Midterm Exam II

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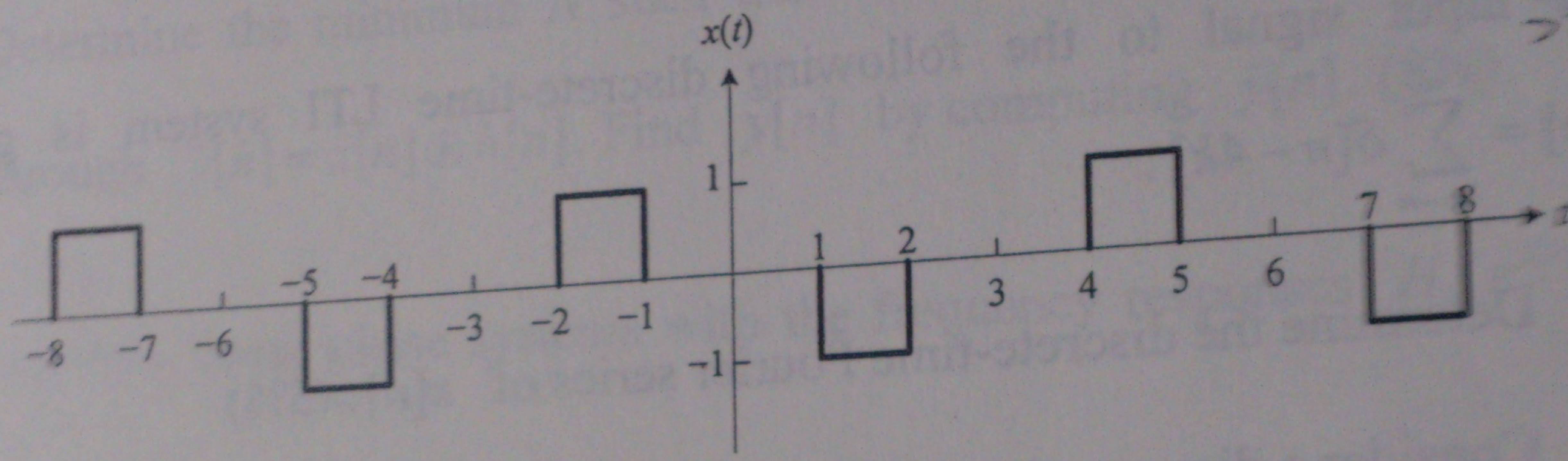
1. Consider an LTI system with frequency response $H(j\omega)=\begin{cases} e^{-j\omega}, & |\omega|\le 7\pi \\ 0, & \text{otherwise} \end{cases}$

   Determine the output $y(t)$ for the input $x(t)=\sum_{k=0}^{\infty}(-1)^k \sin(2\pi k t)$. (10%)

2. Determine the Fourier series coefficients of the following periodic signal. (10%)



3. Consider an LTI system whose response to the input $x(t)=[e^{-t}+e^{-3t}]u(t)$ is $y(t)=[2e^{-t}-2e^{-4t}]u(t)$.

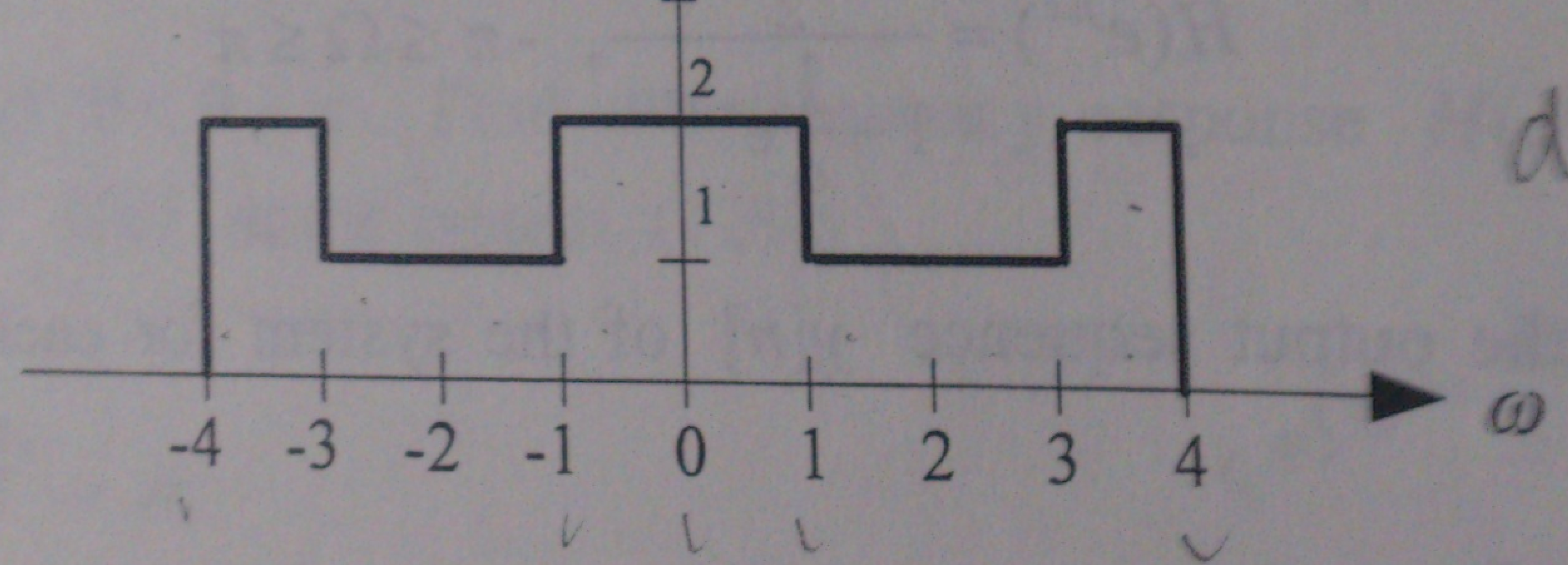
   (1) Determine the frequency response $H(j\omega)$ of the system. (5%)

   (2) Determine the impulse response $h(t)$ of the system. (6%)

4. Evaluate the quantities for the following signal:

   $X(j\omega)=\mathcal{F}\{x(t)\}$

   $X(j\omega)=\int_{-\infty}^{\infty}$



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(1) $\int_{-\infty}^{\infty} x(t)dt$ (2%) (2) $\int_{-\infty}^{\infty} |x(t)|^2 dt$ (3%) (3) $\int_{-\infty}^{\infty} x(t)e^{j2t}dt$ (2%)

(4) $x(0)$ (2%) (5) $\left.\frac{dx(t)}{dt}\right|_{t=0}$ (3%)

$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{-j\omega t}dt$

$\frac{dx(t)}{dt}=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\frac{d(e^{-j\omega t})}{dt}dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot -j\omega\cdot e$

X $-jt$ $\frac{d}{d\omega}$

$e^{j\frac{\pi}{2}n} - \frac{1}{2}$

5. Consider the uniform impulse train $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$ and the two signals

$e^{j\frac{\pi}{2}}$

$x_1(t) = \dfrac{\sin(\omega_1 t)}{\omega_1 t}$ and $x_2(t) = \dfrac{\sin(\omega_2 t)}{\omega_2 t}$, where $2\omega_1 < \omega_2$.

(1) Plot the spectrum of $x(t) = x_1(t) * x_2(t)$. (4%)

(2) Plot the spectrum of $y(t) = x(t)p(t)$ for $T = \dfrac{2\pi}{\omega_2}$. (4%)

(3) Is it possible to recover $x(t)$ from $y(t)$? Why? (4%)

$\frac{\pi}{W_1} \frac{\sin(W_1 t)}{\pi t}$

(*Hint:* $x(t) = \dfrac{\sin(Wt)}{\pi t} \xleftrightarrow{\mathcal{F}} X(j\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$.)

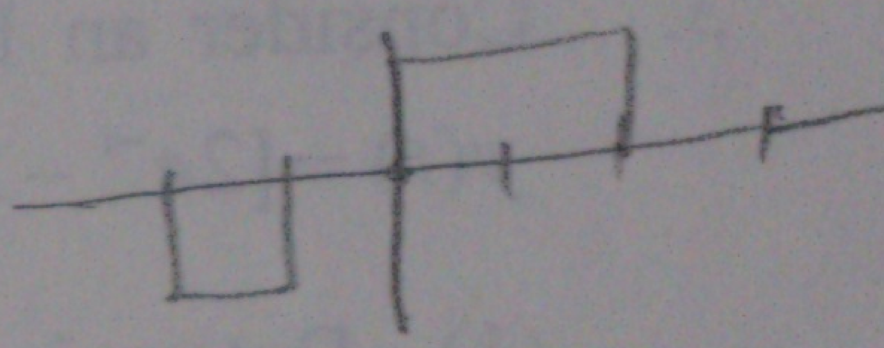
6. The input signal to the following discrete-time LTI system is given by

$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$.

(1) Determine the discrete-time Fourier series of $x[n]$. (5%)

(2) Consider a discrete-time LTI system with impulse response given by

$$h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1. \\ 0, & \text{otherwise} \end{cases}$$



Determine the Fourier series coefficients of the output signal $y[n]$. (6%)

7. Consider a discrete-time LTI system with frequency response given by

$$H(e^{j\Omega}) = \frac{1 - \frac{1}{2}e^{-j\Omega}}{1 - \frac{1}{9}e^{-j2\Omega}}, \quad -\pi \le \Omega \le \pi$$

Determine the output sequence $y[n]$ of the system for each of the following inputs:

$\Omega = \frac{2\pi k}{N}$

(1) $x[n] = (\frac{1}{2})^n u[n]$. (5%)

$e^{j0\Omega} = 1$

(2) $x[n] = \cos(\frac{\pi}{2}n)$. (5%)

$2\pi\delta(\Omega)$

$\frac{2\pi}{T}$

$e^{-j\frac{\pi}{3}n}$

$1 - \frac{1}{9}x^2$

$-j\frac{\pi}{2}k$ 项 $-j\Omega$ $(1+\frac{1}{3}x)(1-$

8. Suppose we have two sequences $x[n]$ and $h[n]$ as follows:

$$x[n]=\begin{cases}1, & 0\le n\le 1\\ 0, & \text{otherwise}\end{cases},\quad h[n]=\begin{cases}1, & 0\le n\le 2\\ 0, & \text{otherwise}\end{cases}.$$

Two periodic sequences $\tilde{x}[n]$ and $\tilde{h}[n]$ are constructed from $x[n]$ and $h[n]$ in the following way:

$$\tilde{x}[n]=\sum_{r=-\infty}^{\infty}x[n+Nr],\ \tilde{h}[n]=\sum_{r=-\infty}^{\infty}h[n+Nr].$$

(1) Compute $\tilde{y}[n]=\tilde{x}[n]\circledast\tilde{h}[n]$ for $N=3$. (6%)

(2) Determine the minimum N such that we can compute $y[n]=x[n]*h[n]$ through $\tilde{y}[n]=\tilde{x}[n]\circledast\tilde{h}[n]$. Find $y[n]$ by computing $\tilde{y}[n]$. (8%)

9. Consider two linear phase systems with the frequency responses $H_1(e^{j\Omega})$ and $H_2(e^{j\Omega})$.

(1) Does a cascade system of $H_1(e^{j\Omega})$ and $H_2(e^{j\Omega})$ have linear phase? Justify your answer. (4%)

(2) Does a parallel system of $H_1(e^{j\Omega})$ and $H_2(e^{j\Omega})$ have linear phase? Justify your answer. (4%)

10. Consider a second-order causal LTI system described by

$$y[n]-2r\cos\theta\, y[n-1]+r^2y[n-2]=x[n]$$

with $r=0.2$ and $0<\theta<\pi$. Find the frequency response $H(e^{j\Omega})$ and the impulse response $h[n]$ of the system. (12%)

$\cos\theta=\frac{1}{2}(e^{j\theta}-e^{-j\theta})$

$(1-re^{-j\Omega})^{-1}$

yT

$\frac{-1}{jre^{-j\Omega}}\frac{d}{d\Omega}\left(\frac{1}{1-re^{-j\Omega}}\right)$

$(n+1)$