omega}) = \sum_{k=-\infty}^{\infty}\left[\pi\,\delta(\Omega-\tfrac{\pi}{2}-2\pi k) + \pi\,\delta(\Omega+\tfrac{\pi}{2}-2\pi k)\right]$$

+2

5.

(a) $a_k = \frac{1}{N}\sum_{\langle N\rangle} x[n]\, e^{j\frac{2\pi}{N}kn}$ $\qquad a_{-k}^* = \frac{1}{N}\sum_{\langle N\rangle}\left(x[n]\, e^{j\frac{2\pi}{N}(-k)n}\right)^*$

$$= \frac{1}{N}\sum_{\langle N\rangle} x^*[n]\, e^{j\frac{2\pi}{N}kn}$$

$\because x[n]$ is real $\therefore x[n] = x^*[n] \qquad \therefore a_k = a_{-k}^*$

+5

$a_0 = \frac{1}{N}\sum_{\langle N\rangle} x[n] = a_0^* = \frac{1}{N}\sum_{\langle N\rangle} x^*[n] \qquad \therefore a_0 = 0 \quad a_0$ is real

(b) $x[n]$ is even and real $\quad ?\; X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$

$\therefore x[n] = x[-n] \qquad ?\; X(e^{-j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{j\Omega n}$ $\qquad$ +4

$\because x[n] = x[-n] \quad \therefore X(e^{-j\Omega}) = \sum_{n=-\infty}^{\infty} x[-n]\, e^{-j\Omega n} = X(e^{j\Omega})$

with part (a) we can conclude that
its Fourier series coefficients are real and even.

1、 $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$ define ?

+13

when $\cancel{x}(t) = e^{st}$ we $y(t) = \int_{-\infty}^{\infty} x(\tau)\, e^{s(t-\tau)} d\tau = e^{st} \int_{-\infty}^{\infty} x(\tau)\, e^{-s\tau} d\tau$

Here e^{st} is the eigenfunction and $\int_{-\infty}^{\infty} x(\tau) e^{-s\tau} d\tau$ is the eigenvalue.

eigenfunction 是一個可以乘上常數倍來表示一個 output 的 function

以 $h(t) = e^{st}$ 去求 $y(t) = h(t) * x(t)$ 時，e^{st} 即為 eigenfunction

eigenvalue 是 eigenfunction 乘上的係數，兩者相乘可得到 output

以上述為例則 $\int_{-\infty}^{\infty} x(\tau) e^{-s\tau} d\tau$ 為 eigenvalue

2、

$$a_k = \frac{1}{T_0} \int_{-T_1}^{T_1} e^{-jk w_0 t} dt = \frac{1}{T_0} \frac{1}{-jk w_0} \left(e^{-jk w_0 T_1} - e^{jk w_0 T_1}\right)$$

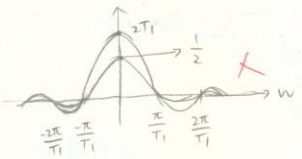
$$= \frac{1}{T_0 jk w_0} \left(e^{jk w_0 T_1} - e^{-jk w_0 T_1}\right) = \frac{2j \sin(k w_0 T_1)}{T_0 j k w_0} = \frac{2 \sin(k w_0 T_1)}{k w_0 T_0}$$

$$X(jw) = \int_{-\infty}^{\infty} e^{-jwt} dt = \int_{-T_1}^{T_1} e^{-jwt} dt = \frac{1}{-jw}\left(e^{-jwT_1} - e^{jwT_1}\right) = \frac{1}{jw}\left(e^{jwT_1} - e^{-jwT_1}\right)$$

$$= \frac{2 \sin(wT_1)}{w} = 2T_1 \operatorname{sinc}\left(\frac{wT_1}{\pi}\right)$$

$\because T_0 = 4T_1 \quad \therefore a_k = \frac{2 \sin(k w_0 T_1)}{k w_0 4T_1} = \frac{1}{2} \operatorname{sinc}\left(\frac{k w_0 T_1}{\pi}\right) = \cdot$

relation ?



+12

3、 $u(t) = \underbrace{\frac{1}{2}u(t)}_{\text{even part}} + \underbrace{\frac{1}{2}}_{\text{odd part}}$ ∵ $\delta[n] \overset{F.T.}{\longleftrightarrow} 1$, $\delta[n] = (u(t))'$

令 $u(t) \overset{F.T.}{\longleftrightarrow} V(jw)$ ∴ $(u(t))' \overset{F.T.}{\longleftrightarrow} = jw\,V(jw)$

∵ $\delta[n] = (u(t))'$ ∴ $jw\,V(jw) = 1$, $V(jw) = \frac{1}{jw}$

$\frac{1}{2} \overset{F.T.}{\longleftrightarrow} \pi\delta(w)$ ∴ $u(t) \overset{F.T.}{\longleftrightarrow} \frac{1}{jw} + \pi\delta(w)$

$\int_0^t e^{-2t}dt = \int_0^t e^{-2t}u(t)\,dt$ we know $e^{-2t}u(t) \overset{F.T.}{\longleftrightarrow} \frac{1}{2+jw}$

∴ $\int_0^t e^{-2t}dt \overset{F.T.}{\longleftrightarrow} \left(\frac{1}{jw} + \pi\delta(w)\right)\frac{1}{2+jw}$

+15

4、(a) $a_k = \frac{1}{T}\int_{-T}^{T}\delta(t-kT)e^{-jwkt}dt = \frac{1}{T}$

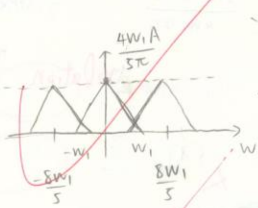
$P(jw) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(w - \frac{2\pi}{T}k)$

+15

(b) $r(t) = s(t)\,p(t)$ ∴ $R(t) = \frac{1}{2\pi}(S(t) * P(t))$

$T = \frac{5\pi}{4w_1}$

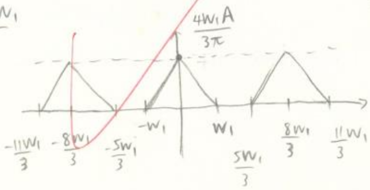
代入 $\frac{2\pi}{T} = \frac{8w_1}{5}$



∵ 重疊有 aliasing

∴ 不能 recover $s(t)$ from $r(t)$

(c) $T = \frac{3\pi}{4w_1}$ 代入 $\frac{2\pi}{T} = \frac{8w_1}{3}$



∵ 沒有重疊

∴ 可以 recover $s(t)$ from $r(t)$